SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
B.Sc.Mathematics - END SEMESTER EXAMINATIONS - NOV'2024
SEMESTER - I
20UMACT1002 - Differential Calculus

Total Duration : 2 Hrs.30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Find y_n, when $y = \dfrac{x^2}{(x-1)^2(x+2)}$.
2. Show that if $y = sin(msin^{-1}x)$, then $(1-x^2)y_2$ - $xy_1 + m^2y = 0$.
3. Compute the maxima and minima of the function $2x^3$ - $3x^2$ - $36x$ + 10.
4. What is the radius of curvature of the curve $x^4 + y^4 = 2$ at the point (1,1).
5. Find the radius of curvature of the curve $x = a(cost + tsint)$, $y = a(sint - tcost)$.
6. Find the co-ordinates of the centre of curvature of the curve $xy = 2$ at the point (2,1).
7. Prove that the $(p-r)$ equation of the cardioid $r = a(1 - cos\theta)$ is $p^2 = \dfrac{r^3}{2a}$.
8. Predict the asymptotes of $x^3 + 2x^2\ y$ - xy^2 - $2y^3 + 4y^2 + 2xy + y$ - 1 = 0.

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. (a) Compute the n^{th} differential coefficient of $cosx\ cos2x\ cos3x$ (4 marks)

   (b) Prove that if $xy = ae^x + be^{-x}$, then $x\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} - xy = 0$ (2 marks)

   (c) Prove that n^{th} differential coefficient of xe^x is $e^x(n+x)$ (4 marks)

10. Apply Lagrange's method for finding the minimum value of $u = a^3x^2 + b^3y^2 + c^3z^2$, where $\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} = 1$

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11. (a) Prove that the radius of curvature at any point on the cycloid $x = a(\theta + sin\theta)$ and $y = a(1 - cos\theta)$ is $4acos\frac{\theta}{2}$.
    (b) Prove that the radius of curvature of the curve $y = e^x$ at the point where it crosses the y-axis is $2\sqrt{2}$.

12. Show that radius of curvature of the curve $r^n = a^n\ cos\ n\theta$ is $\dfrac{a^n r^{-n+1}}{n+1}$. Also find ρ when n = 2, -2, $1/2$, -$1/2$ and 1.

13. Investigate the asymptote of
    (a) $x^3 + y^3 = 3axy$.
    (b) $(x + y)(x - y)(x - 2y - 4) = 3x + 7y - 6$.

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