SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc.(Appl.Maths) END SEMESTER EXAMINATIONS NOVEMBER - 2023
SEMESTER - I
20PAMCT1001 - Algebra -1

Total Duration : 2 Hrs. 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Define Normalizer N(a) and Prove that N(a) is Subgroup of G.
2. If o (G) = p^2 where p is prime number then prove that G is Abelian.
3. Suppose that G is the internal direct product of N_1, N_2,...N_n then for i $\neq$ j, $N_i \cap N_j$= (e) and if a ε N_i, b ε N_j then prove that ab = ba.
4. If A is invertible then prove that det (A) $\neq$ 0 and det (A^{-1}) = (det A)$^{-1}$.
5. Prove that T ε A (V) is unitary iff TT* = 1.
6. Prove that every prime number p and every positive integer m there exists a field having p^m elements.
7. When a group is said to be Solvable. Given the group G and a, b ε G; verify that commutator of a and b is $a^{-1}b^{-1}$ ab.
8. Define norm of x. Prove that for all x, y ε Q,N (xy) = N (x) N (y).

Section C

I - Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. State and prove first Sylow's theorem.
10. Prove that G is solvable if and only if $G^{(k)}$ = (e) for some integer k.
11. For A, B ε F_n and λ ε F Prove that (i) tr (λA) = λ tr (A)
    (ii) tr (A + B) = tr (A) + tr (B) (iii) tr (AB) = tr (BA)
    (iv) if A is invertible then tr (ACA^{-1}) = tr (C).
12. Prove that if F is a field and α, β ε F; α, $\beta \neq 0$ then we can find two elements a, b in F such that 1 + αa^2 + βb^2 = 0.

II - Compulsory question $(1 \times 10 = 10$ Marks)

13. Prove that the Adjoin in Q satisfies
    (i) x** = x; (ii) (αx +βy)* = αx* +βy*;
    (iii) (xy)* = y*x* for all x, y in Q and all real α, β.

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