

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.

M.Sc.Applicable Mathematics - END SEMESTER EXAMINATIONS - NOV'2024

SEMESTER - III

20PAMCT3008 - Differential Equations

Total Duration : 2 Hrs. 30 Mins.

Total Marks : 60

Section B

Answer any **SIX** questions ($6 \times 5 = 30$ Marks)

1. Solve: $x''(t) + k \sin x(t) = 0$; where k is a constant with initial conditions $x(0) = \pi/6$ and $x'(0) = 0$.
2. If P_n is a Legendre polynomial, then prove that $\int_{-1}^1 p_n^2(t) dt = \frac{2}{2n+1}$.
3. Solve: $x'_1 = 5x_1 - 2x_2$; $x'_2 = 2x_1 + x_2$.
4. Determine e^{At} when $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$.
5. Solve: IVP $x' = x^2$; $x(0) = 1$.
6. Solve: $z(xp - yq) = y^2 - x^2$.
7. Use Jacobi's method to solve $p^2x + q^2y = z$.
8. If $u = f(x + iy) + g(x - iy)$, where f and g are arbitrary functions, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

Section C

I - Answer any **TWO** questions ($2 \times 10 = 20$ Marks)

9. Determine the fundamental matrix $\varphi(t)$ for $x' = A(t)x$ where $A = \begin{pmatrix} 3 & -2 \\ -2 & 3 \end{pmatrix}$.
10. State and prove Picard's Theorem.
11. Find the characteristics of the equation $pq = z$ and determine the integral surface which passes through the parabola $x = 0, y^2 = z$.
12. Reduce the equation $(n-1)^2 \frac{\partial^2 z}{\partial x^2} - y^{2n} \frac{\partial^2 z}{\partial y^2} = ny^{2n-1} \frac{\partial z}{\partial y}$ to canonical form and find its general solution.

Contd...

II - Compulsory question ($1 \times 10 = 10$ Marks)

13. Derive the Bessel function of order p,

$$J_p(t) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k! \Gamma(p+k+1)} \left(\frac{t}{2}\right)^{p+2k}; t > 0.$$
