SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
B.Sc.Mathematics - END SEMESTER EXAMINATIONS - NOV'2024
SEMESTER - VI
20UMACT6014 - Complex Analysis

Total Duration : 2 Hrs.30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Prove that a set S is open if and only if each point in S is an interior point.
2. Prove that if a function $f(z)$ is continuous and non - zero at a point z_0, then $f(z) \neq 0$ throughout some neighbour hood of that point.
3. Show that $f(z) = |z|^2$ is differentiable at $z = 0$ but not analytic at $z = 0$.
4. Verify whether $u(x, y) = y^3 - 3x^2y$ is a harmonic conjugate and also construct the analytic function $f(z)$.
5. If a function f is analytic throughout a simply connected domain D, then show that $\int_C f(z)dz = 0$, for every closed contour $'C'$ lying in D.
6. State and prove Liouville's theorem.
7. Let f be continuous on a domain D and if $\int_C f(z)dz = 0$, for every closed contour C in D, then show that f is analytic throughout D.
8. Find Laurent's expansion for $f(z) = \dfrac{1}{z(1-z)^2}$ in the domain
   (i) $0 < |z| < 1$ (ii) $0 < |z-1| < 1$.

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. Suppose that the function $f(z) = u(x, y) + i\ v(x, y)$ be defined throughout some $\in$ neighbourhood of a point $z_0 = x_0 + iy_0$, and suppose that
   (i) the first partial derivatives u_x, u_y,v_x, v_y exists everywhere in the neighbourhood.
   (ii) u_x, u_y,v_x, v_y are continuous at (x_0,y_0) and $u_x = v_y$, u_y, = -v_x then $f'(z_0)$ exists.

Contd...

10. Formulate the proof of Cauchy - Riemann equation in polar co ordinate after stating it.
11. State and prove Cauchy integral formula.
12. Show that every non constant polynomial with complex co-efficients has atleast one zero.
13. Formulate the proof of Laurent's theorem after stating it.

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