SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc.(Maths) END SEMESTER EXAMINATIONS NOVEMBER -2023
SEMESTER - VI
08UMACT6013 - Linear Algebra

Total Duration : 2 Hrs 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Prove that Hom($F^{(n)}$, $F^{(m)}$) is isomorphic to $F^n(m)$ as a vector space.
2. Prove that L(S) is a subspace of V
3. If $v_1, v_2, ..., v_n$ is a basis of V over F and if $w_1, w_2, ..., w_m$ in V are linearly independent over F then prove that m $\leq$ n.
4. Define Inner product space and also prove that $W^\perp$ is a subspace of V.
5. If A is an algebra, with unit element over a field F then show that A is isomorphic to a subalgebra of A(V) for some vector space V over F.
6. Define Characteristic root and prove that the element $\lambda \in$F is a characteristic root of T$\in$A(V) if and only if for some v$\neq$0 in V, vT $= \lambda$v.
7. Prove that, given the matrix A $= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -11 & 6 \end{pmatrix} \in F_3$ (where the characteristic of F is not 2) then $A^3 - 6A^2 + 11A - 6 = 0$.
8. If T$\in$A(V) has all its characteristic roots in F then prove that there is a basis of V in which the matrix of T is triangular.

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. If T is a homomorphism of U onto V with kernel W, then examine V is isomorphic to U/W. Conversely, if U is a vector space and W is a subspace of U, then show that there is a homomorphism of U onto U/W.
10. If V is finite dimensional and if W is a subspace of V, then prove that W is finite dimensional dim W $\leq$ dim V and dim V/W = dim V - dim W.

Contd...

11. Let V be a finite dimensional inner product space then show that V has an orthonormal set as a basis.
12. If V is finite dimensional over F, then prove that T∈A(V) is regular if and only if T maps V onto V.
13. If V is n-dimensional over F and if T∈A(V) has all its characteristic roots in F then show that T satisfies a polynomial of degree n over F.

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