SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc.(App.Maths) - END SEMESTER EXAMINATIONS APRIL - 2023
SEMESTER - I
17PAMCE1001 - Probability and Distributions

Total Duration : 2 Hrs. 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Derive Banach's Matchbox problem.
2. The distribution function of F(x) = 0 if x $<$ 0, F(x) $<$ 1 if x $>$ 0 and $\dfrac{1-F(x+y)}{1-F(y)}$ = 1 - F(x), for all x, y $>$ 0. Then show that there exists a constant $\beta > 0$ such that 1 - F(x) = $e^{-x\beta}$, for all x $>$ 0.
3. If r balls are drawn one at a time without replacement from a bag containing n white and m black balls. If S_r denote the number of black balls drawn, then find the mean and variance of S_r.
4. Find the PMF of U = X+Y for the bivariate negative binomial distribution with PMF P(X=x,Y=y) = $\dfrac{(x+y+k-1)!}{x!y!(k-1)!}$ p_1^x p_2^y $(1\text{-}p_1\text{-} p_2)^k$,where x,y = 0,1,2,...;k $\geq$ 1.
5. Show that X has an n - dimensional normal distribution if and only if every linear function of X has an univariate normal distribution.
6. If x_1,x_2,....,x_n are iid N(μ,σ^2) RVs and $\bar{X}$ and S^2 are independent, then prove that
   $$E\left\{exp\left[(n-1)\frac{S^2}{\sigma^2}t\right]\right\} = (1\text{-}2t)^{-(n-1)/2},\ t < \tfrac{1}{2}\ .$$
7. Show that $X_n \xrightarrow{a.s} X$ if and only if $lim_{n\to\infty}$ P$\{\sup_{m\geq n}|X_m - X| > \in\} = 0$, for all $\in > 0$.
8. Show that $g(X_n) \xrightarrow{P} g(X)$ as $n\to\infty$ if $X_n \xrightarrow{P} X$ and g is continuous function defined on R.

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Section C

I - Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. If X $\sim$ C(μ_1, θ_1) and Y $\sim$ C(μ_2, θ_2) are independent random variables. Then show that X + Y is a C(μ_1+μ_2, θ_1 + θ_2) random variable.
10. Show that M(t_1, t_2) = M(t_1, 0) M(0, t_2), for all t_1,t_2 $\in$ R if and only if X and Y are independent random variables.
11. If (X_1,X_2,...,X_n) be an n - dimensional random variable with a normal distribution and (Y_1,Y_2,...,Y_k),k $\leq$ n is a linear functions of X_j (j = 1,2,3...,n). Then show that (Y_1,Y_2,...,Y_k) also has a multivariate normal distribution.
12. If (X_1,X_2,...,X_n) are identically independent distributed N(μ,σ^2) random variables. Then show that $\bar{X}$ and (X_1 - $\bar{X}$, X_2 - $\bar{X}$,..., X_n - $\bar{X}$) are independent.

II - Compulsory question $(1 \times 10 = 10$ Marks)

13. State and prove Lindeberg – Levy Central Limit Theorem.

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