

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN  
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)  
Chromepet, Chennai - 600 044.

B.Sc.AI - END SEMESTER EXAMINATIONS - NOV'2024  
SEMESTER - I

**22UAIAT1001 - Allied Mathematics - I**

Total Duration : 2 Hrs.30 Mins.

Total Marks : 60

**Section B**

Answer any **SIX** questions ( $6 \times 5 = 30$  Marks)

1. Show that  $\frac{1}{3!} + \frac{2}{5!} + \frac{3}{7!} + \frac{4}{9!} + \dots = \frac{1}{2e}$
2. If the roots of  $x^3 + px^2 + qx + \lambda = 0$  are in G.P., show that  $\lambda p^3 = q^3$
3. Show that  $-2^5 \sin^6 \theta = \cos 6\theta - 6\cos 4\theta + 15\cos 2\theta - 10$ .
4. Find the maximum or minimum values of the function  
 $f(x, y) = 2(x^2 - y^2) - x^4 + y^4$ .
5. Prove that the following matrix is unitary  $\begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}$ .
6. Increase the roots of the equation  $x^4 + 12x^3 + 56x^2 + 120x + 91 = 0$  by 3 and hence solve the equation.
7. If  $\tan \frac{x}{2} = \tan h \frac{x}{2}$ . show that  $\cos x \cosh x = 1$ .
8. Show that the radius of curvature at any point of the cardioids  
 $r = a(1 + \cos \theta)$  is  $\frac{4a}{3} \cos \frac{\theta}{2}$ . Deduce that  $\frac{\rho^2}{r}$  is a constant.

**Section C**

Answer any **THREE** questions ( $3 \times 10 = 30$  Marks)

9. i) Given the following values for x and y

|          |   |    |    |     |     |   |
|----------|---|----|----|-----|-----|---|
| <b>x</b> | 0 | 1  | 2  | 3   | 4   | 5 |
| <b>y</b> | 3 | 12 | 81 | 200 | 100 | 8 |

Find  $\Delta^5 y_0$ .

- ii) Find the missing term from the following data

|          |   |    |    |    |    |    |
|----------|---|----|----|----|----|----|
| <b>x</b> | 0 | 5  | 10 | 15 | 20 | 25 |
| <b>y</b> | 7 | 11 | 14 | -  | 24 | 32 |

Contd...

10. Using Cayley – Hamilton theorem, find  $A^4$  given that  $A = \begin{bmatrix} 2 & -2 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$ .
11. If the sum of the two roots of the equation  $x^4 + px^3 + qx^2 + rx + s = 0$  is equal to the sum of the other two, then prove that  $p^3 + 8r = 4pq$ .
12. If  $\tan(\theta + i\phi) = \cos\alpha + i\sin\alpha$  then show that
- i)  $\theta = \frac{n\pi}{2} + \frac{\pi}{4}$       ii)  $\phi = \left(\frac{1}{2}\right) \log \tan \left[\left(\frac{\pi}{4}\right) + \left(\frac{\alpha}{2}\right)\right]$
13. If  $y = \sin(m\sin^{-1}x)$ , then prove that
- i)  $(1 - x^2) y_2 - xy_1 + m^2 y = 0$ .
- ii)  $(1 - x^2) y_{n+2} - (2n + 1)xy_{n+1} - (n^2 - m^2)y_n = 0$ .

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