SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc.(Maths) END SEMESTER EXAMINATIONS NOVEMBER -2023
SEMESTER - VI
20UMACT6014 - Complex Analysis

Total Duration : 2 Hrs 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Find the value of integral $\int_C \bar{z}dz$ where C is the right-hand half $z = 2e^{i\theta}\left(-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}\right)$ of the circle $\mid z \mid = 2$ from z = -2i to z = 2i.
2. Define limit of a function. Using the definition to show $\lim_{z\to 0}\frac{\bar{z}^2}{z} = 0$.
3. Apply Cauchy's residue theorem to evaluate $\int_C \frac{5z-2}{z(z-1)}dz$ where C is the circle $\mid z \mid = 2$
4. If f is entire and bounded in the complex plane, then prove that $f(z)$ is constant throughout the plane
5. Find the bilinear transformation that maps the points z_1 = -1, z_2 = 0, z_3 = 1 onto the points w_1 = -i, w_2 = 1, w_3 = i.
6. Determine and classify the singular points of
   (i) $f(z) = \frac{z}{e^z - 1}$
   (ii) $g(z) = sin\left(\frac{1}{z}\right)$
7. Show that the function $f(z) = \mid z \mid^2$ is differentiable only at $z = 0$.
8. Without evaluating the integral, justify $\mid \int_C \frac{z+4}{z^3-1}dz \mid \leq \frac{6\pi}{7}$, where C is the arc of the circle $\mid z \mid = 2$ from $z = 2$ to $z = 2i$ that lies in the first quadrant.

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. State and prove sufficient condition for differentiability.

Contd...

10. Prepare the Laurent series expansion for the function $f(z) = -\dfrac{1}{(z-1)(z-2)}$ in the regions
    (i) $\mid z \mid < 1$ (ii) $1 < \mid z \mid < 2$ (iii) $\mid z \mid > 2$
11. State and prove Cauchy integral formula
12. Compute $\displaystyle\int_C \frac{dz}{z^3(z+4)}$ taken counter clockwise around the circle
    (a) $\mid z \mid = 2$ (b) $\mid z+2 \mid = 3$
13. Determine all linear fractional transformations that map the upper half plane Imz>0 onto the open desk $\mid w \mid < 1$ and the boundary Imz = 0 onto the boundary $\mid w \mid = 1$

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