SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc.(Appl.Maths) END SEMESTER EXAMINATIONS NOVEMBER - 2023
SEMESTER - I
20PAMCT1002 - Real Analysis

Total Duration : 2 Hrs. 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Prove that the class $\mathfrak{m}$ is a σ-algebra.
2. Prove that Not every measurable set is a Borel set.
3. State and prove Lebesgue's Dominated Convergence Theorem.
4. Show that if $\alpha > 1$, $\int\limits_0^1 \frac{xsinx}{1+(nx)^\alpha}dx = 0(n^{-1})$ as n $\to \infty$.
5. State and prove Cauchy criterion for uniform convergence.
6. Prove that suppose $\{f_n\}$ is a sequence of functions, differentiable on [a,b] and such $\{f_n(x_0)\}$ that converges uniformly on [a,b], to a function f, and then $f^{'}(x) = \lim\limits_{n\to\infty} f_n^{'}(x)$,$(a \leq x \leq b)$.
7. Prove that suppose **f**maps an open set $E \subset R^n$ into R^m. Then **f** $\in b^{'}(E)$ if and only if the partial derivatives $D_j f_i$ exist and are continuous on E for $1 \leq$ i $\leq m$,$1 \leq$ j $\leq n$.
8. State and prove Stirling's formula.

Section c

I - Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. Prove that let c be any real number and let **f** and **g** be real-valued measurable functions defined on the same measurable set E. Then **f+c**, **cf**, **f+g**, **f-g** and **fg** are also measurable.
10. State and prove the Stone-Weierstrass theorem.
11. State and prove Inverse function theorem.
12. State and prove Parseval's theorem.

II - Compulsory question $(1 \times 10 = 10$ Marks)

13. State and prove Fatou's Lemma.

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