20PAMCT2004

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(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
M.Sc. Appl Maths - END SEMESTER EXAMINATIONS APRIL - 2024
SEMESTER - II
20PAMCT2004 - Algebra II

Total Duration : 2 Hrs. 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Prove that if the element a $\in$ k is algebraic over F if and only if F(a) is a finite extension of F.
2. If L is an algebraic extension of K and if K is an algebraic extension of F, then prove that L is an algebraic of F.
3. Prove that the minimal polynomial p(x) of a$\in$ K over F is irreducible over F.
4. Defend elaborately that a polynomial of degree n over a field can have atmost n roots in any extension field.
5. Prove that the polynomial f(x) $\in$ F[x] has a multiple root if and only if f(x) and f'(x) have a non- trivial common factors.
6. Illustrate the proof of the statement that, the fixed field of G is a subfield of K.
7. If T $\in$ A(v) is nilpotent, then prove that $\alpha_0 + \alpha_1 T + \ldots\ldots + \alpha_m T^m$ where the $\alpha_i \in F$ is invertible if $\alpha_0 \neq 0$.
8. If T $\in$ A(v) has all its characteristic roots in F, then there is a basis if V in which the matrix of T is triangular.

Section C

I - Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. Prove that the number e is transcendental.
10. Formulate the proof of Remainder theorem after stating it.
11. Summarize a detailed proof for the fact if F is of characteristic zero and if a, b are algebraic over F then there exists an element c $\in$ F(a,b) such that F(a,b) = F(c).
12. Examine that if K is a finite extension of F, then prove that G(k, F) is a finite group and its order, o(G(K,F)) satisfies o(G(K,F)) $\leq$ [K:F].

Contd...

II - Compulsory question $(1 \times 10 = 10$ Marks)

13. Demonstrate the proof for the following statement:
    If there exists a subspace w of v, invariant under T, such that v $=$ v_1 $\oplus$w.

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