

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN  
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)  
Chromepet, Chennai — 600 044.

B.Sc.(DS) END SEMESTER EXAMINATIONS NOVEMBER -2023

SEMESTER - II

**22UDSAT2002 - Allied Mathematics - II**

Total Duration : 2 Hrs 30 Mins.

Total Marks : 60

### Section B

Answer any **SIX** questions ( $6 \times 5 = 30$  Marks)

1. If  $I_n = \int \sin^n x dx$  then prove that  $I_n = -\frac{\cos x \sin^{n-1} x}{n} + \frac{n-1}{n} I_{n-2}$ .
2. Find the Fourier series for  $f(x)$  in  $(-\pi, \pi)$  if  $f(x) = \begin{cases} 0, & -\pi < x < 0, \\ \pi, & 0 < x < \pi. \end{cases}$
3. Solve  $(D^2 + 3D + 2)y = e^{-2x} + \sin x$
4. Eliminate the arbitrary function  $f$  from  $z = e^y f(x + y)$ .
5. Find the Laplace transform of  $\sin 2t \sin t$ .
6. Find the inverse Laplace transform of  $\frac{4s^2 - 3s + 5}{(s+1)(s-1)(s-2)}$
7. Prove that if  $\bar{A}, \bar{B}$  are vector functions of  $u$ , then
  - (i)  $\frac{d}{du}(\bar{A} + \bar{B}) = \frac{d\bar{A}}{du} + \frac{d\bar{B}}{du}$
  - (ii)  $\frac{d}{du}(\bar{A} \bullet \bar{B}) = \frac{d\bar{A}}{du} \bullet \bar{B} + \bar{A} \bullet \frac{d\bar{B}}{du}$
8. If  $\bar{F} = xz\bar{i} + yz\bar{j} + z^2\bar{k}$ , evaluate  $\int_C \bar{F} \bullet d\bar{r}$  from the point  $(0, 0, 0)$  to  $(1, 1, 1)$  where  $C$  is given by  $x = t, y = t^2, z = t^3$ .

### Section C

Answer any **THREE** questions ( $3 \times 10 = 30$  Marks)

9. Obtain the Fourier series for the functions

(i)  $f(x) = \pi - x, 0 < x < 2\pi$ .

(ii)  $f(x) = \frac{1}{2}(\pi - x), 0 < x < 2\pi$ .

Deduce that  $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$ .

Contd...

10. Solve  $(y - z)p + (z - x)q = x - y$ .

11. Using Laplace transform, solve  $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 5y = e^{-2t}$ , given that  $y = 0, \frac{dy}{dt} = 1$ , when  $t = 0$ .

12. Prove that if  $\vec{A}$  and  $\vec{B}$  are vector point functions,

(i)  $\nabla(\vec{A} \cdot \vec{B}) = \vec{A} \times (\nabla \times \vec{B}) + (\vec{A} \cdot \nabla)\vec{B} + (\vec{B} \times (\nabla \times \vec{A}) + (\vec{B} \cdot \nabla)\vec{A}$

(ii)  $\nabla \cdot (\vec{A} \times \vec{B}) = (\nabla \times \vec{A}) \cdot \vec{B} - (\nabla \times \vec{B}) \cdot \vec{A}$

13. Evaluate  $\iiint_V \phi \, dV$  where  $\phi = 45x^2y$  and  $V$  is the closed region bounded by the planes  $4x + 2y + z = 8, x = 0, y = 0, z = 0$ .

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