SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc.(Statistics) END SEMESTER EXAMINATIONS NOVEMBER -2023
SEMESTER - I
20USTCT1002 - Probability and Random Variables

Total Duration : 2 Hrs 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Four cards drawn at random from a pack of 5 cards. Find the probability that
   (i) They are a king, a queen, a jack and an ace.
   (ii) Two are kings and two are queens.
   (iii) Two are black and two are red.
   (iv) There are two cards of hearts and two cards of diamonds.
2. Three newspapers A, B and C are published in a certain city. It is estimated from a survey that of the adult population: 20% read A, 16% read B, 14% read C, 8% read both A and B, 5% read both A and C, 4% read both B and C, 2% read all three. Find what percentage read at least one of the papers?
3. The odds that a book on statistics will be favourably reviewed by independent critics are 3 to 2, 4 to 3 and 2 to 3 respectively. Compute the probability of the three reviews:
   a) All will be favourable.
   b) Majority of the reviews will be favourable.
   c) Exactly one review will be favorable.
4. A factory produces a certain type of output by three types of machines.
   The respective daily production values are
   Machine I : 3000 units Machine II : 2500 units Machine III : 4500 units
   Past experience shows that 1% of the output produced by machine I is defective, the corresponding fraction of defectives for other two machines are 1.2% and 2% respectively. An item is drawn at random from the day's production run and is found to be defective. What is the probability that it comes from the output of
   (i) Machine I (ii) Machine II (iii) Machine III ?
5. A variable X is distributed at random between the values 0 and 1 so that its probability function is $f(x) = kx^2(1 - x^3)$, where k is a constant. Find the value of k, the mean and variance of X.
6. State the properties of Expectations.

Contd...

7. Let X and Y be two random variables each taking three values - 1,0 & 1 and having the joint probability distribution

Y \ X	-1	0	1
-1	0	.1	.1
0	.2	.2	.2
1	0	.1	.1

(i) Show that X and Y have different expectations.
(ii) Prove that X and Y are uncorrelated.
(iii) Compute Var (X) and Var (Y).

8. Write the Moment Generating Function (MGF) of a random variable and explain how the MGF is used to calculate moments with illustrations?

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. State the Addition Theorem of Probability for two events. Hence state and prove the same for three events.

10. Write the statement and prove the Bayes' theorem of probability for n events.

11. A random variable X has the following probability distribution:

Value of X, x	0	1	2	3	4	5	6	7	8
p(x)	k	3k	5k	7k	9k	11k	13k	15k	17k

(i) Compute the value of k.
(ii) Find P(X<3), P(X≥3), P(0<X<5)
(iii) What is the smallest value of x for which P(X≤x) $>$ 0.5 ? and
(iv) Find the distribution function of X.

12. State and prove Chebychev's inequality.

13. Justify any three properties of Moment generating Function.

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