SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc.(Appl.Maths) END SEMESTER EXAMINATIONS NOVEMBER - 2023
SEMESTER - III
20PAMCT3008 - Differential Equations

Total Duration : 2 Hrs. 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Show that $\frac{d}{dt}\left[t^p J_p(t)\right] = t^p J_{p-1}(t) and \frac{d}{dt}\left[t^{p-1} J_p(t)\right] = -t^{p-1} J_{p+1}(t)$.
2. Find $e^{At}$when $A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$
3. State and Prove Gronwall inequality.
4. Use Charpit's method to solve $p = (z + qy)^2$.
5. Reduce the PDE $y^2\frac{\partial^2 z}{\partial x^2} - 2xy\frac{\partial^2 z}{\partial x \partial y} + x^2\frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x}\frac{\partial z}{\partial x} + \frac{x^2}{y}\frac{\partial z}{\partial y}$ to canonical form and hence solve it.
6. If $P_n$is a Legendre polynomial then, Prove that $\int_{-1}^{1} P_n^2(t)dt = \frac{2}{2n+1}$.
7. Use Jacobi's method to solve $p^2x + q^2y = z$.
8. If $u = f(x + iy) + g(x - iy)$, where $f$and g are arbitrary functions, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

Section C

I - Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. Solve the Hermite equation $x'' - 2tx' + 2x = 0$ has an ordinary point at t=0, since -2t. and 2 are analytic functions at t=0 with positive radius of convergence. Also determine the constants a_k and series solution of z(t).
10. Let $A : I \to M_n(\Re)$ be continuous. Suppose a matrix Φ satisfies $X' = A(t)X, t \in I$. Then, Prove that $\det \Phi$satisfies the equation $(\det \Phi)' = (trA)(\det \Phi)$
11. State and prove Picard's theorem.

Contd...

12. Find the equation of the integral surface of the differential equation.
$2y(z-3)p+(2x-z)q=y(2x-3)$, which passes through the circle
$z=0, x^2+y^2=2x$.

II - Compulsory question $(1\times 10=10$ Marks)

13. Reduce the equation $(n-1)^2\dfrac{\partial^2 z}{\partial x^2}-y^{2n}\dfrac{\partial^2 z}{\partial y^2}=ny^{2n-1}\dfrac{\partial z}{\partial y}$ to canonical form and find its general solution.

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