SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS) (Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC) Chromepet, Chennai - 600 044. B.Sc.Mathematics - END SEMESTER EXAMINATIONS - NOV'2024 SEMESTER - V 20UMACT5010 - Real Analysis

Total Duration : 2 Hrs.30 Mins.

Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30 \text{ Marks})$

- 1. Prove that a non-decreasing sequence which is bounded above is convergent.
- 2. Show that the sequence $\left\{ \left(1 + \frac{1}{n}\right)^n \right\}_{n=1}^{\infty}$ is convergent.
- 3. Define Cauchy sequence and prove that if $\{s_n\}_{n=1}^{\infty}$ is a Cauchy sequence of real numbers, then $\{s_n\}_{n=1}^{\infty}$ is convergent.
- 4. Discuss conditional convergence and absolute convergence.
- 5. Prove that every subset of R_d is open.
- 6. If $\langle M, \rho \rangle$ be a complete metric space and A is a closed subset of M, then prove that $\langle A, \rho \rangle$ is also complete.
- 7. State and prove Rolle's theorem.
- 8. State and prove the second fundamental theorem of calculus.

Section C

Answer any **THREE** questions $(3 \times 10 = 30 \text{ Marks})$

- 9. Prove that countable union of countable sets is countable.
- 10. State and prove nested-interval theorem.
- 11. If G_1 and G_2 are open subsets of the metric space M, then prove that $G_1 \cap G_2$ is also open.
- 12. State and prove Picard fixed-point theorem.
- 13. If f is continuous on the closed bounded interval [a, b], and if $F(x) = \int_a^x f(t)dt$, $(a \le x \le b)$, then F'(x) = f(x) $(a \le x \le b)$.
