20PAMCT4010

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc.(App.Maths) - END SEMESTER EXAMINATIONS APRIL - 2023
SEMESTER - IV
20PAMCT4010 - Functional Analysis

Total Duration : 2 Hrs. 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Let M be closed linear space of Normed Linear Space N. If the norm of a coset x + M in the quotient space $\frac{N}{M}$ is $\|x + M\| = inf\{\|x + m\| : m \in M\}$. Then prove that $\frac{N}{M}$ is a Normed Linear Space
2. If N is a normed linear space, then show that $|\ \|x\| - \|y\|| \leq \|xy\|$, for all x, y $\in$ N
3. Prove that if T is a one-one, continuous linear transformation of a Banach space B onto B', then T is a homeomorphism.
4. State and prove the open mapping theorem.
5. If P is a projection on a closed linear space M of H, then prove that M is invariant under an operator T iff TP = PTP.
6. If P and Q are the projections on the closed linear subspaces M and N on H, then prove that M $\pm$ N iff PQ = 0 $\Leftrightarrow$ QP = 0.
7. If 0 is the only topological divisor of zero in A, then A= C.
8. Define resolvent set and derive resolvent equation.

Section C

I - Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. State and prove Hahn – Banach Theorem.
10. State and Prove Uniform boundedness theorem.
11. Let H be a Hilbert Space and let f be an arbitrary functional in H*, then prove that there exists a unique vector y $\in$ H such that f(x)=$\langle x, y\rangle$ for all x $\in$ H.
12. If r is an element of R, then show that 1-r is left regular and regular.

II - Compulsory question $(1 \times 10 = 10$ Marks)

13. With usual notation, prove that $r(x) = lim\|x^n\|^{1/n}$.

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