

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN  
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)

Chromepet, Chennai - 600 044.

B.Sc. Statistics - END SEMESTER EXAMINATIONS APRIL - 2024

SEMESTER - II

**20USTCT2004 - Matrix Algebra**

Total Duration : 2 Hrs. 30 Mins.

Total Marks : 60

### Section B

Answer any **SIX** questions ( $6 \times 5 = 30$  Marks)

1. Prove a necessary and sufficient condition for a matrix  $A$  to be a symmetric is that  $A$  and  $A'$  are equal.
2. If  $A$  be an  $n \times n$  matrix, prove that  $|adj A| = |A|^{n-1}$ .
3. Prove that the characteristic roots of a Hermitian matrix are real.
4. Show that the relation of congruence of matrices is an equivalence relation in the set of all  $n \times n$  matrices over a field  $F$ .
5. If  $AB = A$  and  $BA = B$  then  $B'A' = A'$  and  $A'B' = B'$  and hence prove that  $A'$  and  $B'$  are idempotent.
6. Show that if  $A$  be an  $n$ -rowed non-singular matrix,  $X$  be an  $n \times 1$  matrix,  $B$  be an  $n \times 1$  matrix, the system of equations  $AX = B$  has a unique solution.
7. If  $A$  and  $B$  are two square matrices of the same order, then prove that  $AB$  and  $BA$  have the same characteristic roots.
8. Write short notes on the signature and index of a real quadratic form.

### Section C

Answer any **THREE** questions ( $3 \times 10 = 30$  Marks)

9. Prove that every Hermitian matrix  $A$  can be written as  $A = B + iC$  where  $B$  is real and symmetric and  $C$  is real and skew-symmetric.
10. Show that every  $m \times n$  matrix of rank  $r$  can be reduced to the form  $\begin{pmatrix} I_r & O \\ O & O \end{pmatrix}$  by a finite chain of  $E$ -operations, where  $I_r$  is the  $r$ -rowed unit matrix.
11. Prove that the system of equations  $AX = B$  is consistent i.e. possesses a solution, if and only if the coefficient matrix  $A$  and the augmented matrix  $[A \ B]$  are of the same rank.

12. State and prove the Cayley-Hamilton theorem.
13. To prove that the number of positive terms in any two normal reductions of a real quadratic form is the same.

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