SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
B.Sc.Mathematics - END SEMESTER EXAMINATIONS - NOV'2024
SEMESTER - IV
20UMACT4007 - Vector Calculus and Fourier Transforms

Total Duration : 2 Hrs.30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Find the directional derivative of $f(x, y) = x^2y^3 + xy$ at $(2, 1)$ in the direction of a unit vector which makes an angel of $\pi/3$ with x-axis.
2. Verify Stokes theorem for $\overline{F} = (x^2 + y^2)\bar{i}$ - $2xy\bar{j}$ taken around the rectangle bounded by the lines $x = a$, $y = 0$, $y = b$.
3. State and prove convolution theorem for Fourier transform.
4. Find the Fourier transform of $f(x)$ if
   $f(x) = \begin{cases} 1, & |x| < a; \\ 0, & |x| > a > 0 \end{cases}$
   Deduce that $\int_0^\infty \frac{sint}{t} dt = \frac{\pi}{2}$.
5. Evaluate $\iint_S \overline{F}.\overline{n}\ ds$ where $\overline{F} = yz\bar{i} + zx\bar{j} + xy\bar{k}$ and S is the part of the surface of the sphere $x^2 + y^2 + z^2 = 1$ which lies in the first octant.
6. Using divergence theorem of Gauss evaluate $\int_S \int \overline{F}.\overline{n}\ ds$ where $\overline{F} = x^3\bar{i} + y^3\bar{j} + z^3\bar{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$.
7. Find the fourier transform of $f(x) = xe^{-x^2/2}$.
8. Find the Fourier sine and cosine transform of e^{-ax} $(a > 0)$.

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. Prove that $\bar{f} = (2x + yz)\bar{i} + (4y + xz)\bar{j} - (6z - xy)\bar{k}$ is solenoidal as well as irrotational. Also, find the scalar potential of $\vec{f}$.

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10. Verify Green's theorem in a plane for the integral $\int_C (x-2y)dx + xdy$ taken around the circle $C : x^2 + y^2 = 1$.

11. Let V be the solid region between the paraboloid $z = 4 - x^2 - y^2$ and the xoy plane. Verify the divergence theorem for $\vec{F} = 2z\bar{i} + x\bar{j} + y^2\bar{k}$.

12. Show that the Fourier transform of $f(x) = \begin{cases} a^2 - x^2, & |x| < a \\ 0, & |x| > a \end{cases}$ is $2\sqrt{\frac{2}{\pi}} \left(\frac{sin\, as \, - \, as\, cos\, as}{s^3} \right)$ and hence deduce $\int_0^\infty \frac{sint - t\,cost}{t^3} dt = \frac{\pi}{4}$.

13. Solve the integral equation $\int_0^\infty f(x)\ cos\ sx\ dx = se^{-x}$.

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