

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.

B.Sc. Maths - END SEMESTER EXAMINATIONS APRIL - 2024
SEMESTER - II

20UMACT2004 - Integral Calculus and Fourier Series

Total Duration : 2 Hrs. 30 Mins.

Total Marks : 60

Section B

Answer any **SIX** questions ($6 \times 5 = 30$ Marks)

1. Evaluate $\int x^2 e^{-3x} dx$.
2. Show that $\int_0^{\pi/2} \sin^6 x \cos^5 x dx = \frac{4}{11} \cdot \frac{2}{9} \cdot \frac{1}{7} = \frac{8}{693}$
3. Determine the value of $\int e^{2x} \cos 3x dx$.
4. Construct $\Gamma(1/2) = \sqrt{\pi}$.
5. Evaluate $\int_0^{\infty} e^{-x^2} dx$.
6. Simplify $\int x^2 \sin 3x dx$.
7. Show that $x^2 = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} (-1)^n \frac{\cos nx}{n^2}$ in the interval $(-\pi \leq x \leq \pi)$.
8. Find Fourier series for the function $f(x) = x$ in the interval $[0, 2\pi]$.

Section C

Answer any **THREE** questions ($3 \times 10 = 30$ Marks)

9. Evaluate $\int (\log x)^3 x^4 dx$.
10. Simplify $\int x^3 \cos 2x dx$.
11. Evaluate $\int_0^{\pi/2} \sqrt{\sin \theta} d\theta \times \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin \theta}}$.
12. Find the Fourier series for the function $f(x) = \frac{1}{2}(\pi - x)$ in the interval $[0, 2\pi]$.
13. If $f(x) = \begin{cases} -x & \text{in } -\pi < x < 0 \\ x & \text{in } 0 < x < \pi \end{cases}$

Expand $f(x)$ as Fourier series in the interval $-\pi$ to π .

Deduce that $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots$
