SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc.(Appl.Maths) - END SEMESTER EXAMINATIONS APRIL - 2023
SEMESTER - III
20PAMCT3007 - Complex Analysis

Total Duration : 2 Hrs. 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. If G is an open set and let $f : G \to C$ be a differentiable function, then show that f is analytic on G.
2. Show that $\displaystyle\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx = \frac{\pi}{\sqrt{2}}$.
3. Use Gamma function, show that $\sqrt{\pi}\ \Gamma(2z) = 2^{2z-1}\ \Gamma(z)\Gamma(z + \frac{1}{2})$.
4. If D = {z:|z|<1} and suppose that $f : \partial D \to R$ is a continuous function, then show that there is a continuous function u: $D^- \to R$ such that
   (a) $u(z) = f(z)$ for z in D,
   (b) u is harmonic in D , Moreover, u is unique and is defined by the formula
   $$u(re^{i\theta}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} P_r(\theta - t) f(e^{it}) dt.$$
5. State and prove Little Picard theorem.
6. If $p(z)$ is a non-constant polynomial, then prove that there is a complex number 'a' with $p(a) = 0$.
7. State and prove Schwarz's Lemma.
8. State and prove Jensen's formula.

Section C

I - Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. If f is an analytic function on a region containing the closure of the disk D $= \{z : |z| < 1\}$ and satisfying f(0) = 0,f' (0) = 1, then prove that there is a disc S $\subset$ D on which f is one to one and such that $f(S)$ contains a disk of radius 1/72.

Contd...

10. Show that for a > 1, $\displaystyle\int_0^{\pi} \frac{d\theta}{a + cos\theta} = \frac{\pi}{\sqrt{a^2-1}}$.

11. State and prove Riemann Mapping theorem.

12. If G is a region, then prove that

    (a) the metric space $Har\ (G)$ is complete.

    (b) if $\{$u$_n\}$ is a sequence in $Har(G)$ such that $u_1 \leq u_2 \leq$ ... then either u$_n(z) \rightarrow \infty$ uniformly on compact subsets of G or $\{$u$_n\}$ converges in $Har(G)$ to a harmonic function.

II - Compulsory question $(1 \times 10 = 10$ Marks)

13. If G is an open subset of the plane and $f : G \rightarrow C$ an analytic function. If γ is a closed rectifiable curve in G such that $n(\gamma;$ w$)$ $=$ 0, for all w in $C - G$, then prove that for 'a' in $G - \{\gamma\}$, $n(\gamma; a)f(a) = \dfrac{1}{2\pi i}\displaystyle\int \frac{f(g)}{z-a}dz$.

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