

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.

M.Sc.(Appl.Maths) - END SEMESTER EXAMINATIONS APRIL - 2023

SEMESTER - IV

20PAMCT4011 - Differential Geometry and Tensor Calculus

Total Duration : 2 Hrs. 30 Mins.

Total Marks : 60

Section B

Answer any **SIX** questions ($6 \times 5 = 30$ Marks)

1. Define (i) Curvature (ii) Torsion (iii) Involutives.
2. Compute the length of the curve given as the intersection of the surfaces $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $x = a \cosh(z/a)$, from the point $(a, 0, 0)$ to the point (x, y, z) .
3. Explain about right Helicoid and general Helicoid.
4. Prove that the curves of the family $v^3/u^2 = \text{constant}$ are geodesics on a surface with metric $v^2 du^2 - 2uv du dv + 2u^2 v^2 (u > 0, v > 0)$.
5. Show that the transformations of tensors form a group.
6. Determine that a skew-symmetric tensor of the second order has only $n(n-1)/2$ different non-zero components.
7. State and prove Ricci tensor theorem.
8. Find the metric tensor and the expression for the line element in cylindrical coordinates.

Section C

I - Answer any **TWO** questions ($2 \times 10 = 20$ Marks)

9. (i) Show that if a curve is given in terms of a general parameter u , then the equation of the osculating plane corresponding to $\left[\bar{R} - \bar{r}(0), \bar{r}'(0), \bar{r}''(0) \right] = 0$ is $\left[\bar{R} - \bar{r}, \dot{\bar{r}}, \ddot{\bar{r}} \right] = 0$.
(ii) Show that the involutes of a circular helix are plane curves.
10. Determine the coefficients of the direction which makes an angle $1/2\pi$ with the direction whose coefficients are (l, m) .
11. Compute that the equations for Geodesics.

Contd...

12. (i) Conclude $A_{iJ} B^i C^J$ is invariant, if B^i and C^J are contravariant vectors and also A_{iJ} is a covariant tensor.
- (ii) Show that, if a tensor is symmetric with respect to two indices in any coordinate system, it will remain symmetric with respect to these two indices in any other coordinate system.

II - Compulsory question ($1 \times 10 = 10$ Marks)

13. Determine the derivatives of the fundamental tensor.
