

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN  
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)

Chromepet, Chennai — 600 044.

M.Sc.(Appl.Maths) END SEMESTER EXAMINATIONS NOVEMBER - 2023

SEMESTER - II

**20PAMET2002 - Mathematical Statistics**

Total Duration : 2 Hrs. 30 Mins.

Total Marks : 60

**Section B**

Answer any **SIX** questions ( $6 \times 5 = 30$  Marks)

- Define (i) Parameter Space (ii) Estimator (iii) consistent (iv) LMVUE (v) UMVUE
- (i) State a necessary and sufficient condition for an unbiased estimator to be a UMVUE.  
(ii) Give FCR inequality.
- Define Maximum likelihood estimator and obtain MLE for Hyper geometric function.
- Let  $T(x)$  be maximal invariant with respect to  $\zeta$ . Then show that  $\varphi$  is invariant under  $\zeta$  if and only if  $\varphi$  is a function of  $T$ .
- Calculate level  $\alpha$  generalized likelihood ratio test of  $H_0 : p \leq p_0$  against  $H_1 : p > p_0$  for  $X \sim b(n, p)$ .
- A die is rolled 120 times with the following results

Result	1	2	3	4	5	6
Frequency	20	30	20	25	15	10

Test the hypothesis that the die is fair or not at level  $\alpha = 0.05$ .

- The following table gives the yield of three varieties of wheat, obtained with four different kinds of fertilizers

Fertilizer	Variety of Wheat		
	A	B	C
$\alpha$	8	3	7
$\beta$	10	4	8
$\gamma$	6	5	6
$\delta$	8	4	7

Test the hypothesis that the three varieties of wheat yield the same average yield and that the three fertilizers are equally effective.

**Contd...**

8. Describe the two way analysis of variance table with interaction.

### Section C

I - Answer any **TWO** questions ( $2 \times 10 = 20$  Marks)

9. State and prove Factorization Criterion.
10. Construct shortest-length confidence interval and also obtain minimizing length of the confidence interval for the sample  $X_1, X_2, \dots, X_n$  from  $N(\mu, \sigma^2)$ , where  $\sigma^2$  is known.
11. State and Prove Neman- Pearson Fundamental Lemma.
12. Twenty observations on shear strength and the age of the corresponding batch of propellant is shown below in the table. Draw scatter diagram, fit a straight model and also estimate  $\sigma^2$ .

Observation	Shear Strength	Age
i	$y_i(\text{psi})$	$x_i(\text{weeks})$
1	2158.70	15.50
2	1678.15	23.75
3	2316.00	8.00
4	2051.30	17.00
5	2207.50	5.50
6	1708.30	19.00
7	2575.00	2.50
8	1784.70	24.00
9	2357.90	7.50
10	2256.20	11.00
11	2165.20	13.00
12	2399.55	3.75
13	1779.80	25.00
14	2336.75	9.75
15	1765.30	22.00
16	2053.50	18.00
17	2414.40	6.00
18	2200.50	12.50
19	2654.20	2.00
20	1753.70	21.50

Contd...

**SEMESTER - II**  
**20PAMET2002 - Mathematical Statistics**

II - Compulsory question ( $1 \times 10 = 10$  Marks)

13. The mean life of a sample of 9 light bulbs was observed to be 1309 hours with standard deviation of 420 hours. A second sample of 16 bulbs chosen from different batch showed a mean life of 1205 hours with standard deviation of 390 hours. Test the hypothesis whether that the two batches have same mean, assuming that the population variances are the same? Is the assumption of equality of variances on which the t-test is based, valid?

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