SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc.(App.maths) - END SEMESTER EXAMINATIONS APRIL - 2023
SEMESTER - IV
20PAMET4004 - Calculus of Variations and Integral Equations

Total Duration : 2 Hrs. 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. On what curves the functional $V(y(x)) = \int_0^1 \left((y')^2 + 12xy\right) dx$ be extremized?

2. Find the extremals of the functional $v[y(x), z(x)] = \int_0^{\pi/2} (y'^2 + z'^2 + 2yz)dx$,

   y(0) =0, y($\frac{\pi}{2}$) $= 1$, z(0) $= 0$, z($\frac{\pi}{2}$) $= -1$.

3. Find the transversality condition for functional v $= \int_{x_0}^{x_1}$ A(x,y)$\sqrt{1 + y'^2}$

4. Test for an extremum, the functional v[y(x)] $= \int_0^x \frac{\sqrt{1 + y'^3}}{\sqrt{y}} dx$,

   y(0)=0, y(x_1)=y_1

5. Solve the homogeneous Fredholm integral equation $g(s) = \lambda \int_0^1 e^5 e^t g(t)dt$

6. Prove that the mth iterated kernel k$_m$(s,t) satisfies the following relation k$_m$(s,t) $= \int k_r(s, x)k_{m-r}(x, t)dx$, where r is any positive integer less than m.

7. Solve the integral equation $g(s) = f(s) + \lambda \int_0^s e^{s-t} g(t)dt$

8. Prove that, a non-null symmetric L_2-kernel K is non-negative if and only if all its eigen values are positive, it is positive-definite iff the above condition is satisfied and in-addition some (and therefore every)full orthogonal system of eigen functions of K is complete.

Contd...

Section C

I - Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. Derive the differential equation of free vibrations of a string.

10. Find the extremal distance between two surfaces $z = \phi(x, y)$and $z = \psi(x, y)$.

11. Show that the integral equation $g(s) = f(s) + \frac{1}{\pi}\int\limits_0^{2\pi}[\sin(s+t)]g(t)dt$

    possesses no solution for f(s) = s, but that it possesses infinitely many solutions when f(s) = 1.

12. Evaluate the resolvent for the integral equation $g(s) = f(s) + \lambda\int\limits_0^1 (s+t)g(t)dt$.

II - Compulsory question $(1 \times 10 = 10$ Marks)

13. State and prove Riesz-Fischer theorem.

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