SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc.(Appl.Maths) - END SEMESTER EXAMINATIONS APRIL - 2023
SEMESTER - II
20PAMET2002 - Mathematical Statistics

Total Duration : 2 Hrs. 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Let X $\sim$ b(1, θ^2) Does there exist an unbiased estimator of θ?

2. Find the CRK bound for the variance of an unbiased estimator of θ in sampling from N(0, 1).

3. Let X_1, X_2,..., X_n be a sample from the p.d.f f(x) $= \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & otherwise \end{cases}$,
   Find E(X) and Var(X)

4. Let X_1, X_2,..., X_n be a sample from G(I, θ). Find the shortest-length confidence interval for θ at level 1 - α, based on a sufficient statistic for θ.

5. An urn contains 10 marbles, of which M are white and 10 - M are black. To test that M = 5 against the alternative hypothesis that M = 6, one draws 3 marbles from the urn without replacement. The null hypothesis is rejected if the sample contains 2 or 3 white marbles; otherwise, it is accepted. Find the size of the test and its power.

6. Find the GLR test of H_0: p = p_0 against H_1: p $\neq$ p_0, based on a sample of size 1 from b(n, p).

7. A manufacturer recorded the cutoff bias (volts) of a sample of 10 tubes as follows:
   12.1, 12.3, 11.8, 12.0, 12.4, 12.0, 12.1, 11.9, 12.2, 12.2.
   The variability of cutoff bias for tubes of a standard type as measured by the standard deviation is 0.208 volt. Is the variability of the new tube with respect to cutoff bias less than that of the standard type?

Contd...

8. The following are the coded values of the amounts of corn (in bushels per acre) obtained from four varieties, using unequal number of plots for the different varieties:

A:	2,1,3,2
B:	3,4, 2, 3,4, 2
C:	6,4,8
D:	7,6,7,4

Test whether there is a significant difference between the yields of the varieties.

Section C

I - Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. If a sample consists of n independent observations X_1, X_2, ..., X_n from the same distribution, if UMVUE exists, prove that it is a symmetric function of the X_i's.

10. Suppose that n observations are taken on an RV X with distribution N(μ, 1), but instead of recording all the observations, one notes only whether or not the observation is less than 0. If $\{X < 0\}$ occurs m(<n) times, find the MLE of μ.

11. If a sufficient statistic T exists for the family $\{f_0: \theta \in \Theta\}$, $\Theta=\{\theta_0, \theta_1\}$, critically analyse the Neyman-Pearson MP test is a function of T or not.

12. The following table shows the yield (pounds per plot) of four varieties of wheat obtained with three different kinds of fertilizers.

	VARIETY OF WHEAT			
FERTILIZER	A	B	C	D
α	8	3	6	7
β	10	4	5	8
γ	8	4	6	7

Test the hypotheses that the four varieties of wheat yield the same average yield and that the three fertilizers are equally effective.

II - Compulsory question $(1 \times 10 = 10$ Marks)

13. The performance of each of two different dive-bombing methods is measured adozen times. The sample variances for the two methods are computed to be 5545 and 4073, respectively.
    i) Do the two methods differ in variability?
    ii) Does the variability of the first method exceed that of the second method?

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