SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc.(Maths) END SEMESTER EXAMINATIONS NOVEMBER -2023
SEMESTER - V
20UMACT5012 - Graph Theory

Total Duration : 2 Hrs 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Prove that, let G be a graph on atleast 6 vertices, then G or $\bar{G}$ contains a triangle.
2. Show that an edge e = (u,v) of a graph G is not a cut-edge iff e belongs to a cycle in G.
3. Prove that in a connected graph G there is an Eulerian trail iff the number of vertices of odd degree is either zero or two.
4. Prove that for a (p,q) graph G, the following statements are equivalent.
   (a) G is a tree.
   (b) G is connected and q = p - 1.
   (c) G is acyclic and q = p - 1.
5. Prove that if G is a connected graph, then the distance between v_i and v_j is the smallest integer $n(\geq 0)$ such that $[A^n]_{i,j} \neq 0$.
6. State and prove Kuratowaski theorem on planar graph.
7. Prove that for any graph G, $\chi(G) \leq \Delta(G) + 1$
8. Prove that if G is a bipartite graph with q(G) ≥ 1, then $\chi_1(G) = \Delta(G)$

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. Prove that for any graph G, $q(G) \geq p(G) - \omega(G)$
10. Prove that a nontrivial connected graph is Eulerian iff it has no vertex of odd degree.
11. State and prove Hall's theorem.
12. Prove that if G is a plane (p,q) graph in which every face is bounded by a cycle of length at least n, then $q \leq \dfrac{n(p-2)}{n-2}$
13. Prove that if G is a graph on p vertices, then
    (a) $2\sqrt{p} \leq \chi(G) + \chi(\bar{G}) \leq p + 1$ (b) $p \leq \chi(G)\chi(\bar{G}) \leq \dfrac{(p+1)^2}{4}$

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