

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.

M.Sc.Applicable Mathematics - END SEMESTER EXAMINATIONS - NOV' 2024
SEMESTER - I

20PAMCT1001 - Algebra - I

Total Duration : 2 Hrs. 30 Mins.

Total Marks : 60

Section B

Answer any **SIX** questions ($6 \times 5 = 30$ Marks)

1. With usual notation prove that $n(k) = 1 + p + \dots + p^{k-1}$.
2. Suppose that G is the internal direct product of N_1, N_2, \dots, N_n . Then show that for $i \neq j$, $N_i \cap N_j = (e)$, and if $a \in N_i$, $b \in N_j$ then $ab = ba$.
3. If F is of characteristic 0 and if S and T are in $A_F(V)$, such that $ST - TS$ commutes with S , then $ST - TS$ is nilpotent.
4. Prove that for every prime number p and every integer m there exists a field having p^m elements.
5. The adjoint in Q satisfies
 1. $x^{**} = x$;
 2. $(\delta x + \gamma y)^* = \delta x^* + \gamma y^*$;
 3. $(xy)^* = y^* x^*$;
 for all x, y in Q and all real δ and γ .
6. Let R be a Euclidean ring, then prove that any finitely generated R -module M is the direct sum of a finite number of cyclic submodules.
7. Prove that $\det A = \det(A')$.
8. Show that G is solvable if and only if $G(k) = (e)$ for some integer k .

Section C

I - Answer any **TWO** questions ($2 \times 10 = 20$ Marks)

9. State and prove Sylow's theorem.
10. Prove that "Every finite abelian group is the direct product of cyclic groups".
11. Prove the following for a normal transformation N is
 1. Hermitian if and only if its characteristic roots are real.
 2. Unitary if and only if its characteristic roots are all of absolute value 1.
 3. If N is normal and $AN = NA$, then $AN^* = N^*A$.

Contd...

12. Prove that "Every positive integer can be expressed as the sum of squares of four integers".

II - Compulsory question ($1 \times 10 = 10$ Marks)

13. State and prove Wedderburn theorem
