SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc.(Maths) END SEMESTER EXAMINATIONS NOVEMBER -2023
SEMESTER - II
20UMACT2004 - Integral Calculus and Fourier Series

Total Duration : 2 Hrs 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks$)$

1. Find $\int x^3 cos2x dx$

2. Compute $\int\int (x^2 + y^2)dxdy$ over the region for which x, y are each ≥ 0 and $x + y \leq 1$

3. Compute $\int_0^\infty e^{-x^2} dx$

4. Express $f(x) = x (-\pi < x < \pi)$ as a Fourier series with period 2π

5. Express $f(x) = c - x$ where $0 < x < c$ as a half range cosine series with period $2c$.

6. By changing the order of integration , compute $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dxdy$

7. Show that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$

8. Evaluate $\int_0^1 x^n e^x dx$

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks$)$

9. Compute the reduction formula for the integral $I_{m,n} = \int sin^m x cos^n x dx$ and hence evaluate $\int sin^6 x cos^3 x dx$.

10. Compute $\iiint \frac{dx}{(x + y + z + 1)^3}$ taken over the volume bounded by the panes $x = 0, y = 0, z = 0, x + y + z = 1$

11. Determine $\int_0^1 x^m(1 - x^n)^p dx$ in terms of Gamma functions and evaluate the integral $\int_0^1 x^5(1 - x^3)^{10} dx$

12. Compute the Fourier expansion of the functions $f(x) = \pi^2 - x^2$ in the interval $-\pi < x < \pi$

13. If $f(x) = \begin{cases} x & \text{when} \quad 0 < x < \frac{\pi}{2} \\ \pi - x & \text{when} \quad x > \frac{\pi}{2} \end{cases}$.
    Develop $f(x)$ as a sine series in the interval $(0,\pi)$.

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