SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
B.Sc. Maths - END SEMESTER EXAMINATIONS APRIL - 2024
SEMESTER - IV
20UMACT4007 - Vector Calculus and Fourier Transforms

Total Duration : 2 Hrs. 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Find the directional derivative of the f= xy+yz+zx in the direction of $\overline{z} = \overline{i} + 2\overline{j} + 2\,\overline{k}$ at the point (1,2,0)
2. Find the curl$\overline{f}$ at the point if $\overline{f}$ = xy^2 $\overline{i}$+2 x^{2}y$\overline{j}$ -3yz$^2\overline{k}$
3. Evaluate along $\int_C \overline{f}.\overline{dr}$ where $\overline{c}$ is the curve y=2x^{2}in the xy plane from (0,0) to (1,2) if $\overline{f}$ = xy $\overline{i}$ - y^2 $\overline{j}$
4. Evaluate $\iint_s \bar{F} = \hat{n}$ ds where $\bar{F} = 4xz\bar{i} - y^2\bar{j} + yz\bar{k}$ taken over the cube bounded by x = y = z = 0, x = y = z = 1.
5. Write any four properties of Fourier transforms.
6. Find the Fourier transform of F(x) $= \begin{cases} 1 \; in \; |x| < a \\ 0 \; in \; |x| > a \end{cases}$
7. Find the Fourier sine transform of f(x) $= \begin{cases} x^2, 0 < x < 1 \\ 0, x > 1 \end{cases}$
8. Prove that (i) $F_s[f(x)cosax]$ = 1/2 $[\bar{f}s(s+a) + [\bar{f}s(s-a)]$
   (ii) $F_c[f(x)cosax]$ = 1/2 $[\bar{f}c(a+s) + [\bar{f}c(a-s)]$

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. Find the constants a & b so that the surfaces $5x^2 - 2yz - 9x = 0$ and $ax^2y + bz^3 = 4$ may cut on the gonally at the point (1, -1,2).
10. Evaluate surface integral $\iint_s \overline{f.n}ds$ where f is the surface of the cube bounded by x=0, y=0, z=0 and x=a, y=a,z=a if $\overline{f}$ = 4xz $\overline{i}$ - y$^2\overline{j}$+ yz$\overline{k}$

Contd...

11. Apply Gauss divergence to evaluate $\iint_s (x+z)dy\ dz\ +(y+z)dz\ dx\ +(x+y)\ dx\ dy$ where s is the surface of the sphere $x^2+y^2+z^2=4$

12. Show that the Fourier transform of f(x) $= \begin{cases} a^2-x^2, |x|<a \\ 0, |x|>a \end{cases}$ is $2\sqrt{\frac{2}{\pi}}\left(\frac{sinsa-sacossa}{s^3}\right)$ Hence deduce $\int_0^\infty \frac{sint-tcost}{t^3} = \frac{\pi}{4}$

13. Evaluate fourier cosine and sine transform of $f(x)=e^{-ax}$

*****