20PAMCT2004

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(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc.(App.Maths) - END SEMESTER EXAMINATIONS APRIL - 2023
SEMESTER - II
20PAMCT2004 - Algebra - II

Total Duration : 2 Hrs. 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. If L is an algebraic extension of K and if K is an algebraic extension of F, then prove that L is an algebraic extension of F.
2. (i) Describe algebraic and transcendental numbers.
   (ii) Show that , if L is a finite extension of F and K is a subfield of L which contains F, then [K :F] | [L :F].
3. State and prove Remainder Theorem.
4. Prove that if K is a normal extension of F, then K is the splitting field of some polynomial over F.
5. Prove that if V is n-dimensional over F and if T $\in$ A(V) has all its characteristic roots in F, then T satisfies a polynomial of degree n over F.
6. Prove that if T $\in$ A(V) is nilpotent, then $\alpha_0 + \alpha_1 T + ... + \alpha_m T^m$, where the $\alpha_i \in F$, is invertible if $\alpha_0 \neq 0$.
7. Prove that $V_i \neq (0)$ and $V = V_1 \oplus V_2 \oplus ... \oplus V_k$, for each i = 1, 2, ... , k.
8. Define characteristic polynomial of T and prove that every linear transformation T $\in$ $A_F(V)$ satisfies its characteristic polynomial and every characteristic root of T is a root of $P_T(x)$.

Section C

I - Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. Show that if F is of characteristic 0 and if a, b are algebraic over F, then there exists an element c$\in$F(a, b) such that F(a, b) = F (c).
10. Prove if K is a finite extension of F, then G(K, F) is a finite group and its order, o(G(K, F)) satisfies o(G(K, F)) $\leq$ [K:F].
11. Prove that if T $\in$ A(V) has all its characteristic roots in F, then there is a basis of V in which the matrix of T is triangular.

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12. Show that the elements S and T in $A_F(V)$ are similar in $A_F(V)$ if and only if they have the same elementary divisors.

II - Compulsory question $(1 \times 10 = 10$ Marks)

13. Prove that if L is a finite extension of K and if K is a finite extension of F, then L is a finite extension of F. Moreover, [L:F] = [L:K][K:F].

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