SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc.(App.Maths) - END SEMESTER EXAMINATIONS APRIL - 2023
SEMESTER - I
22PAMET1001 - Probability and Distributions

Total Duration : 2 Hrs. 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. State and prove Bonferroni's Inequality.
2. Prove that if f is a nonnegative function satisfying $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dxdy = 1$, then f is the joint density function of some random variable.
3. Let (X_1,X_2) have uniform distribution on the triangle $\{0 \leq x_1 \leq x_2 \leq 1\}$; that is, ($X_1$,$X_2$) has joint density function $f(x_1,x_2) = \begin{cases} 2, & 0 \leq x_1 \leq x_2 \leq 1 \\ 0, & Elsewhere \end{cases}$
   Find the density function of $X_1 + X_2$.
4. Let X_1,X_2 be independent random variables with common density given by $f(x) = \begin{cases} 1 & if 0 < x < 1 \\ 0 & Otherwise \end{cases}$ Find the marginal PDF's of $X_1 + X_2$ and $X_1 - X_2$.
5. Explain Bivariate Binomial distributions.
6. Let X_1, X_2, ...,X_n be iid $N(\mu,\sigma^2)$random variables. Then prove that $\overline{X}$and $(X_1 - \overline{X}, X_2 - \overline{X}, ..., X_n - \overline{X})$are independent.
7. Prove that the set of discontinuity points of a distribution function F is at most countable.
8. Let (X_1, X_2, ...X_n) be an n-dimensional random variable with a normal distribution. Let Y_1, Y_2, ...$Y_{k\leq n}$, be linear functions of X_j (j=1,2,3,...n). Then prove that (Y_1, Y_2, ...Y_k) also has a multivariate normal distribution.

Section C

I - Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. If X_1,X_2,...,X_n are independent normal random variables and $\sum_{i=1}^{n} a_i b_i$ var(X_i) = 0, Then prove that $L_1 = \sum_{i=1}^{n} a_i X_i$ and $L_2 = \sum_{i=1}^{n} b_i X_i$ are independent. where a_1, a_2, ...,$a_n$and b_1, b_2, ...b_n are fixed (nonzero) real numbers.

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10. Prove that X be an random variable with standard normal PDF $f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$

11. Explain Bivariate Poisson distributions.

12. State and prove Slutsky's theorem.

II - Compulsory question $(1 \times 10 = 10$ Marks)

13. State and prove Lindeberg-Levy Central Limit theorem.

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