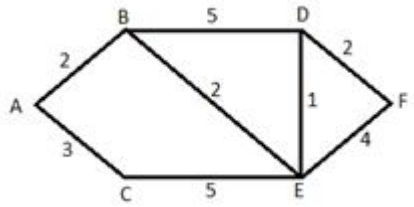
SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc.(CGS) END SEMESTER EXAMINATIONS NOVEMBER -2023
SEMESTER - I
21UCGAT1001 - Discrete Mathematics

Total Duration : 2 Hrs 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Using truth table, prove that the statement formula (P∧Q)∨(P∧∼Q) is logically equivalent to P.
2. Let X = {1,2,.....7} and R = $\{(x, y)|x - y$ is divisible by 3$\}$. Show that R is an equivalent relation.
3. For any commutative monoid (M,*), prove that the set of idempotent elements of M forms a submonoid.
4. Prove the following :
   i. Let $G = (V, E)$ be a graph with 'e' edges then $\sum_{v \in V} deg(v) = 2e$
   ii. In a graph G the number of odd degree vertices is always even.
5. Prove that a complete graph k_{33} and k_5 are non planar
6. In (Q-{1}, *), * is defined by $a * b = a + b + ab$, for all $a, b \in Q$ then prove that (Q-{1}, *) is a group.
7. Without using truth table, prove that (P→R) ∧ (Q→R) ⇔ (P∨Q) → R
8. Using Dijkstra's Algorithm find the shortest path from vertex A to the vertex F in the figure given below.



Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. Show that ((P∨Q) ∧ ∼ (∼P∧(∼Q∨∼R))) ∨ (∼P∧∼Q) ∨ (∼P∧∼R) is a tautology.

Contd...

10. a) Find $f \circ g$, $g \circ f$, f^2 and g^2 when $f : i \to i$ and $g : i \to i$ defined by $f(x) = 2x-1$, $g(x) = x^2-2$ (5)

    b) Show that $f : i \to i$ defined by $f(x) = 3x - 1$, $x \in i$ is both 1-1 and onto (bijective). Also find its inverse if it exists. (5)

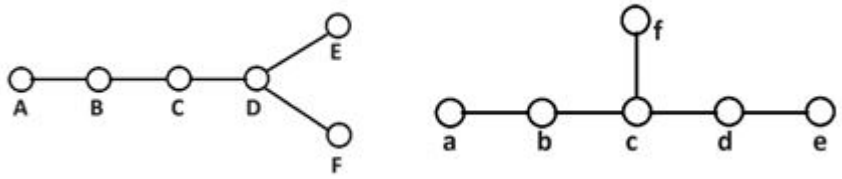
11. Let (G, *) be given group. Then prove the following :

    i. The identity element of a group is unique. (3)

    ii. $(a * b)^{-1} = b^{-1} * a^{-1}$ for all $a, b \in g$ (3)

    iii. A non empty subset H of G is a subgroup of G, then $a, b \in H \implies a * b^{-1} \in$ H for all a, b (4)

12. i. Check the given two graphs are isomorphic or not? (5)



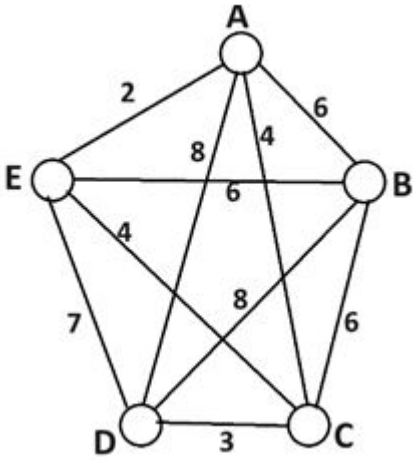
    ii. Give an example of a graph which is (5)

    a) Euler but not Hamiltonian

    b) Hamiltonian but not Euler

    And justify.

13. Find the minimum spanning tree for the weighted graph given below using Kruskal algorithm.



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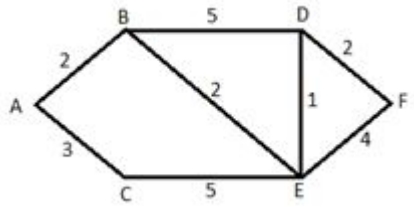

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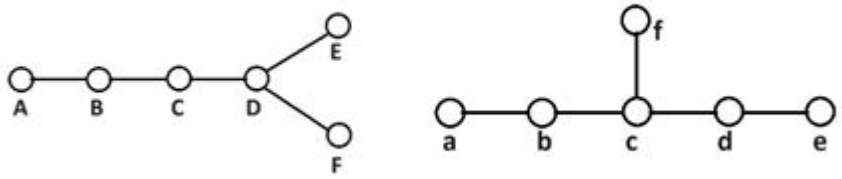

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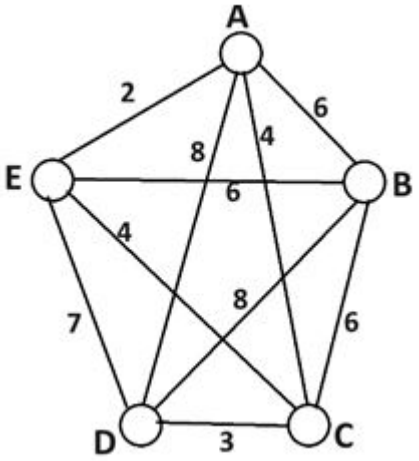

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