SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
B.Sc. Maths - END SEMESTER EXAMINATIONS APRIL - 2024
SEMESTER - VI
20UMACT6013 -Linear Algebra

Total Duration : 2 Hrs. 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. If V is a vector space over F then prove that
   (a) $\alpha 0 = 0$ for $\alpha \in F$
   (b) $ov = 0 \; for \; v \in F$
   (c) $(-\alpha)\, v = -(\alpha v) \; for \; \alpha \in F, v \in V.$
   (d) $if \; v \neq 0, \; then \; \alpha v = 0 \; implies \; that \; \alpha = 0.$
2. Examine whether L(S) is a subspaces of V. Interpret your result.
3. If $v_1, v_2, \ldots . \, v_n \in V$are linearly independent, then determine that every element in their linear span has a unique representation in the form $\lambda_1 v_1 + \ldots \ldots + \lambda_n v_n$with $\lambda_i \in F$.
4. Let V be a finite-dimensional over F and if $u_1, \; \ldots \ldots, u_m \in V$ are linearly independent, then construct vectors $u_{m+1}, \ldots \ldots \; u_{m+r}$ in V such that $u_1, \; \ldots \ldots \ldots u_m$, $u_{m+1}, \ldots \ldots \; u_{m+r}$ is a basis of V.
5. If $u, v \; \in V$ and $\alpha, \beta \; \in F$, Demonstrate $(\alpha u + \beta v, \alpha u + \beta v) = \alpha\overline{\alpha}\,(u, u) + \alpha\overline{\beta}\,(u, v) + \overline{\alpha}\beta\,(v, u) + \beta\overline{\beta}\;(v, v)\,.$
6. Let $u, v \; \in V$ then simplify $|u, v| \leq ||u|| \; ||v||\,.$
7. Develop that A is isomorphic to a subalgebra of A (V) for some vector space V over F, if A is an algebra with unit element, over F.
8. The element $\lambda \in F$ is a characteristic root of $T \in A\,(V)$ if and only if for some $v \neq 0$ in V, $vT = \lambda v$. Justify the result.

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. If T is a homomorphism of U onto V with kernel W, then prove that V is isomorphic to U/W. Conversely, if U is a vector space and W a subspace of U, categorize that then there is a homomorphism of U onto U/W.

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10. Test whether W is finite-dimensional, if V is finite-dimensional and if W is a subspace of V. Also, Examine dim W $\leq$dim V and dim V/W = dim V - dim W.

11. Let V be a finite-dimensional inner product space; then V has an orthonormal set as a basis.

12. Measure the linear transformation $T \in A\,(V)$is regular if and only if T maps V onto V, if V is finite-dimensional over F.

13. Construct $v_1, v_2, \ldots .\ v_k$ are linearly independent over F if $\lambda_1, \ldots \ldots .\lambda_k$ in F are distinct characteristic roots of $T \in A(V)$ and if $v_1, v_2, \ldots .\ v_k$ are characteristic vectors of T belonging to $\lambda_1, \ldots \ldots .\lambda_k$ respectively.

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