SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
M.Sc.Applicable Mathematics - END SEMESTER EXAMINATIONS - NOV' 2024
SEMESTER - II
20PAMCT2004 - Algebra - II

Total Duration : 2 Hrs. 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. If L is an algebraic extension of K and if K is an algebraic extension of F, then prove that L is an algebraic extension of F.
2. Show that a polynomial of degree n over a field can have at most n roots in any extension field.
3. For any$f(x), g(x) \in F[x]$ and any $\alpha \in F$, then prove that
   i) $(f(x) + g(x))' = f'(x) + g'(x)$
   ii) $(af(x))' = af'(x)$
   iii) $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$
4. Prove that G(K,F) is a finite group and its order o(G(K,F)) satisfies o(G(K,F)) $\leq$ [K:F] where K is finite extension of F.
5. If V is finite - dimentional over F and if T A(V) has all its characteristic roots in F, then prove that T satisfies a polynomial of degree n over F.
6. The polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a nontrivial common factor Justify.
7. If $T \in A(V)$ has all its characteristic roots in F, then prove that there is a basis of V in which the matrix of T is triangular.
8. Show that the polynomial $x^2 + x$ +4 is irreducible over the field F of integers modulo 11.

Section C

I - Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. Prove that the number $'e'$ is transcendental.
10. Prove if p(x) is irreducible in F[x] and if v is a root of p(x), then F(v) is isomorphic to F'(w) where w is a root of p'(t): more over, this isomorphism can so be chosen that
    i) $\nu\sigma = w$ ii) $\alpha\sigma = \alpha \ forevery \ \alpha \in F$

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11. If F is of characteristic 0 and if a, b, are algebraic over F, then prove there exists an element $c \in F(a,b)$ such that $F(a,b) = F(c)$.

12. If K is a field and if $\sigma_1, \sigma_2, ..., \sigma_n$ are distinct automorphisms of K, then show it is impossible to find elements $a_1, a_2, ..., a_n$, not all 0, in K such that $a_1\sigma_1(u) + a_2\sigma_2(u) + ... + a_n\sigma_n(u) = 0$ for all $u \in K$.

II - Compulsory question $(1 \times 10 = 10$ Marks)

13. If $T \in A(V)$ is nilpotent of index of nilpotent n, then prove that there exists a basis W of V, invariant under T, such that $V = V_1 \oplus W$, Where V, is the subspace of V spanned by v, vT,...,vT^{n-1}

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