SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
M.Sc. Appl. Maths - END SEMESTER EXAMINATIONS APRIL - 2024
SEMESTER - III

20PAMCT3007 - Complex Analysis

Total Duration : 2 Hrs. 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. State and prove Fundamental theorem of algebra.
2. If G is simply connected and f : G $\longrightarrow$ ₵ is analytic in G then prove that f has a primitive in G.
3. State and prove Schwarz's lemma.
4. Prove that, If $|z| \leq 1 and \ \ p \geq 0 \ then \quad |1 - E_p(z)| \leq |z|^{P+1}$
5. State and prove Weierstrass factorization theorem.
6. Show that, If G is a bounded Dirichlet region then for each 'a' in G there is a Green's function on G with singularity at a.
7. Let f be an entire function of finite order λ where λ is not an integer; then prove that f has infinitely many zeros.
8. Prove that, If f is an entire function that omits two values then f is a constant.

Section C

I - Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. If f has an isolated singularity at 'a' then prove that the point z = a is a removable singularity iff $\lim_{z \longrightarrow a} (z - a) f(z) = 0$
10. Let f be a function defined on (0,∞) such that f(x)>0 for all x>0. Suppose that f has the following properties:
    (a) log f(x) is a convex function
    (b) f(x+1) = xf(x) for all x
    (c) f(1) = 1
    Then f(x) = $\Gamma(\mathrm{x})$ for all x.
11. State and prove Harnack's theorem.

Contd...

12. For each α and β, $0 < \alpha < \infty \; and \;\; 0 \leq \beta \leq 1$, there is a constant C($\alpha$, β) such that if f is an analytic function on some simply connected region containing $\overline{B}$(0;1) that omits the values 0 and 1, and such that $|f\,(0)| \leq \alpha; then \;\; |f(z) \leq C\,(\alpha, \;\; \beta)\, for\; |z| \leq \beta$.

II - Compulsory question $(1 \times 10 = 10$ Marks)

13. State and prove Morera's Theorem.

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