SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc.(Maths) END SEMESTER EXAMINATIONS NOVEMBER -2023
SEMESTER - I

20UMACT1001 - Trignometry and Analytical Geometry of 2 Dimensions

Total Duration : 2 Hrs 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Prove that $32\cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$
2. If $\tan(\alpha + i\beta) = x + iy$ then prove that $x^2 + y^2 + 2x\cot 2\alpha = 1$
3. Prove that $\log\frac{\cos(x-iy)}{\cos(x+iy)} = 2i\tan^{-1}(\tan x\tan hy)$
4. Show that $\frac{\pi}{2\sqrt{3}} = 1 - \frac{1}{3.3} + \frac{1}{5.3^2} - \frac{1}{7.3^3} + \ldots\ldots\ldots\ldots\ldots + \infty$
5. Find the locus of the midpoint of a variable chord of the parabola $y^2 = 4x$ if the chord passes through the fixed point (3,4) always.
6. Separate into real and imaginary parts of $\tan^{-1}(x + iy)$
7. Prove that $\frac{\sin 7\theta}{\sin\theta} = 64\cos^6\theta - 8\cos^4\theta + 24\cos^2\theta - 1$
8. Show that
   $\log\tan\left(\frac{\pi}{4} + \frac{ix}{2}\right) = i\tan^{-1}(\sin hx)$

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. Show that $2^5\sin^4\theta\cos^2\theta = \cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2$
10. If $\sin(A + iB) = x + iy$ prove that
    (i) $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$
    (ii) $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$
11. Find the general value of $(x + iy)^{\alpha+i\beta}$ and show that the sum of the moduli of the values less than unity of $(1 + i)^{1+i}$ is $\frac{1}{\sqrt{2}}e^{\frac{3\pi}{4}}\cos ec\pi$
12. Sum to infinity the series
    $1 + \frac{c^2}{2!}\cos 2\theta + \frac{c^4}{4!}\cos 4\theta + \ldots\ldots\ldots\infty$
13. A chord PQ of an ellipse subtends a right angle at the center of the ellipse. Show that the locus of the intersection of the tangents at P and Q is the ellipse $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{a^2} + \frac{1}{b^2}$

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