

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN  
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)  
Chromepet, Chennai - 600 044.

M.Sc.Applicable Mathematics - END SEMESTER EXAMINATIONS - NOV'2024  
SEMESTER - I

**22PAMET1001 - Probability and Distributions**

Total Duration : 2 Hrs. 30 Mins.

Total Marks : 60

**Section B**

Answer any **SIX** questions ( $6 \times 5 = 30$  Marks)

1. Find the moment generating function of Normal distribution.
2. Derive the mean and variance of uniform distribution.
3. Let  $(X, Y)$  be jointly distributed with p.d.f  $f(x, y) = 2, 0 < x < 1, 0 < y < x$  and 0 otherwise. Compute the marginal and conditional density functions of  $x$  and  $y$ . Check whether they are independent.
4. If  $X$  and  $Y$  follow bivariate normal distributions  $N(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$  write joint and marginal density functions of  $X$  and  $Y$ .
5. State and prove the additive property of chi-square distribution.
6. Show that if  $X_n$  converges to  $X$  almost surely then  $X_n$  converges to  $X$  in Probability.
7. Explain the properties of joint distribution function.
8. If  $X_1, X_2, \dots, X_n$  be a random sample from a normal population with mean  $\mu$  and variance  $\sigma^2$  and if the sample mean  $\bar{X}$  and variance  $S^2$  are independent then show that  $\frac{\sqrt{n}(\bar{X} - \mu)}{S} \sim t_{(n-1)}$

**Section C**

I - Answer any **TWO** questions ( $2 \times 10 = 20$  Marks)

9. If  $X$  and  $Y$  are independent random variables with p.m.f  $P(\mu_1)$  and  $P(\mu_2)$  then show that the conditional distribution of  $X$  given  $X + Y$  is binomial.
10. If  $X_1, X_2$  and  $X_3$  are identical and independent random variables with the common density function  $f(x) = e^{-x}, x > 0$  and 0 otherwise and if  $Y_1 = X_1 + X_2 + X_3, Y_2 = \frac{X_1 + X_2}{X_1 + X_2 + X_3}$  and  $Y_3 = \frac{X_1}{(X_1 + X_2)}$  then show that  $Y_1, Y_2$  and  $Y_3$  are independent.

**Contd...**

11. Obtain the marginal mean and variance of bivariate binomial distribution
12. Derive the p.d.f of student 't' distribution.

II - Compulsory question ( $1 \times 10 = 10$  Marks)

13. If  $X_1, X_2, \dots, X_n$  be a random sample from a normal population with mean  $E(X_i) = \mu_1$ ,  $V(X_i) = \sigma_1^2$ ,  $i = 1, 2, \dots, n$  then show that the sum  $S_n = X_1 + X_2 + X_3 + \dots + X_n$  is asymptotically normal with mean  $\mu = n\mu_1$  and variance  $\sigma^2 = n\sigma_1^2$ .

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