SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
B.Sc.Mathematics - END SEMESTER EXAMINATIONS - NOV'2024
SEMESTER - V
20UMACT5012 - Graph Theory

Total Duration : 2 Hrs.30 Mins.   Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks$)$

1. Prove the following :
   i. Let $G = (V, E)$ be a graph with 'e' edges then $\sum_{v \in V} \deg(v) = 2e$.

   ii. In a graph G the number of odd degree vertices is always even.

2. Give an example of a graph and justify.
   i. Both Euler and Hamiltonian graph.
   ii. Neither Euler nor Hamiltonian graph.

3. Prove that a graph G is a tree if and only if every two vertices of G are connected by a unique path.

4. If any connected plane (p, q) graph $(p \geq 3)$ with r faces then prove that $q \geq \dfrac{3r}{2}$ and $q \leq 3p - 6$. Hence deduce K_5 is non planar.
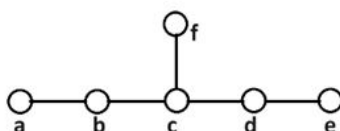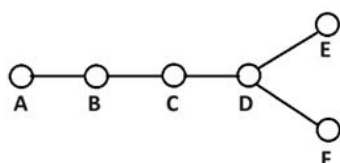
5. Prove that the following statements are equivalent for any graph.
   i. G is 2 colourable.
   ii. G is bipartite.
   iii. Every cycle of G has even length.

6. Define isomorphism of graphs. Check the given two graphs are isomorphic or not?

7. If G is a graph with $p \geq 3$ vertices, and $\delta \geq \dfrac{p}{2}$, then prove that G is Hamiltonian.

Contd...

8. What is a connector problem? Explain how the connector problem can be translated into graph theoretic problem with suitable example.

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks$)$

9. a) If G is a graph with $\delta(G) \geq 2$ then prove that G contains a cycle of length atleast $\delta + 1$.

   b) Prove that a vertex in a connected graph G is a cut vertex if and only if there exists a vertices u and w such that every path connecting u and w contains v.

10. If G is a (p, q) graph $(p \geq 3)$ such that $\deg(u) + \deg(v) \geq p$ for every pair (u, v) of a non adjacent vertices in G then prove that G is a Hamiltonian graph.

11. Prove that a (p, q) graph G is a Bi-partite if and only if it contains of no odd cycle.

12. If G is a connected graph having V, E and F as the set of vertices edges and faces respectively, then prove that $|V| - |E| + |F| = 2$.

13. For any graph G prove that $\chi(G) = \Delta(G) + 1$.

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