20UMACT5010

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc.(Maths) END SEMESTER EXAMINATIONS NOVEMBER -2023
SEMESTER - V
20UMACT5010 - Real Analysis

Total Duration : 2 Hrs 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Prove that Countable union of countable sets is countable.
2. If $\{S_n\}_{n=1}^{\infty}$ is a convergent sequence of real numbers, then Prove that $\lim_{n\to\infty} \sup S_n = \lim_{n\to\infty} S_n$
3. Let (M,r) be a metric space and a be a point in M. Let F and g are real valued functions whose domains are subsets of M. If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = N$, then prove that $\lim_{x\to a} \left[f(x)g(x)\right] = LN$
4. If the subset A of the metric space (M,ρ) is totally bounded, then prove that A is bounded.
5. State and prove Rolle's theorem.
6. State and prove Ratio test.
7. State and prove second fundamental theorem of calculus.
8. Prove that the continuous function of a continuous function is continuous

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. Prove that a increasing sequence which is bounded above is convergent. Hence, deduce $\left(1 + \frac{1}{n}\right)_{n=1}^{\infty}$ is convergent
10. If $\{a_n\}_{n=1}$ is a sequence of positive numbers such that
 i) $a_1 \geq a_2 \geq ... \geq a_n \geq a_{n+1} \geq ...$ and
 ii) $\lim_{n\to\infty} a_n = 0$, then prove that the alternating series $\sum_{n=1}^{\infty}(-1)^{n+1} a_n$ convergent.
11. Prove that a function $f : M_1 \to M_2$ is continuous if and only if, the inverse image of every open set is open.

Contd...

12. Let (M,ρ) be a metric space. The subset A of M is totally bounded iff every sequence of points of A contains a Cauchy subsequence.

13. Suppose $'f'$ has a derivative at 'c' and g has a derivative at $f(c)$.
Then prove $\phi = g^{\circ}f$ has a derivative at c and $\phi'(c) = g'[f(c)]f'(c)$

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