SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
M.Sc. Appl. Maths - END SEMESTER EXAMINATIONS APRIL - 2024
SEMESTER - IV
20PAMET4004 - Calculus of Variations and Integral Equations

Total Duration : 2 Hrs. 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Find the extremals of the functional
   $v\,[y(x),\, z(x)] = \int_0^{\frac{\pi}{2}} \left[y'^2 + z'^2 + 2yz\right]\, dx,$
   $y(0) = 0,\ y(\frac{\pi}{2}) = 1\, z(0) = 0,\, z(\frac{\pi}{2}) = -1$

2. Solve the integral equation
   g(s) = f(s) + $\lambda \int_0^1 e^{s-t}$ g(t) dt.

3. Test for an extremum of a functional $v = \int_{x_0}^{x_1} (y'^2 + z'^2 + 2yz)\, dx$
   Given y(0)=0, z(0)=0, and the point (x_1,y_1,z_1) can move over the plane x=x_1.

4. Invert the integral equation g(s) = f(s) + $\lambda \int_0^{2\pi}$ (sin s cos t)g(t) dt.

5. Write the Fredholm integral equation and Volterra equation of first and second type.

6. Derive Euler's equation.

7. Find the Neumann series for the solution of the integral equation
   g(s) = (1+s)+ $\lambda \int_0^s (s-t)g(t)dt.$

8. Solve the integral equation $f(s) = \int_a^s \frac{g(t)dt}{(cost - coss)^{1/2}}$ 0 $\leq$a $<$ s $<$ b $\leq \pi$

Contd...

Section C

I - Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. State and prove fundamental lemma of calculus of variation.
10. Find the transferability condition in the problem of investigating the functional $v = \int_{x_0}^{x_1}$ F(x, y, z, y', z') dx. for an extremum.
11. Show that the integral equation
    g(s) = f(s) + $(1/\pi) \int_0^{2\pi}$ sin(s+t) g(t) dt Possesses no solution for f(s)=s, but that possesses infinitely many solutions when f(s)=1.
12. Solve the Fredholm integral equation of the second kind
    g(s) = s + $\lambda \int_0^1$ (st^2 + s^2 t) g(t) dt.

II - Compulsory question $(1 \times 10 = 10$ Marks)

13. a) Solve the volterra equation g(s) = 1 + $\int_0^s$ st g (t) dt.

    b) Solve the integral equation $s = \int_0^s \frac{g(t)dt}{(s-t)^{1/2}}$

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