SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc.(Maths) END SEMESTER EXAMINATIONS NOVEMBER -2023
SEMESTER - IV
20UMACT4007 - Vector Calculus and Fourier Transforms

Total Duration : 2 Hrs 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Compute the directional derivative of the function ϕ = $xy^2 + yz^3$ at the point (2, -1, 1) in the direction of the normal to the surface xLogz - y^2 + 4 = 0 at the point (-1, 2, 1).
2. If $\vec{a}$ is a constant vector and $\vec{r}$ is the positional vector of the point (x, y, z) with respect to the origin. Show that
   (i) Div ($\vec{a}\ X\ \vec{r}$) = 0 and (ii) Curl ($\vec{a}\ X\ \vec{r}) = 2\ \vec{a}$
3. If $\vec{F} = xy\ \vec{i} - z\ \vec{j} + x^2\ \vec{k}$, Evaluate $\int_C \vec{F}\ X\ \vec{dr}$, where C is the curve
   x = t^2, y = 2t, z = t^3 from (0, 0, 0) to (1, 2, 1).
4. Compute the work done when a force $\vec{F} = (x^2 - y^2 + x)\ \vec{i} - (2xy + y)\ \vec{j}$ displaces a particle in the xy-plane from (0, 0) to (1, 1) along the curve y = x.
5. Verify Stokes theorem when $\vec{F} = (2xy - x^2)\ \vec{i} - (x^2 - y^2)\ \vec{j}$ and C is the boundary of the region bounded by the parabolas y^2 = x and x^2 = y.
6. If F [f(x)] = F(s) show that F [f(x - a)] = e^{ias} F(s).
7. Compute the Fourier Sine transform of 1/x.
8. Determine the Fourier Sine transform and Fourier Cosine transform of f(x) = e^{-ax}, a > 0.

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. If $\vec{F} = (x^2 - y^2 + 2xz)\ \vec{i} + (xz - xy + yz)\ \vec{j} + (z^2 + x^2)\ \vec{k}$ at the point (1,1,1) determine $\nabla .\ \vec{F}; \nabla \times\ \vec{F}; \nabla .(\nabla \times\ \vec{F})$ and $\nabla \times (\nabla \times\ \vec{F})$.

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10. Verify Stoke's theorem for $\vec{F} = xy\,\vec{i} - 2yz\,\vec{j} - zx\,\vec{k}$ where S is the open surface of the rectangular parallelepiped formed by the planes x=0, x=1, y=0, y=2, and z=3 above the xoy-plane.

11. Determine the Fourier transform of $f(x) = \begin{cases} 1 - x^2 & |\, x \,| < 1 \\ 0 & |\, x \,| > 1 \end{cases}$ and hence deduce

$$\int_0^\infty \frac{sins - sCoss}{s^3} Cos\frac{s}{2} ds$$

12. Determine the Fourier transform of $f(x) = \begin{cases} 1 & |\, x \,| < a \\ 0 & |\, x \,| > a \end{cases}$ and hence deduce

(i) $\int_0^\infty \frac{Sint}{t} dt = \frac{\pi}{2}$

(ii) $\int_0^\infty \left(\frac{Sint}{t}\right)^2 dt = \frac{\pi}{2}$

13. Verify Green's theorem in a plane $\int_C [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$ where C is the boundary of the region defined by the lines x=0, y=0 and x+y=1.

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