SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
M.Sc. Appl Maths - END SEMESTER EXAMINATIONS APRIL - 2024
SEMESTER - IV
20PAMCT4010 - Functional Analysis

Total Duration : 2 Hrs. 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. If M is a closed linear subspace of a normed linear space N and the norm of a coset x+M in the quotient space N/M is defined by $\|x+M\|$=$\inf\{\|x+m\|\,; m \in M\epsilon M\}$, Prove that N/M is a normed linear space.Further,if N is a Banach space,then so is N/M.
2. If M is a closed linear subspace of a normed linear space N and x_0 is a vector not in M, then show that there exists a functional f_0 in N^* such that $f_0\,(M) = 0$ and $f_0(x_0) \neq 0$.
3. State and prove open mapping theorem.
4. If x and y are any two vectors in a Hilbert space then show that $|(x, y)| \leq \|x\|\,\|y\|$
5. Prove that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.
6. If M and N are closed linear subspaces of a Hilbert space H such that M $\perp$N, then prove that the linear subspace M+N is also closed.
7. State and prove Bessel's inequality for finite orthonormal set.
8. Prove that an operator T on H is normal $\Leftrightarrow$ $\|T^*x\| = \|Tx\|$ for every x.

Section C

I - Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. State and prove the closed graph theorem.
10. State and prove the uniform boundedness theorem.
11. Show that there exists a unique vector y in H such that f(x) = (x,y) for every x in H when His a Hilbert space and f is an arbitrary functional in H*.
12. Prove that an operator T on H is unitary $\Leftrightarrow$ it is an isometric isomorphism of H onto itself.

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II - Compulsory question $(1 \times 10 = 10$ Marks)

13. Let N and N' be normed linear spaces and T a linear transformation of N into N'. Prove that the following conditions on T are all equivalent to one another.
    (i) T is continuous.
    (ii) T is continuous at the origin in the sense that $x_n \to 0 \Rightarrow T(x_n) \to 0.$
    (iii) There exists a real number K $\geq$ 0 with the property that $\|T(x)\| \leq$ K$\|x\|$ for every $x \in N$.
    (iv) if S $=\{\ x$: $\|x\| \ \leq \mathbf{1}\ \}$ is the closed unit sphere in N, then its image T(S) is a bounded set in N'.

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