SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
M.Sc.Applicable Mathematics - END SEMESTER EXAMINATIONS - NOV'2024
SEMESTER - I
20PAMCT1002 - Real Analysis

Total Duration : 2 Hrs. 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Let $\{E_i\}$ be a sequence of measurable sets, then prove that
   i) If $E_1 \subseteq E_2 \subseteq ...$, we have $m(limE_i) = limm(E_i)$.
   ii) If $E_1 \supseteq E_2 \supseteq ...$, and $m(E_i) < \infty$ for each i,
   then we have $m(limE_i) = limm(E_i)$.
2. State and prove Lebesque's dominated convergence theorem.
3. State and prove Cauchy criterion for uniform convergence.
4. If X is a complete metric space and if ϕ is a contraction of X into X then prove that there exist one and only one $x \in X$ such that $\phi(x) = x$.
5. Prove that the exponential function e^x in R^1
   i. is continuous and differentiable for all x.
   ii. $\lim\limits_{x\to\infty} x^n e^{-x} = 0$ for every n.
6. Let $\{\phi_n\}$ be orthonomal on [a,b] and if $f(x) \sim \sum\limits_{n=1}^{\infty} c_n\phi_n(x)$. Then prove that $\sum\limits_{n=1}^{\infty} |c_n|^2 \leq \|f\|^2$ and $\lim\limits_{n\to\infty} c_n = 0$.
7. Let α be monotonically increasing on $[a,b]$. Suppose $f_n \in \Re(\alpha)$ on [a,b] for n = 1,2,3,..... and $f_n \to f$ uniformly on $[a,b]$. Then prove that $f \in \Re(\alpha)$ on [a,b] and $\int\limits_a^b f d\alpha = \lim\limits_{n\to\infty} \int\limits_a^b f_n d\alpha$.
8. a. Show that $m*(E) = 0$, then E is measurable.
   b. Show that $\int\limits_1^{\infty} \frac{dx}{x} = \infty$.

Contd...

Section C

I - Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. Prove that Lebesque outer measure of an interval equals to its length.
10. State and prove Stone-Weierstrass theorem.
11. State and prove inverse function theorem.
12. Suppose f and g are Riemann -Integrable functions with period 2π and $f(x) \sim \sum_{-\alpha}^{\infty} c_n e^{inx}$, $g(x) \sim \sum_{-\alpha}^{\infty} \gamma_n e^{inx}$ Then prove that

    i. $\lim_{N \to \infty} \frac{1}{2\pi} \int_{-\pi}^{\pi} |f(x) - S_N(f(x))|^2 dx = 0.$

    ii. $\frac{1}{2\pi} \int_{-\pi}^{\pi} |f(x)|^2 dx = \sum_{-\infty}^{\infty} |c_n|^2.$

II - Compulsory question $(1 \times 10 = 10$ Marks)

13. State and prove Fatou's lemma.

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