SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
M.Sc.Applicable Mathematics - END SEMESTER EXAMINATIONS - NOV' 2024
SEMESTER - II

20PAMCT2005 - Topology

Total Duration : 2 Hrs. 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Let X be a complete metric space, and let Y be a subspace of X. Prove that Y is complete $\Leftrightarrow$ it is closed.
2. State and Prove Cauchy's inequality.
3. Prove that every separable metric space is second countable.
4. Show that any continuous mapping defined on a compact metric space into a metric space is uniformly continuous.
5. Prove that a one-to-one continuous mapping defined on a compact space onto a Hausdorff space is a homeomorphism.
6. State and prove Cantor's intersection theorem.
7. Show that every compact metric space has the Bolzano-Weierstrass property.
8. Define topological space and give the topology for the set $\{$a,b,c$\}$. Define discrete topology.

Section C

I - Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. Let X be a metric space. Let Y be a complete metric space, and let A be a dense subspace of X. If f is a uniformly continuous mapping of A into Y. Show that f can be extended uniquely to a uniformly continuous mapping g of X into Y.
10. The set R^n of all n-tuples $x = (x_1, x_2, x_3, ..., x_n)$ of real numbers is a real Banach space with respect to coordinatewise addition and scalar multiplication and the norm defined by $\|x\| = \left(\sum_{i=1}^{n} |x_i|^2\right)^{1/2}$. Show that R^n is a Banach space.
11. State and prove Heine - Borel Theorem
12. Define normal space and show that every compact Hausdorff space is normal.

II - Compulsory question $(1 \times 10 = 10$ Marks)

13. State and Prove Tychonoff's theorem

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