SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc. - END SEMESTER EXAMINATIONS NOVEMBER - 2022
SEMESTER - III
20PAMCT3008 - Differential Equations

Total Duration : 2 Hrs 30 Mins. **Total Marks : 60**

Section A

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Find the power series solution of the equation $y' = t^2 - y^2$, $y = 1$ when $t = 1$.
2. Prove that if P_n (t) and P_m (t) are Legendre polynomials then $\int_{-1}^{1} P_n(t)P_m(t)$ dt = 0, if m $\neq$ n
3. Compute e^{At} when A=$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.
4. Use the fundamental matrix to solve the initial value problem $x' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} x$, $x(0) = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$
5. Compute the first 4 successive approximations for the solution of $x' = x^2$, $x(0)$=1
6. Using Charpit's method to compute the complete integral of the PDE $z^2 = pqxy$
7. Find the surface which intersects the surface of the system $z(x+y) = c(3z+1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1$, $z = 1$
8. Solve $x^2 \dfrac{\partial^2 z}{\partial x^2} - y^2 \dfrac{\partial^2 z}{\partial y^2} - y \dfrac{\partial y}{\partial x} + x \dfrac{\partial z}{\partial x} = 0$.

Section B

Part A

Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. (i) Show that $\dfrac{d}{dt}[t^p J_p(t)] = t^p J_{p-1}(t)$
   (ii) Compute a regular singular point and roots of the indicial equation for the following differential equation $t^2 x''$ - $(1 + t)x$=0

Contd...

10. Determine the fundamental matrix for the system $x' =$Ax where
A= $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -11 & 6 \end{bmatrix}$

11. (i) Define Lipschitz condition for the function $f(t, x)$. Illustrate with an example that the existence of the partial differential of f is not necessary for the function f to be a Lipschitz function.
(ii) State Picard's Theorem.

12. Solve (D^2- DD'+ D'-1) z = cos$(x + 2y) + e^y + xy + 1$.

Part B

Compulsory question $(1 \times 10 = 10$ Marks)

13. (i) Determine the PDE by eliminating the arbitrary constants h and k from the equation $(x - h)^2 + (y - k)^2 + z^2 = \lambda^2$
(ii) Solve the partial differential equation $\left(\frac{y^2 z}{x}\right) p + xzq = y^2$ by Lagrange's method.

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