

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN  
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)  
Chromepet, Chennai - 600 044.

B.Sc. Maths - END SEMESTER EXAMINATIONS APRIL - 2024

SEMESTER - III

**20UMAAT3003 - Mathematical Statistics I**

Total Duration : 2 Hrs. 30 Mins.

Total Marks : 60

**Section B**

Answer any **SIX** questions ( $6 \times 5 = 30$  Marks)

- Define (i) Mutually exclusive events  
(ii) Exhaustive events  
(iii) Equally likely events
- Prove that if A and B are any two events (subsets of sample space S) and are not disjoint, then  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- Write a short note on discrete and continuous random variables?
- A continuous random variable X has a probability density function  $f(x) = 3x^2$ ,  $0 \leq x \leq 1$ . Find a and b such that (i)  $P(X \leq a) = P(X > a)$  and (ii)  $P(X > b) = 0.05$ .
- A coin is tossed until a head appears. What is the expectation of the number of tosses required?
- State and Prove Chebychev's inequality.
- Define normal distribution mention its properties.
- Derive moments and moment generating function of uniform distribution.

**Section C**

Answer any **THREE** questions ( $3 \times 10 = 30$  Marks)

- The probabilities of X, Y and Z becoming managers are  $\frac{4}{9}$ ,  $\frac{2}{9}$  and  $\frac{1}{3}$  respectively. The probabilities that the bonus scheme will be introduced if X, Y and Z becomes managers are  $\frac{3}{10}$ ,  $\frac{1}{2}$  and  $\frac{4}{5}$  respectively. (i) what is the probability that bonus scheme will be introduced and (ii) If the bonus scheme has been introduced, what is the probability that the manager appointed was X?
- A random variable X has the following probability function:

<b>x</b>	0	1	2	3	4	5	6	7
<b>p(x)</b>	0	k	2k	2k	3k	k <sup>2</sup>	2k <sup>2</sup>	7k <sup>2</sup> +k

- Find k
- Evaluate  $P(X < 6)$ ,  $P(X \geq 6)$  and  $P(0 < X < 5)$
- Determine the distribution function of X.

**Contd...**

11. Prove the following results:
- (i) If  $X$  and  $Y$  are random variables, then  $E(X+Y) = E(X) + E(Y)$
  - (ii) If  $X$  and  $Y$  are independent random variables, then  $E(XY) = E(X) \cdot E(Y)$
12. A random variable  $X$  is normally distributed with mean 12 and standard deviation 4. Find out the probability of the following:
- (i)  $X \geq 20$  (ii)  $X \leq 20$  and (iii)  $0 \leq X \leq 12$ .
13. If  $X$  and  $Y$  are independent gamma variates with parameters  $\mu$  and  $\nu$  respectively. Show that the variables  $U = X+Y$ ,  $Z = \frac{X}{X+Y}$  are independent and that  $U$  is a  $\gamma(\mu+\nu)$  variate and  $Z$  is a  $\beta_1(\mu, \nu)$  variate.

\*\*\*\*\*