SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
B.Sc.Mathematics - END SEMESTER EXAMINATIONS - NOV'2024
SEMESTER - V
20UMACT5009 - Modern Algebra

Total Duration : 2 Hrs.30 Mins.   Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks$)$

1. If H and K are finite subgroups of G of order O(H) and O(K) respectively then show that O(HK)$=\dfrac{O(H)O(K)}{O(H \cap K)}$

2. Prove that N is a normal subgroup of G if and only if gNg^{-1}=N for every g $\in$ G.

3. If G is a group, then show that $A(G)$, the set of all automorphisms of G, is also a group.

4. Prove that a finite integral domain is a field.

5. Let R be a commutative ring with unit element whose only ideals are (0) and R itself, then prove that R is a field.

6. Define ideal. And also prove that if U is an ideal of the ring R, then R/U is a ring and is a homomorphic image of R.

7. Prove that the ideal A = (a_0) is a maximal ideal of the Euclidean ring R if and only if a_0 is a prime element of R.

8. Let R be an Euclidean ring, suppose that for a,b,c $\in$ R, a|bc but (a,b) = 1 then show that a|c.

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks$)$

9. (i) If G is a finite group and a$\in$ G, then show that O(a)|O(G).
   (ii) Prove that if n is a positive integer and a is relatively prime to n,
        then $a^{\phi(n)} \equiv (1 \bmod n)$.

10. State and prove Cayley's theorem for abelian group.

11. If Φ is a homomorphism of R into R' with kernel I(Φ), then show that
    (i) I(Φ) is a subgroup of R under addition.
    (ii) If $a \in$ I(Φ) and $r \in$ R then both ar and ra are in I(Φ).

Contd...

12. If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R if and only if R/M is a field.

13. Prove that every integral domain can be imbedded in a field.

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