SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
B.Sc. Maths - END SEMESTER EXAMINATIONS APRIL - 2024
SEMESTER - VI
08UMACT6013 -Linear Algebra

Total Duration : 2 Hrs. 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. a) Define Homomorphism. b) Show that L(S) is a subspace of V.
2. a) Define Linearly dependent over F.
   b) If $v_1, v_2, \ldots, v_n \epsilon$ V are linearly independent, then show that every element in their linear span has a unique representation in the form $\lambda_1 v_1 + \lambda_2 v_2 + \ldots + \lambda_n v_n$ with the $\lambda_i \ \epsilon$ F.
3. Prove that $F^{(n)}$ is isomorphic $F^{(m)}$ if and only if n = m.
4. Suppose that V is a finite-dimensional inner product space and W is a subspace of V, then prove that (W $\perp$) $\perp$ = W.
5. a) Define Algebra over F.
   b) Prove that if V is a finite-dimensional over F, then T ϵ A(V) is singular if and only if there exists a v$\neq$0 in V such that vT = 0.
6. a) Define Characteristic root.
   b) If $\lambda \ \epsilon$ F is a characteristic root of T ϵ A(V), then for any polynomial q(x) ϵ F[x], prove that q(λ) is a characteristic root of q(T).
7. Suppose that V is a n-dimensional over F and if T ϵ A(V) has the matrix m_1(T) in the basis $v_1, v_2, \ldots, v_n$ and the matrix m_2(T) in the basis $w_1, w_2, \ldots, w_n$ of V over F, then show that there is an element C ϵ F_n such that m_2(T) = C m_1(T) C^{-1}.
8. If u, $\nu \ \epsilon$ V and $\alpha_1 \ \beta \ \epsilon$ F, then show that (αu + βv, αu$\epsilon\beta$v = $\alpha \ \overline{\alpha}$ (u,u) + $\alpha \ \overline{\alpha}$ (u,u) + $\alpha \ \overline{\beta}$ (u,v) + $\overline{\alpha}\beta$(v,u) + $\beta \ \overline{\beta}$ (v,v)

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. a) Define Vector Space.
   b) If V is the internal direct sum of $U_1, U_2, \ldots, U_n$, then prove that V is isomorphic to the external direct sum of $U_1, U_2, \ldots, U_n$.

Contd...

10. Suppose that V and W are of dimensions m and n, respectively, over F, then prove that Hom(V,W) is of dimension *mn* over F.

11. a) Define Inner product space.
    b) If u,v ϵ V, then prove that $|(u,v)| \leq |u||v|$.

12. Prove that if V is a finite-dimensional over F, then T ϵ A(V) is invertible if and only if the constant term of the minimal polynomial for T is not 0.

13. If T ϵ A(V) has all its characteristic roots in F, then show that there is a basis of V in which the matrix of T is triangular.

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