SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
B.Sc. Maths - END SEMESTER EXAMINATIONS APRIL - 2024
SEMESTER - II
20UMACT2003 - Classical Algebra

Total Duration : 2 Hrs. 30 Mins.   Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks$)$

1. Find the sum to infinity of the series $1 + \dfrac{3}{4} + \dfrac{3.5}{4.8} + \dfrac{3.5.7}{4.8.12} + \ldots\ldots\ldots$

2. Solve the equation $x^4 - 5x^3 + 4x^2 + 8x - 8 = 0$
   given that one of the roots is $1 - \sqrt{5}$

3. Remove the fractional coefficients from the equation $x^3 + \dfrac{1}{4}x^2 - \dfrac{1}{16}x + \dfrac{1}{72} = 0$

4. Show that any matrix A can be represented as the sum of a symmetric matrix and a skew symmetric matrix.

5. Calculate A^4 when $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

6. Sum the series $\displaystyle\sum_{n=0}^{\infty} \dfrac{(n+1)^3}{n!} x^n$

7. Solve the equation $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$ given that two of its roots are equal in magnitude and opposite in sign

8. If $A = \begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix}$ determine A^n in terms of A.

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks$)$

9. a) Sum the series $1 + \dfrac{1+3}{2!} + \dfrac{1+3+3^2}{3!} + \dfrac{1+3+3^2+3^3}{4!} + \ldots\ldots\ldots to\ \infty$

   b) Sum the series $\displaystyle\sum_{n=1}^{\infty} \dfrac{1}{(2n-1)2n(2n+1)}$

10. a) Show that $\dfrac{a^2}{x-\alpha} + \dfrac{b^2}{x-\beta} + \dfrac{c^2}{x-\gamma} - x + \delta = 0$ has only real roots if
    $a, b, c, \alpha, \beta, \gamma, \delta$ are all real.

    b) Form a rational cubic equation which shall have for roots $1, 3 - \sqrt{-2}$.

Contd...

11. If the sum of two roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ equals the sum of the other two then prove that $p^3 + 8r = 4pq$

12. a) show that $A = \begin{bmatrix} \dfrac{1}{3} & \dfrac{2}{3} & \dfrac{2}{3} \\[2mm] \dfrac{2}{3} & \dfrac{1}{3} & \dfrac{-2}{3} \\[2mm] \dfrac{-2}{3} & \dfrac{2}{3} & \dfrac{-1}{3} \end{bmatrix}$ is orthogonal.

b) Prove that product of two orthogonal matrices is orthogonal.

13. Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$

and hence determine its inverse.

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