Electricity
and Magnetism 2

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NOTICE

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I. Electricity and Magnetism 2

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II. Prerequisite Course or knowledge

In order to be able to complete this course, you must have successfully passed the magnitude Electricityand Magnetism 1 and you must have a clear understanding of the following notions:

Ohm's Law;

Kirchoff's laws;

Effect of an electric field on a charged particle

Effect of a magnetic field on a moving charged particle

Magnetic flux

Ampere's theorems

III. Time

You will need approximately 120 hours to complete the entire course, divided into the following sections:

1. Circuits in alternative current- Bridges in alternative current   **30 hours**

2. Magnetic fields and charged particles – Inductance   **30 hours**

3. Magnetic properties of matter   **30 hours**

4. Maxwell's equations – electromagnetic waves   **30 hours**

IV. Material

You will need the following materials: Computer with internet connection, printer, Microsoft office, MathType software.

V. Module Rationale

Electromagnetism is a part of physics of which the applications in ordinary, day-to-day life are countless and fascinating. Among just a few, we can mention television, radio and mobile telephone systems which use electromagnetic waves in order to function. This magnitude will help the learner acquire a few basic and indispensable notions in order to understand this field better. In addition, it will allow the learner to reinforce his/her pedagogical competencies with a view to teaching this discipline.

VI. Content

6.1   Overview

The first part of this magnitude will examine alternative powered circuits, circuits which display diverse combinations of resistance, inductance and capacity.

The second part will examine the momentum of charged particles in both electric and magnetic fields. We will describe and explain J.J. Thompson's experiment, the Hall effect and the principle of accelerating particles such as cyclotrons and synchrotrons. Next we will study the electromagnetic induction phenomenon, Lenz's law, auto- and mutual induction phenomena and their resulting applications. The third section will describe and analyze the properties of the three classes of magnetic substances: the paramagnetic, the ferromagnetic and the diamagnetic. The last part of the magnitude explains the four fundamental equations (Maxwell's equations) which underlie electromagnetism and examine electromagnetic waves, their polarization and the energy they transport.
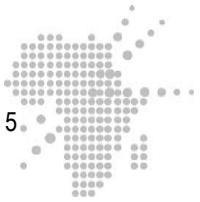
6.2   **Outline**

The magnitude consists of four parts which progress as follows:

1. Circuits in alternating current – Bridges in alternating current
2. Magnetic fields and charged particles – Inductance
3. Magnetic properties of matter
4. Maxwell's equations – electromagnetic waves

6.3   **Graphic Organizer**

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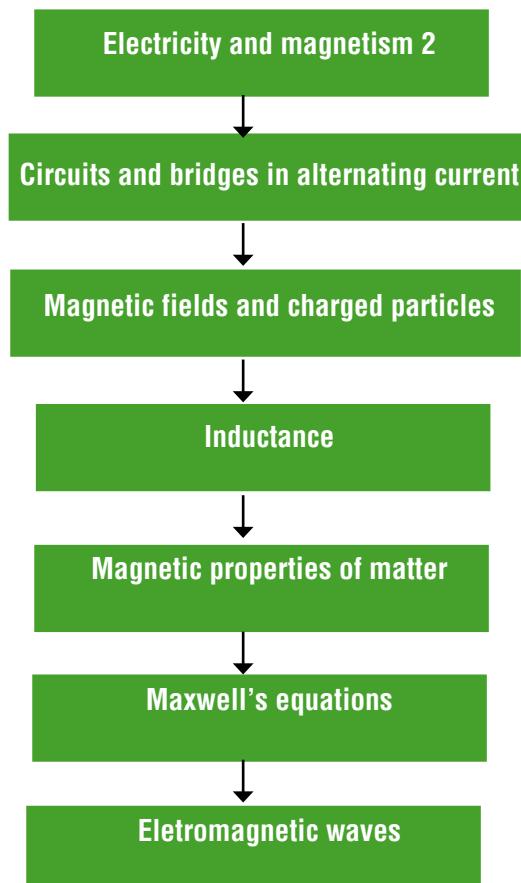
VII. General Objectives

The learner should have an understanding of the following:

- The phase relation between the tension applied to the limits of a circuit in alternative voltage and its current
- The notion of impedance
- The resonance phenomenon in an RLC circuit series
- The Hall effect
- The principle of a cyclotron
- Lenz's law
- The auto-induction phenomenon
- The mutual induction phenomenon
- The magnetic properties of matter
- Maxwell's equations
- The phenomenon of energy transmission through an electromagnetic wave
- The notion of polarization of an electromagnetic wave
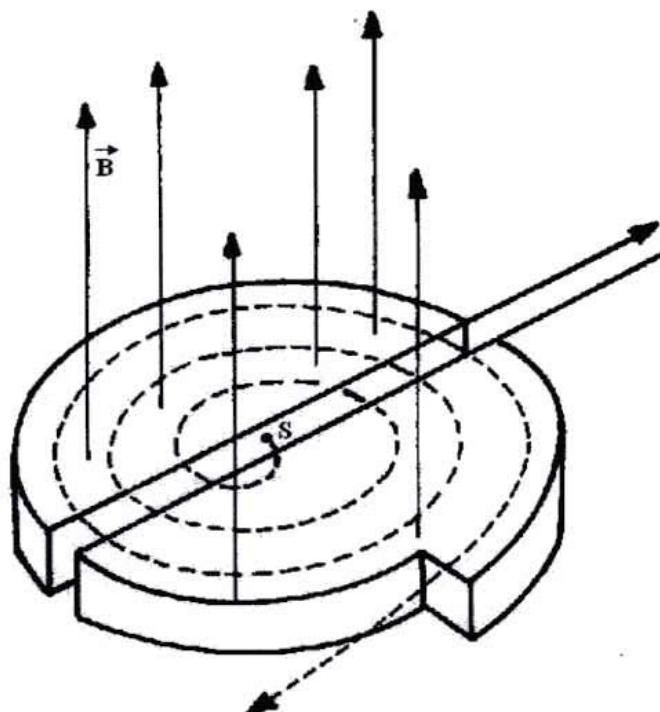
VIII. Specific Learning Objectives

Unit	Learning Objective
1. Circuits in Alternative current - Bridges in Alternative current **30 hours**	- Determine the outcome of two or more vectors - Establish the phase difference between tension according to Fresnel's construction and current in a circuit with resistance - Establish the phase difference between tension and current in a coiled circuit - Establish the phase difference between tension and current in a circuit with condenser - Establish the phase difference between tension and current in an RLC circuit - Calculate the impedance of an RLC circuit - Calculate the quality factor of an RLC circuit series - Calculate the band-width of an RLC circuit series - Calculate the frequency resonance - Distinguish efficient and instant values of the tension and the intensity of the electric current - Calculate the instant value of the intensity of a current passing through a circuit - Calculate the instant value of the tension at the limits of a circuit - Recall the relation between the efficient value of the intensity and amplitude of the current passing through a circuit - Recall the relation between the efficient value and the amplitude of tension at the limits of a circuit
2. Magnetic field and charged particle - Inductance **30 hours**	- Recall Lenz's law - Apply Lenz's law - Describe the Hall effect - Interpret the Hall effect - Determine the relation $\dfrac{e}{m}$ - Calculate the radius of a charge carrier's trajectory in a cyclotron - Calculate the inductance of a circuit - Calculate the mutual coefficient induction of two circuits

3. Magnetic properties Of matter: **30 hours**	- Describe the phenomenon of paramagnetism - Interpret the phenomenon of paramagnetism - Describe the phenomenon of diamagnetism - Interpret the phenomenon of diamagnetism - Describe the phenomenon of ferromagnetism - Interpret the phenomenon of ferromagnetism - Recall the relationship between a magnetic induction field's vectors, magnetic activity and magnetization - Recall the Hysterisis phenomenon - Calculate the magnetization of a magnetic substance
4. Maxwell's Equations - Electromagnetic waves: **30 hours**	- Apply Maxwell's equations - Determine the polarization of a plane electro-magnetic wave - Determine the Poynting vector associated with a plane electromagnetic wave - Determine the energy density of a plane electro-magnetic wave - Determine the intensity of a plane electro-Magnetic wave - Determine the radiation pressure of electro-magnetic wave

IX. Teaching and Learning Activities

Pre-Assesment Evaluation

Prerequisite Testing

Electricityand magnetism 2

Learning Objective : This test will allow you to verify your knowledge of previous material in order to be able to complete this magnitude. It is therefore compulsory for all learners.

Questions

Check the appropriate box in all of the following

Exercise 1

Consider the circuit below :
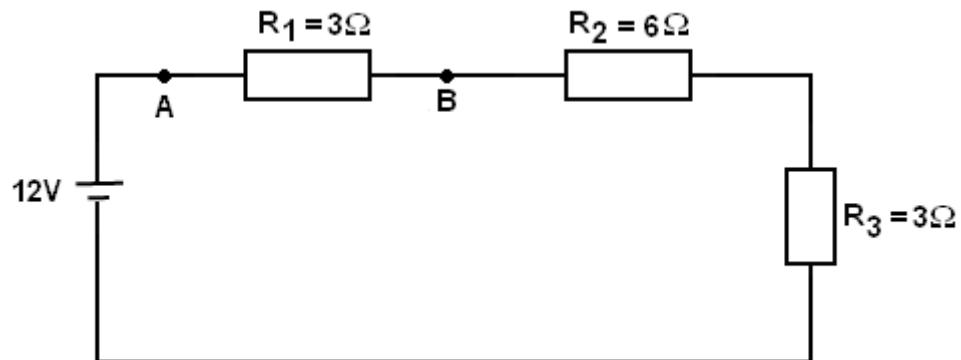
The intensity of the current I which passes through the circuit and the difference in potential of V_{AB} at the limits of R_1 are :

1 : I = 2A and V_{AB} = 6 V

2 : I = 1A and V_{AB} = 3 V

3 : I = 1,33A and V_{AB} = 3,99 V

Exercice 2 :

Consider the circuit below:
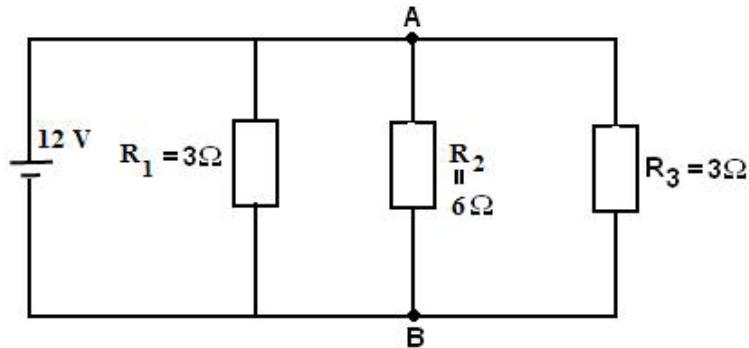
The difference in potential V_{AB} at the limits of resistance R_2 and the intensity of the current I_2 which passes through this resistance are :

1 : $V_{AB} = 2$ V and $I_2 = 12$A

2 : $V_{AB} = 4$ V and $I_2 = 24$A

3 : $V_{AB} = 12$ V and $I_2 = 2$A

Exercice 3 :

Consider the circuit below:
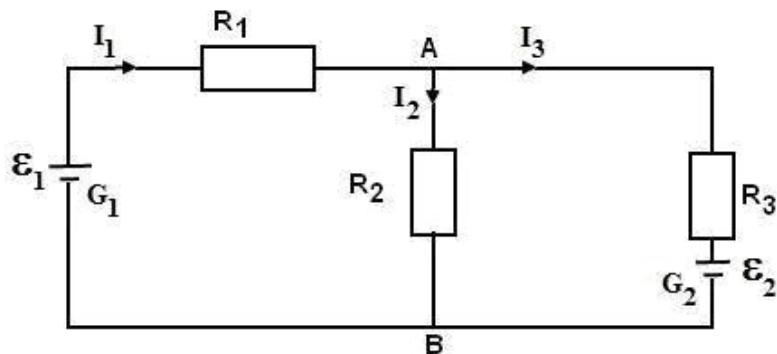
G_1 and G_2 are two generators with f.e.m. ε_1 and ε_2. $G_1AG_2BG_1$ are linked as follows:

$G_1 \rightarrow$ A $\rightarrow$ G_2 $\rightarrow$ B $\rightarrow$ G_1: The application of Kirchhoff's law in this link results in the following

1 : $\varepsilon_1 - \varepsilon_2 = R_1 \, I_1 + R_3 \, I_3$

2 : $\varepsilon_1 + \varepsilon_2 = R_1 \, I_1 + R_3 \, I_3$

3 : $\varepsilon_2 - \varepsilon_1 = R_1 \, I_1 + R_3 \, I_3$

Exercise 4
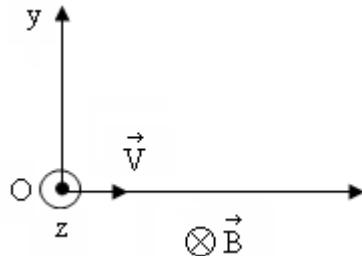
Consider the figure above. An electron with a horizontal speed $\vec{V}$ penetrates a region comprised of an electric field $\vec{E}$ and a magnetic field $\vec{B}$. B is perpendicular to the direction of $\vec{V}$ as the figure shows. If the electron does not deviate from its initial trajectory, $\vec{E}$ will travel according to the following:

1 : Positively along the y axis

2 : Negatively along the y axis

3 : Negatively along the z axis

Exercise 5

An electron is placed within the infrastructure of a condenser charged plan as the figure indicates. Let us assume that the electron is initially at rest and let us disregard its weight. The force which is applied on the electron is $\vec{F} = e\ \vec{E}$.
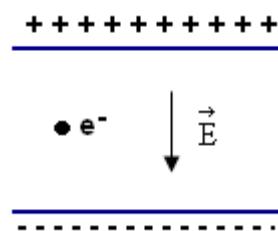
1 : true

2 : false

Exercise 6

In the international system, magnetic flux is measured in Wb

    1 : true

    2 : false

Exercise 7

In the international system, magnetic inductance $\vec{B}$ is measured in Wb

    1 : true

    2 : false

Exercise 8

A spiral with an area equal to 5,0cm² is placed in a magnetic field $\vec{B} =$

$(40\ \vec{i}\ -\ 18\ \vec{k})\ 10^{-4}\ T$ and parallel to the xy plane. The flux which passes this spiral is 900 nWb

    1 : true

    2 : false

Exercise 9

A spiral with section $\vec{S} = 30\ \vec{k}\ m^{2}$ is placed in a magnetic field $\vec{B} = 2\ x10^{-2}\ \vec{i}\ T$

. The flux of $\vec{B}$ through this spiral is 60Wb.

    1 : true

    2 : false

Exercise 10

A circular coil with a radius of 3cm, comprising 50 spirals is placed in a uniform magnetic field B = 0,10 T and parallel in the normal at its section. The magnetic flux which passes through this coil is $2,83 \times 10^{-4}$ Wb

    1 : true
    2 : false

Exercise 11

Ampere's law states that : $\oint_{(C)} \vec{B} \cdot \vec{dl} = 0$

    1 : true
    2 : false

Exercise 12

A current of intensity I passes through a rectilinear, lineic, and infinite conductor. The magnitude of the magnetic field at a point situated at distance r of

the wire is : $B = \dfrac{\mu_0\, I}{2\,\pi\,r}$

    1 : true
    2 : false

Exercise 13

A current of intensity I passes through a cylindrical conductor C_1 of infinite length and with radius a. The conductor is placed in a hollow metallic cylinder C_2 with the same axis. The interior radius of C_2 is b, and the exterior radius is c (b < c).

The current of intensity I circulating in C_1 travels with the same intensity in C_2. The magnetic field created by this system is naught at a point situated at distance r of the axis such that r > c.

    1 : true
    2 : false

Exercise 14

Let us return to the previous exercise (exercise n°13). The field at a point situated at a distance r of the axis such that a < r < b is naught

  1 : true
  2 : false

Exercise 15

Let us return to exercise n° 13. The field at a point situated at distance r such that r < a is naught.

  1 : true
  2 : false

ANSWER KEY

Solution to exercise 1

The correct answer is « 2 : I = 1A and V_{AB} = 3 V »

Here we are presented with series in a grouping. The equivalent resistance of the circuit is $R = R_1 + R_2 + R_3 = 12\ \Omega$. The current which passes through the circuit is $I = \dfrac{12\ V}{12\ \Omega} = 1\ A$ and the difference in potential at the limits of R_1 is $V_{AB} = R_1\ I = (3\Omega)(1A) = 3V$.

Solution to exercise 2 :

The correct answer is « 1 : V_{AB} = 12 V and I_2 = 2A ».

The resistances are connected in a parallel way to the limits of the generator f.e.m. 12V. The difference of the potential V_{AB} is therefore 12 V and the current I_2 is solved as following:

$$I_2 = \frac{V_{AB}}{R_2} = \frac{12V}{6\Omega} = 2A$$

Solution to exercise 3 :

The correct answer is « 1 : $\varepsilon_1 - \varepsilon_2 = R_1 I_1 + R_3 I_3$ »

In the chain $G_1 A G_2 B G_1$ linked in the direction $G_1 \circledR A \circledR G_2 \circledR B \circledR G_1$, we come out of generator G_1 by its positive limit and from generator G_2 by its negative limit. The two resistances R_1 and R_3 are crossed in the direction of the current. Therefore we have:

$$\varepsilon_1 - \varepsilon_2 - R_1 I_1 - R_3 I_3 = 0 \Rightarrow \varepsilon_1 - \varepsilon_2 = R_1 I_1 + R_3 I_3 .$$

Solution to exercise 4 :

The answer is « 2 : negatively along the y axis »

The force $\vec{F} = -e\,\vec{E} - e\,\vec{v} \wedge \vec{B}$ is applied to the electron. Since it stays on its initial trajectory $\Rightarrow e\,\vec{E} = -e\,\vec{v} \wedge \vec{B}$. it follows that $\vec{E} = -\,\vec{v} \wedge \vec{B}$. $\vec{E}$ is perpendicular to the defined by $\vec{v}$ et $\vec{B}$ therefore it follows the y axis and because of the 'minus' sign it follows the y axis negatively.

Solution to exercise 5 :

The correct answer is « 2 : false »

The electron is negatively charged therefore the force applied to it is : $\vec{F} = -e\,\vec{E}$

Solution to exercise 6 :

The answer is « 1 : true »

The magnetic flux is equal to $\Phi = \vec{B} \cdot \vec{S}$, but the field B is in $\dfrac{Wb}{m^2}$ and the surface S is m^2

Solution to exercise 7 :

The Weber is the magnetic flux unity and the unity of the magnetic field is the Tesla or the Weber in square meters. The correct answer is therefore « 2 : false »

Solution to exercise 8 :

The flux which crosses the spiral is given by $\Phi = \vec{B} \cdot \vec{S}$. The vector $\vec{S}$ here follows

z, since the spiral is placed in parallel with the xy plan. We express the vector $\vec{S}$

therefore as : $\vec{S} = (5,0) \times 10^{-4} \ \vec{k} \ m^2$. We thus obtain :

$$\Phi = \vec{B} \cdot \vec{S} = (40 \ \vec{i} - 18 \ \vec{k}) \ 10^{-4} \cdot (5,0) \times 10^{-4} \ \vec{k} = - \ 900nWb.$$

The correct answwser is therefore « 2 : false ». (Please note the 'minus' sign)

Solution to exercise 9 :

The correct answer is « 2 : false »

The flux is $\Phi = \vec{B} \cdot \vec{S}$. Since $\vec{B}$ et $\vec{S}$ are perpendicular between themselves, $\Phi = 0$

Solution to exercise 10 :

Let Φ_1 be equal to the flux passing through a spiral. The flux passing through the coil is equal to $\Phi = 50 \ \Phi_1$. The value

$(2,83 \times 10^{-4}Wb)$ is equal to the flux Φ_1. Therefore, $\vec{B}$ et $\vec{S}$ being parallel to each other, the flux Φ_1 is equal to:

$$\Phi_1 = B \ S = (0,10T) \ (3,14) \ (0,03)^2 m^2 = 2,83 \times 10^{-4}Wb$$

The correct answer is therefore « 2 : false ».

Solution to exercise 11 :

Ampere's law states that : the circulation of the magnetic field $\vec{B}$ along a closed curve (C) is equal to the product by μ_0 of the algebraic sum of the intensities I of the currents which pass through a surface Σ defined by (C). The sign I is linked to the direction along (C). An example is given below.
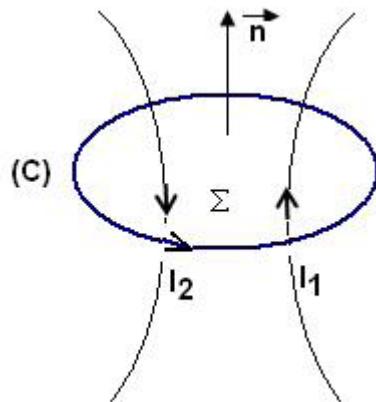
$$\oint_{(C)} \vec{B}.\vec{dl} = \mu_0 (I_1 - I_2)$$

The correct answer here is therefore « 2 : false ».

Solution to exercise 12 :

Let us apply Ampere's theorem. We take as closed curve (C) a line of field : a circle having the wire as axis and radius r.

We have : $\oint_{(C)} \vec{B} \cdot \vec{dl} = \mu_0 \ I$. Here $\vec{B}$ et $\vec{dl}$ are parallel. The magnitude $\vec{B}$ is the same along all points of (C) $\Rightarrow B \oint_{(C)} dl = \mu_0 \ I$

After the integration, we get :

$$B \ (2\pi \ r) = \mu_0 \ I \Rightarrow \ B = \frac{\mu_0 \ I}{2 \ \pi \ r}$$

The answer is therefore « 1 : true »

Solution to exercise 13

Let us apply Ampere's theorem on the circumference (C) of radius $r > c$. The algebraic

sum of the currents which intertwine (C) is here naught, such that: $\oint_{(C)} \vec{B} \cdot \vec{dl} = 0$
$\Rightarrow B = 0$

The answer is therefore « 1 : true »
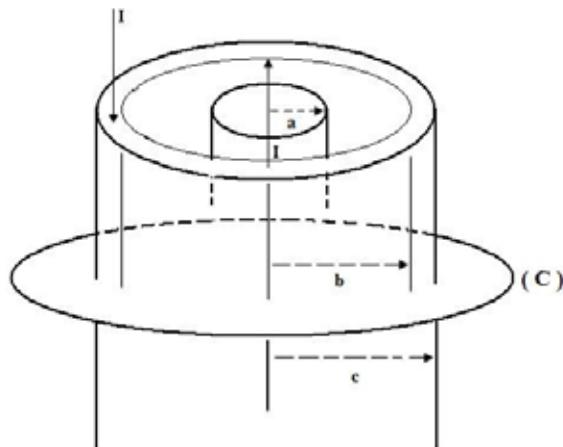
Solution to exercise 14

Let us apply the theorem on the circumference (C) of radius r such that $a < r < b$.

The whole current I is intertwined by (C), therefore, we get $\oint_{(C)} \vec{B} \cdot \vec{dl} = \mu_0 I \Rightarrow$
B cannot be naught.
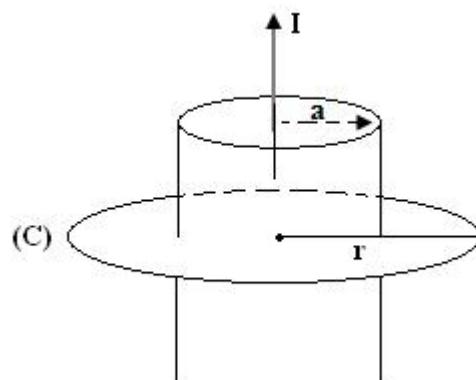
The correct answer here is therefore « 2 : false ».

Solution to exercise 15

The field B is not naught since a fraction of the current I is intertwined by (C).
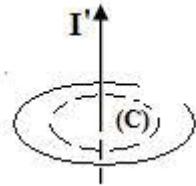
The correct answer is   « 2 : false ».

Pedagogical Comments for Students (100-200 words)

You have at least 75%, your interest in the magnitude is obvious. I encourage you to persevere in your work.

You have between 50% and 75%, your results are very encouraging. So keep up the good work.

You have between 35% and 50%, of course, your results are not perfect. But it seems to me you really have the will to succeed in this field. It's this will which we will have need of. I won't hide it from you, the field you have chosen is really fascinating, but it requires a lot of work. To start with, you will have some catching up to do. It is in this manner that we will be able to succeed.

You have less than 35%, you will have to make a big effort, since in addition to the magnitude you will have to review your preceding courses…

X. Learning Activites

Learning Activity 1.1

Unit 1: Circuits in alternative current

Name of the activity

Phase difference between the tension applied to the limits of a circuit and its current – Impedance of a circuit.

Time it will take : 15 hours

Note: For this activity, if you have **at least ¾ of the points**, you have done very good work, you can continue.

If you have **less than half the points,** you must redo the readings and repeat the exercises.

If you have **more than half the points and less than ¾ of the points**, you have done good work, but you must make more efforts in the future.

Specific Objectives

Following this activity, the learner must be able to:
Represent through Fresnel's construction the phase difference between the tension applied to the limits of a circuit and its current

- Calculate the phase difference between the tension applied to the limits of a circuit and its current
- Establish the expression of an instant current passing through a circuit consisting of diverse combinations of resistance, inductance and capacity
- Calculate the amplitudes of tension
- Calculate the amplitudes of a current passing through a circuit
- Recall the relation describing capacitive reactance
- Recall the relation describing inductive reactance
- Calculate the impedance of a circuit consisting of diverse combinations of resistance, inductance and capacity
- Determine the impedance of a circuit according to Fresnel's representation
- Establish the expression of a complex impedance of a circuit
- Deduct the impedance of a circuit understanding its complex impedance

Outline of the activity

This activity will examine circuits in alternative current consisting of diverse combinations of resistance, inductance and capacitance. You will examine the currents I which pass through these circuits when they are supplied by a sinusoidal e.m.f V. Particular attention will be directed to the oscillations of I and V and to relate them to the phase difference between them. When looking at circuits in alternative current, impedance plays the same role as resistance in continual current and there exists several approaches to find it. You will learn to determine the impedance of a circuit by:

- Considering the relation which links the amplitudes of the tension V and of the current I or the relation which links the efficient value
- Constructing the Fresnel representation which corresponds to it
- Using its complex impedance

Appropriate Readings

RASOLONDRAMANITRA, H. (2007). Circuits à courant alternatif. Madagascar. Université d'Antananarivo. Cours inédit.

Appropriate Resources

1. BRAMAND, P., FAYE, P. et THOMASSIER, G. (1983). Physique- Terminales C et E. Collection Eurin-Gié. Hachette, Paris.
2. BEISER, A.(1979). Physique appliquée : Cours et problèmes.Serie Schaum . Traduction française par LOBENBERG M. McGraw-Hill, Paris
3. BREITHAUPT, J. (2000). New Understanding : Physics for Advanced Level. Fourth Edition, Stanley Thornes Publishers Ltd,England ,
4. FRANCIS W. S., ZEMANSKY, M. W. and YOUNG. H. D. (1987). University Physics. Seventh Edition, Addison-Wesley Publishing Company, USA.
5. FRANCIS W. S., ZEMANSKY, M. W. and YOUNG. H. D. (1974). College Physics. Fourth Edition. Addison-Wesley Publishing Company, USA.
6. HALPERN, A. (1991) Maxi Schaum Physique 2. Traduction française par BABES,V. McGraw-Hill, Paris
7. KALACHNIKOV, S. (1980). Electricité. Traduction française. Editions Mir, Moscou, Réédition :1983
8. NELKON, M. and PARKER, P. (1998). Advanced Level Physics. Seventh Edition, Heinemann, (Oxford ?)
9. RESNICK, R. et HALLIDAY, D. (1979). Electricité et magnétisme, physique 2. Traduit par André Lebel et Claudine Thériault. Editions du Renouveau Pédagogique. Inc. Montréal (Québec) Canada

10. TIPLER, P.-A. and Company. (1999). Physics for Scientists and Engineers. Fourth Edition.   W.H. FREEMAN and company. Worth Publishers Inc., New York, USA

11. TIPLER, P. A. (1982). Physics. Second Edition. Worth Publishers Inc, New York, USA

Useful Links

http://perso.orange.fr/daniel.robert9/tension_alternative0.html
http://perso.orange.fr/daniel.robert9/tension_alternative1.html
http://www.walter-fendt.de/ph11f/accircuit_f.htm
http://www.univ-lemans.fr/enseignements/physique/02/electri/rlcsinus.html

Description of the activity

This activity is comprised of two exercises in the form of MCQ's and of exercises to be corrected subjectively (exercises with more than one answer) which will allow the learner to judge his/her mastery of the study themes: Alternative current circuits – *phase difference between the tension applied to the limits of a circuit and its current* – *impedance,* and outlines of what the learner must retain after each solved exercise. Exercise 1 consists of two parts.

The exercises are compulsory. Each wrong answer to the first two exercises is associated to a remedial exercise allowing the learning to understand the root of his/her mistake.

Formative Evaluation

It is compulsory for the learners to do the exercises as a collaborative team. The group grade is the same for all individual members of the team.

Exercise 1 counts for 60% of the points and exercise 2 counts for 40% of the points.

Exercise 1

Check the right answer.

We have an inductance coil L = 0,3H, with a resistance R = 80W and a capacity condensor C = 50 μF.

a – The coil and the resistance are constructed as a series with a tension $V = V_m$ sinwt with a efficiency value of 120V and a frequency of 60Hz. The impedance of this circuit is :

❑ 113 Ω

❑ 53 Ω

❑ 80 Ω

❑ 138,5 Ω

❑ 96 Ω

❑ 100 Ω

b- The coil, the resistance and the condenser are constructed in series with the source V. The current passing through this circuit is :

❑ I = 1,70 sin (120 πt – 0,6458)

❑ I = 1,70 sin (120 πt + 0,6458)

Exercise 2

A resistance R and an inductnace L are constructed in series at the limits of a source V= V_m sinwt. The complex impedance of this circuit is :

❑ $R + \dfrac{1}{jC\grave{u}}$

❑ $R + jL\omega$

❑ $\dfrac{1}{R} + \dfrac{1}{jL\grave{u}}$

Learning Activities

- <u>consult the objectives</u> to reach in order to identify the essential points in the course readings
- carefully read the « appropriate readings »
- perform the work collaboratively to solve the exercises

The learners are put in groups of activity according to their numbers. Each group designates a delegate. The groups all solve exercise 1 at the same time. After one hour and under the direction of a tutor, the delegates exchange their group's solutions through a chat-room. The other members of the group have access to the chat-room. The chat-room conversation is recorded. The course professor or the tutor correct the answers given in the chat-room after which they provide the answers to exercise 1 in the work space reserved for the learners.

For exercise 2, the groups each solve the problem separately after which the delegates send the group's answer to the course Professor by attached e-mail document. After correcting the different answers, the Professor allows each of the groups to view the corrected work in a space reserved for the learners.

The following diagram outlines the itinerary which we propose in this activity.

Gaining a general understanding by the means of compulsory readings

Exercises with Multiple Choice Questions (MCQ)

↓

Corrections with comments on each choice

↓

A series of complementary exercises

↓

Corrections + a visit to various sites

↓

A summary

↓

An Evaluation of the work

Answer Key

Exercise 1-a

❏ 113 Ω. This is not the correct answer. This value corresponds to the inductive reactance of the inductance coil L = 0,3H connected to the limits of the alternative tension source $U = U_m \sin\omega t$ with frequency 60Hz.

To increase your knowledge and to reinforce your understanding :

- Establish the phase relation between the tension at the limits of the inductance L and the current passing through it.
- Give expression to the inductive reactance and calculate it.

❏ 53 Ω. Wrong answer. This value is the capacitive reactance of the condenser with capacity C = 50μF linked to the source of the tension $U = U_m \sin\omega t$.

To increase your knowledge and to reinforce your understanding :

- Establish the phase relation between the tension at the limits of the condenserr - - and the current that passes through the circuit.
- Give expression to the capacitive reactance and calculate it.

❏ 80 Ω This value corresponds in the case where the circuit contains only resistance R = 80 Ω connected to the limits $U = U_m \sin\omega t$.

To increase your knowledge and to reinforce your understanding :

- Establish the phase relation between the tension at the limits of R and the current passing through it .

❑138,5 Ω. <u>Right answer</u>. It is, indeed, the impedance we are looking for.

To reinforce your understanding :

- Construct Fresnel's representation corresponding to this circuit and deduce its impedance Z and the phase relation between the applied tension U and the current.

❑ 96 Ω. It is the impedance of the circuit below

$U = U_m \sin\omega t$; f = 60Hz.
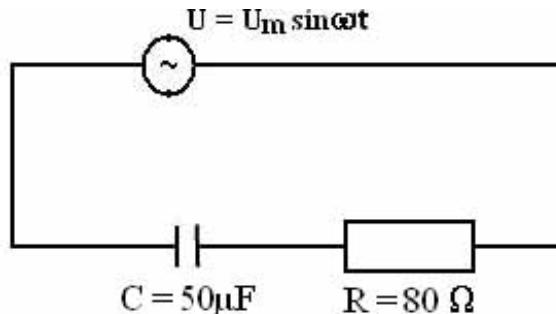
To reinforce your understanding, use Fresnel's representation to establish the expression of its impedance and describe the phase relation beween the applied tension U and the current.

❑ 100 Ω. It is the impedance of the circuit series R-L-C as shown below

$U = U_m \sin\omega t$; f = 60Hz.
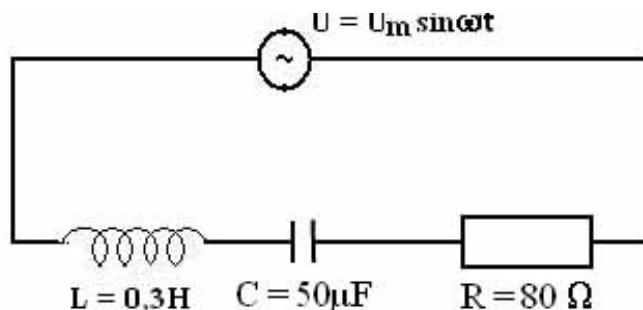
To reinforce your understanding, construct the Fresnel representation which corresponds to this circuit and deduct its impedance Z and the phase difference between the applied tension and the current.

Remedial Exercises

❏113W. Phase relation between the tension at the limits of inductance L and the current passing through it.

$$U = U_m \sin \omega t$$

$$L = 0,3H$$

$$V_L$$
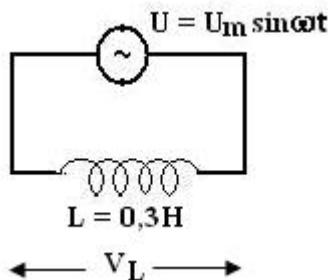
The difference of the instant potential at the limits is:

$$V_L = L \frac{dI_L}{dt} = U_m \sin \omega t$$

I_L is the instantaneous current which passes through the circuit.

When we solve the equation, we get :

$$I_L = - \frac{U_m}{\omega L} \cos \omega t = \frac{U_m}{\omega L} \sin (\omega t - \frac{\pi}{2}) = I_{Lm} \sin (\omega t - \frac{\pi}{2}).$$

The tension is ahead of the current by 90°.

$$I_{Lm} = \frac{U_m}{\omega L} = \frac{U_m}{X_L}$$ (1) is the amplitude of the current. This relation (1) is due to the

form of Ohm's law (U=R1).

$X_L = \omega L$ is called inductive Reactance and acts like a resistance

A.N. $X_L = 2\pi fL = 2\pi \times 60Hz \times 0,3H = 113 \ \Omega$.

53W. Phase relation between the tension at the limits of the condenser and the current which passes through the circuit.
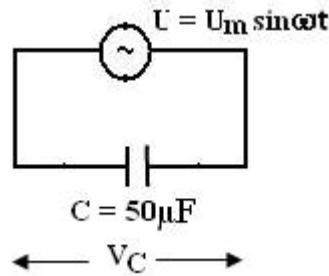
$$V_C = \frac{q}{C} = U_m \sin wt \text{ (d.d.p at the limits of the condensator)}$$

The instantaneous current which passes through this circuit is : $I_C = \dfrac{dq}{dt}$

We get:

$$I_C = \omega C U_m \cos \omega t = \omega C U_m \sin (\omega t + \frac{\pi}{2})$$

The current is ahead of the tension by 90°.

$$I_{Cm} = w C U_m = \frac{U_m}{(\frac{1}{\omega C})} = \frac{U_m}{X_C} \text{ (amplitude of the current).}$$

We call X_C the capacative reactance.

$$\text{A.N. } X_C = \frac{1}{\omega\ C} = \frac{1}{2\pi\ f C} = \frac{1}{2\pi \text{ x } 60Hz \text{ x } 5010^{-6}F} \approx 53\,\Omega.$$

80W. Phase relation between the tension at the limits of the resistance and the current which passes through it.
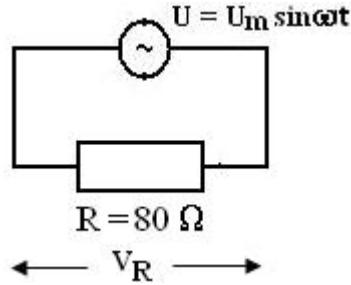
The d.d.p at the limits of this resistance in this case is:

$$V_R = RI_R = U_m \sin\omega t \Rightarrow I_R = \frac{U_m}{R} \quad \text{The current and the tension are in phase.}$$

Or $\quad I_R = I_{Rm} \sin\omega t$ whith $I_{Rm} = \frac{U_m}{R}$

R = 80W according to the equation

138,5 W : Examination of the LR circuit : Fresnel's representation, impedance and phase relation.
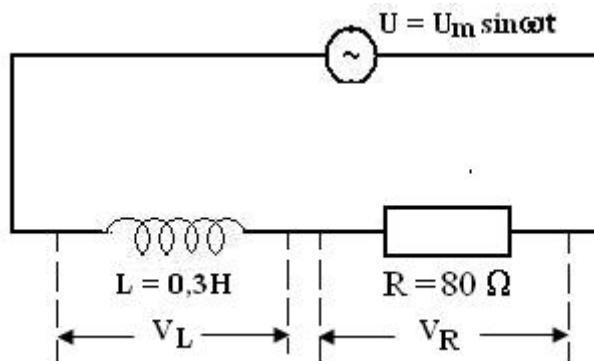
V_L is ahead of the current by 90°.

V_R is in phase with the current.

The Fresnel representation corresponding to this circuit is:
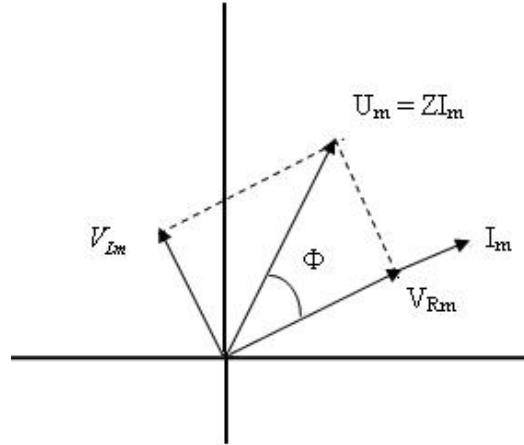
When we use the Pythagorean model, we obtain:

$$U_m^2 = V_{Rm}^2 + V_{Lm}^2$$

But $V_{Rm} = R\,I_m$ and $V_{Lm} = X_L\,I_m$

$$\Rightarrow U_m^2 = I_m^2\,(R^2 + X_L^2)$$

$$U_m = I_m\,\sqrt{R^2 + X_L^2} = I_m Z \text{ with}$$

$$Z = \sqrt{R^2 + X_L^2}$$

A.N. : $Z = \sqrt{(80)^2 + (113)^2} = 138,5\,\Omega$

$$\tan F = \frac{V_{Lm}}{V_{Rm}} = \frac{X_L}{R} = \frac{113}{80} = 1,4125.$$

The current has a delay of $\Phi = 54°,70$ on the applied tension

96 W : Examination of the CR circuit
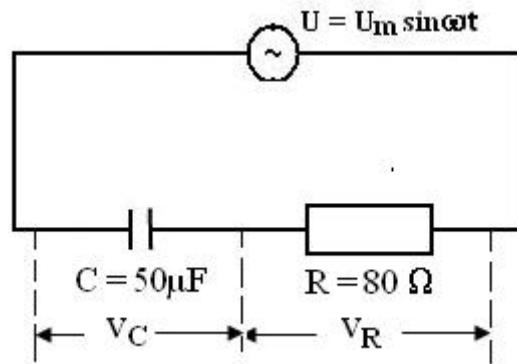
V_C has a delay of 90° on the current

V_R is in phase with the current

Fresnel's representation shows :
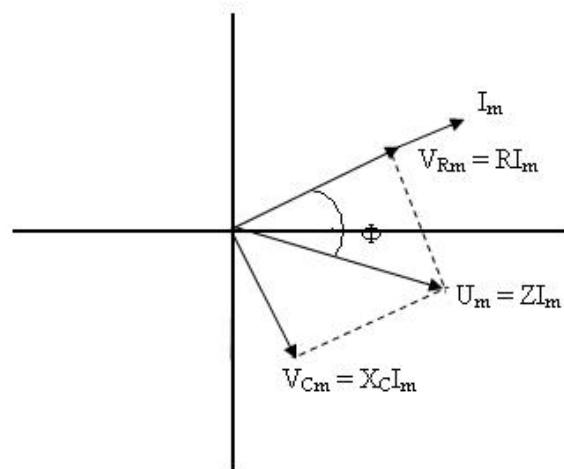
$$U_m^2 = I_m^2 (R^2 + X_C^2)$$

$$U_m = I_m \sqrt{R^2 + X_C^2} = I_m Z \text{ avec}$$

$$Z = \sqrt{R^2 + X_C^2}$$

A.N. :

A.N. : $Z = \sqrt{(80)^2 + (53)^2} \approx 96\Omega$.

$$\tan\Phi = \frac{V_{Cm}}{V_{Rm}} = \frac{X_C}{R} = \frac{53}{80} = 0,6625.$$

The current is ahead by $\Phi = 33°52$ on the applied tension

= 100 W : Examination of the R-L-C series
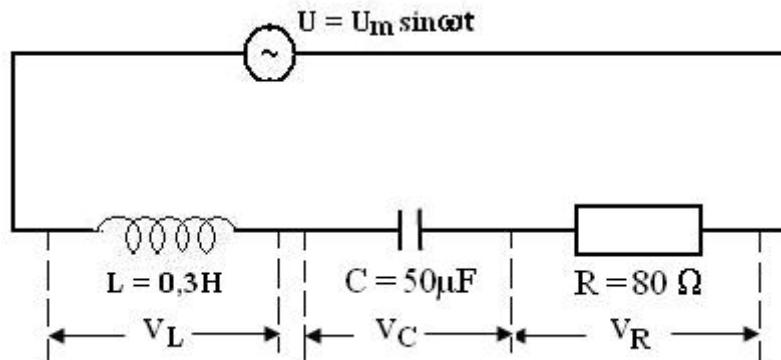
V_L is ahead of the current by 90°

V_C has a delay of 90° on the current.

V_R is in phase with the current

$X_L = 113\ \Omega$ et $X_C = 53\Omega \Rightarrow X_L > X_C$

The Fresnel representation corresponding to this circuit is :
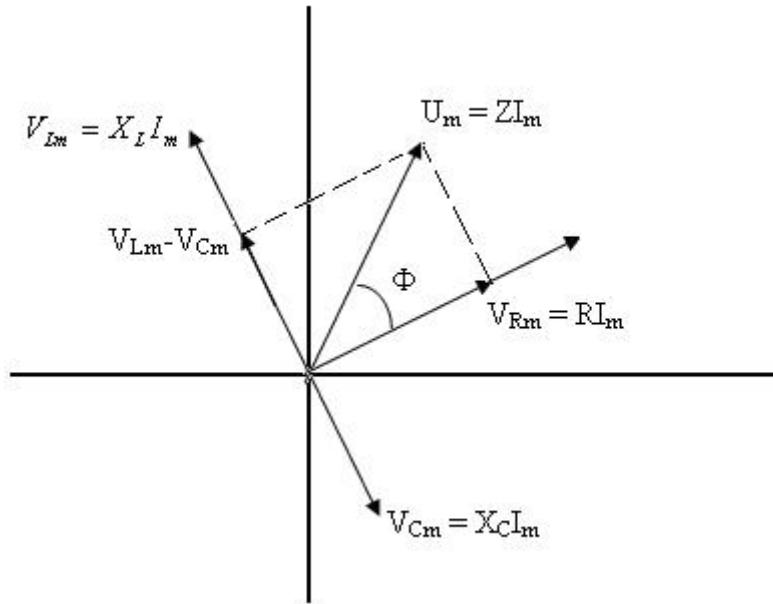
$$V_{Lm} - V_{Cm} = (X_L - X_C)\, I_m$$

Using Pythagoras theorem we obtain:

$$U_m^2 = [R^2 + (X_L - X_C)^2]\, I_m^2$$

$$U_m = I_m \sqrt{R^2 + (X_L - X_C)^2} = I_m Z \quad \text{avec} \quad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\text{A.N.} : Z + \sqrt{(80)^2 + (113 - 53)^2} = 100\Omega$$

$$\tan\Phi$$

$$= \frac{V_{Lm} - V_{Cm}}{V_{Rm}}$$

$$= \frac{X_L - X_C}{R} = \frac{113 - 53}{80} = 0{,}75$$

The current has a delay of $\Phi = 37°$ on the applied tension.

Please also note that $\cos\Phi = \dfrac{R}{Z}$

Exercise 1-b

$I = 1,70\sin(120\pi t – 0,6458)$. Correct answer. Compare your answer with the solution given below.

Detailed Solution

Let us re-examine the solutions we found when looking at the R-L-C circuit series :

$Z = 100W$

$\tan \Phi = 0,75 \implies \Phi = 37°$

or $\Phi = 0,6458$ rad

$\tan \Phi$ is positive, therefore the tension is ahead by Φ on the current

The instantaneous current which passes through the circuit is therefore $I = I_m \sin(\omega t - \Phi)$ (1) with Φ here expressed in radian : $\omega = 2\pi f = 2\pi \times 60$ Hz $= 120\pi$ rads^{-1}..

$I_m = \dfrac{U_m}{Z}$

(I_m : amplitude of the current ; U_m : amplitude of the tension)

We obtain : $U_m = U_{eff} \times \sqrt{2} = (120\ V) \times \sqrt{2}$

Thus $I_m = \dfrac{120\sqrt{2}}{100} = 1,70$ A

Relation (1) becomes :

$I = 1,70 \sin(120\pi t -0,6458)$

$I = 1,70\sin(120\pi t + 0,6458)$ Wrong! Indeed, this expression describes a current which is ahead of the phase in relation to the applied tension. However, just the opposite is happening here: the current has a delay of $\Phi = 0,6458$ rad on the tension.

Sites to visit

The following sites explore some animations of physics phenomena which relate to your study themes. **Two recommendations to help you progress:**

- **observe** each animation attentively
- try to **apply** each animation to everything you have learned during the course of the previous stages of learning

http://perso.orange.fr/daniel.robert9/tension_alternative0.html

http://perso.orange.fr/daniel.robert9/tension_alternative1.html

These two sites respectively explore the themes of capacity circuit characteristics and inductives and explain in detail the capacitive and inductive reactances and the phase difference between the tension and the current in the circuits.

http://perso.orange.fr/daniel.robert9/tension_alternative1.html

http://www.walter-fendt.de/ph11f/accircuit_f.htm

On these sites you can visualize Fresnel's diagrams and the oscillations of tension and current which correspond to:

- a circuit consisting only of a resistance
- a circuit consisting only of an inductance
- a circuit consisting only a capacitor

http://www.univ-lemans.fr/enseignements/physique/02/electri/rlcsinus.html

This site consists of an examination of RLC circuit series. It demonstrates:

- an animation of the tension and intensity oscillations
- intensity and phases difference curves

Summary 1

Let us consider a circuit linked to an alternative tension source $U = U_m \sin\omega t$ and I being the amplitude current I_m which passes through it.

What you need to remember:

The circuit contains only a resitance	R. - U and I are in phase
	- $I_m = \dfrac{U_m}{R}$
The circuit contains only an inductance L whose resistance is negligible	- U is ahead by 90° on I.
	- $I_m = \dfrac{U_m}{X_L}$; $X_L = L\omega =$ inductive reactance
The circuit contains only a capacity C	- U has a delay of 90° on I.
	- $I_m = \dfrac{U_m}{X_C}$; $X_C = \dfrac{1}{C\omega} =$ capacitive reactance
The circuit is a resistance R and an rence inductance L constructed in series with U	- The angle of the phase diffe- between the tension U and the current I is Φ such that
	$\tan\Phi = \dfrac{X_L}{R}$
	- U is ahead of Φ by 1
	- $I_m = \dfrac{U_m}{Z}$; impedance $Z = \sqrt{R^2 + X_L^2}$
	- $\cos\Phi = \dfrac{R}{Z}$

The circuit is a resistance R and a capacity C constructed in a series with U	- The angle of the phase difference between U and I is F such that $$\tan\Phi = = \frac{X_C}{R}$$ U has a delay of Φ on I $$I_m = \frac{U_m}{Z} \; ; \text{impedance } Z = \sqrt{R^2 + X_C^2}$$ $$\cos\Phi = \frac{R}{Z}$$
The circuit consists of a resistance R, an inductance L and a capacity C constructed in series with U	- The angle of the phase difference between the tension U and the current I is such that $$\tan\Phi = \frac{X_L - X_C}{R} \; . \text{ If } \tan\Phi \text{ is positive}$$ then U is ahead of Φ on I; If tanF is negative, then U has a delay of Φ on I. $$I_m = \frac{U_m}{Z} \; ; Z = \sqrt{R^2 + (X_L - X_C)^2}$$ $$\cos\Phi = \frac{R}{Z}$$

Corrections for exercise 2

$R + \dfrac{1}{jCn}$. This cannot be the right answer as the circuit does not contain any capacitor.

This expression represents rather the complex impedance of a circuit composed of a resistance and a capacitor linked in a series and connected to the limits of a sources of tension $U = U_m$ sinwt.

$R + jL\omega$. Well done. Right answer. We have here a construction in series. In a series construction, the complex impedances of the different elements add up. Remember also that the complex impedance magnitude indicates the impedance of the circuit.

This magnitude is here $\sqrt{R^2 + L^2 n^2} = \sqrt{R^2 + X_L^2}$: This is indeed the result we found in exercise 1 (please review the solution to this exercise).

$\dfrac{1}{R} + \dfrac{1}{jLn}$ is linked to the complex impedance of a circuit consisting of a resistance

and an inductance constructed in a parallel at the limits of a source of alternative tension

$U = U_m$ sin wt. It is therefore not the correct answer.

Supplementary Exercise: To increase your knowledge, find the impedance of this circuit.

Correction of the supplementary Exercise
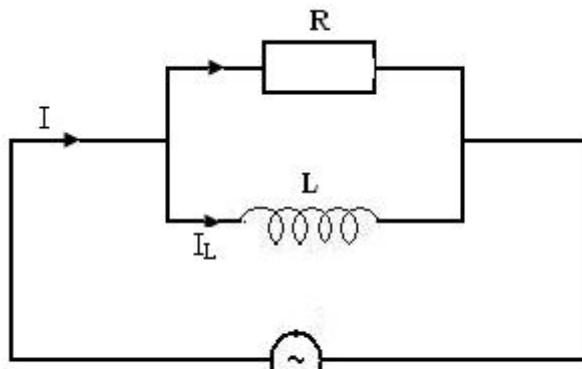
$$U = U_m \sin\omega t$$

1st method

Let us use the complex impedance $\overline{Z}$. We obtain $\dfrac{1}{\overline{Z}} = \dfrac{1}{R} + \dfrac{1}{jL\omega} = \dfrac{1}{R} + \dfrac{1}{jX_L}$

➔ $$\overline{Z} = \dfrac{1}{\dfrac{1}{R} + \dfrac{1}{jX_L}} = \dfrac{jRX_L}{R + jX_L} = \dfrac{jRX_L(R - jX_L)}{R^2 + X_L^2}$$

$$\overline{Z} = \dfrac{RX_L^2 + jR^2X_L}{R^2 + X_L^2}$$

The impedance of Z is the magnitude of $\overline{Z}$

$$Z = \sqrt{\left(\dfrac{RX_L^2}{R^2 + X_L^2}\right)^2 + \left(\dfrac{R^2X_L}{R^2 + X_L^2}\right)^2} = \sqrt{\dfrac{R^2X_L^2(R^2 + X_L^2)}{(R^2 + X_L^2)^2}}$$

$$Z = \dfrac{RX_L}{\sqrt{R^2 + X_L^2}}$$

2nd method

We use Fresnel's representation for the currents. Note that $V_L = V_R = U$, V_L and V_R representing respectively the differences in potential at the limits of the inductance and of the resistance.

I_R and V_R are in phase

V_L is ahead of I_L by 90° therefore it follows that I_L has a delay of 90° on I_R (see $V_L = V_R$ and I_R and V_R are in phase).

Thus we obtain: : $I^2 = I_R^2 + I_L^2 = \left(\dfrac{U}{R}\right)^2 + \left(\dfrac{U}{X_L}\right)^2$

➔ $I = U \sqrt{\dfrac{1}{R^2} + \dfrac{1}{X_L^2}}$

We deduct the impedance Z:

$$Z = \frac{1}{\sqrt{\dfrac{1}{R^2} + \dfrac{1}{X_L^2}}} = \frac{1}{\sqrt{\dfrac{R^2 + X_L^2}{R^2 X_L^2}}} = \sqrt{\dfrac{R^2 X_L^2}{R^2 + X_L^2}} = \frac{R X_L}{\sqrt{R^2 + X_L^2}}$$

Summary 2

What you need to remember:

To find the complex impedance $\overline{Z}$ of an alternative current circuit, you need to

- Replace each inductance L by its complex impedance $jL\omega$.

- Replace each capacitor C by its complex impedance $\dfrac{1}{j\omega C}$.
- Keep unchanged the Ohmic resistances R.
- Add the complex impedances of these different elements for a series. Construction.
- Add the inverse complex impedances of these different elements for a parallel construction.

The $\overline{Z}$ magnitude indicates the impedance of the circuit. The argument of Φ on $\overline{Z}$ is the advance in applied tension phase on the current: if $\overline{Z} = x + j\,y$ then $\tan\Phi$

$= \dfrac{y}{x}$.

Auto Evaluation

The learners record the difficulties they encountered and the mistakes they made while finding the solutions of the exercises in order to avoid them later on. They revue the parts of the course that they had difficulty understanding and prepare a summary evaluation.

Teacher's Guide

The Professor corrects the group exercises. He/She puts the corrections in a work-space accessible to the learners. The corrections are accompanied by adequate feedback. The grades obtained by each group are attributed to the members of the group and count for 10% of the final evaluation of the magnitude.

Learning Activity 1-2

UNIT 1: Alternative current circuits

Title of the Activity

Resonance Series: bandwidth and quality factor

Approximate number of hours: 15

Directions: For this activity, if you have **at least ¾ of the points**, you have done very good work, you may continue.

If you have **less than half the points**, you must do the readings over again and re-do the activities.

If you have **more than half the points and less than ¾ of the points**, you have done good work, but you must make some efforts in the future.

Specific Objectives

Following this activity, you must be able to:

- Describe the characteristics of the resonance of an RLC series circuit
- Describe the variation of the current passing through an RLC series circuit by way of the pulsation of the applied tension
- Describe the variation of the impedance of the circuit by way of the pulsation of the applied tension
- Graphically determine the bandwith of an RLC circuit series
- Trace the capacitive reactance by way of the pulsation
- Trace the inductive reactance by way of the pulsation
- Establish the literal expression of the bandwidth by way of R and L
- Establish the literal expression of the quality factor by way of R, L, and C

Outline of the Activity

This activity still relates to alternative current circuits. However, here we are concerned with the examination of the phenomenon of resonance in a circuit series consisting of a resistance, a capacitance and an inductance and at the limits of which we place a sinusoidal tension whose pulsation we vary while keeping its amplitude constant. Your task will be to analyze the effects of this variation on the amplitude of the current and on the impedance of the circuit. You will have to extricate the characteristics of the state of resonance. A course complement on the notions "bandwidth" and of "quality factor" will also be included.

Appropriate Readings

- RASOLONDRAMANITRA, H. (2007). Circuits à courant alternatif. Madagascar. Université d'Antananarivo. Cours inédit.

Relevant Resources

1. BRAMAND, P., FAYE, P. et THOMASSIER, G. (1983). Physique- Terminales C et E. Collection Eurin-Gié. Hachette, Paris.
2. BEISER, A.(1979). Physique appliquée : Cours et problèmes.Serie Schaum . Traduction française par LOBENBERG M. McGraw-Hill, Paris
3. BREITHAUPT, J. (2000). New Understanding : Physics for Advanced Level. Fourth Edition, Stanley Thornes Publishers Ltd,England ,
4. FRANCIS W. S., ZEMANSKY, M. W. and YOUNG. H. D. (1987). University Physics. Seventh Edition, Addison-Wesley Publishing Company, USA.
5. FRANCIS W. S., ZEMANSKY, M. W. and YOUNG. H. D. (1974). College Physics. Fourth Edition. Addison-Wesley Publishing Company, USA.
6. HALPERN, A. (1991) Maxi Schaum Physique 2. Traduction française par BABES,V. McGraw-Hill, Paris
7. KALACHNIKOV, S. (1980). Electricité. Traduction française. Editions Mir, Moscou, Réédition :1983
8. NELKON, M. and PARKER, P. (1998). Advanced Level Physics. Seventh Edition, Heinemann, (Oxford ?)
9. RESNICK, R. et HALLIDAY, D. (1979). Electricité et magnétisme, physique 2. Traduit par André Lebel et Claudine Thériault. Editions du Renouveau Pédagogique. Inc. Montréal (Québec) Canada
10. TIPLER, P.-A. and Company. (1999). Physics for Scientists and Engineers. Fourth Edition.   W.H. FREEMAN and company. Worth Publishers Inc., New York, USA
11. TIPLER, P. A. (1982). Physics. Second Edition. Worth Publishers Inc, New York, USA

Useful Links

http://labo.ntic.org/RLC_serie/RLC.html

http://hebergement.ac-poitiers.fr/l-cc-angouleme/coulomb-exos-phy/applets/RLC_serie/RLC.htm

http://perso.orange.fr/olivier.granier/electro/simul/cour_rlc/circuitRLC4.htm

http://www.fundp.ac.be/sciences/physique/didactique/elec/RLC4.php

http://www.fundp.ac.be/sciences/physique/didactique/elec/RLC3.php

http://perso.orange.fr/olivier.granier/electro/simul/cour_rlc/circuitRLC3.htm

http://sitelec.free.fr/cours/rlcseries.pdf

http://www.physique-eea.unicaen.fr/enseignement/deug-st/sm/dsm153/poly/dsm153-e.pdf

http://www.unilim.fr/pages_perso/frederic.louradour/Oscillo_2.PDF

Description of the Activity

In this activity :

1. a reading on the theme of "resonance" is suggested as an introductory basis

2. following this reading a course complement on the notion of "bandwidth" and "quality factor" is suggested in order to consolidate the different notions of the preceding stages of learning; you will take an active part in the course of this stage by establishing the mathematical expressions corresponding to these two notions

3. you will then be asked to solve a certain number of exercises with the goal of verifying what you have learned during the preceding two stages

4. after this you will be guided toward various sites which deal with the theme of your studies and/or which suggest some animations

5. a summary of what you need to remember will be proposed

Your itinerary may visualized thus:

Read your study theme

Establish some mathematical relations

↓

(course complement)

↓

Do some exercises

↓

Evaluate your work
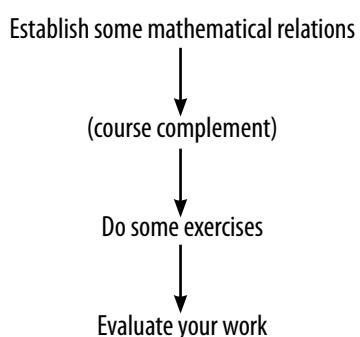
Formative Evaluation

It is compulsory for the learners to do all the exercises as a collaborative team. The grade for the group is the same as that given to its different members.

**Exercise 1 counts for 30% of the points and exercise 2 for 70% of the points.**

Course complement: bandwidth and quality factor

Let us consider the following circuit. The pulsation ω varies while keeping the amplitude of the tension constant
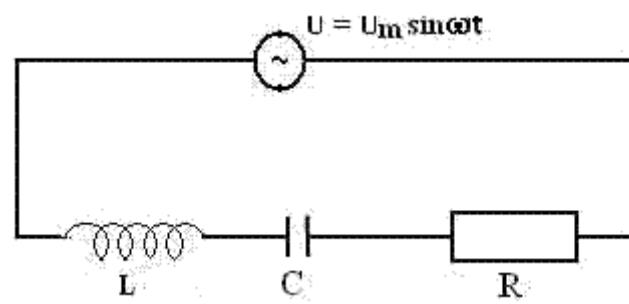
In the following, we will examine efficient values. The curve seen below describes I_{eff} as a function of w. This curve presents a maximum for w = ω_0.
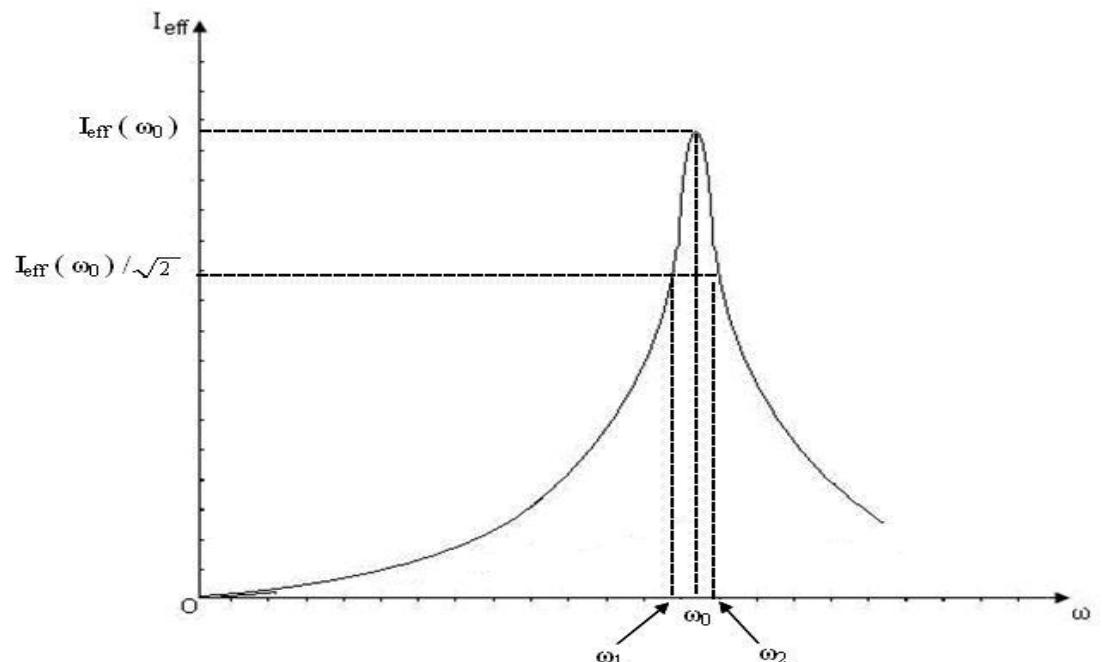
A- Bandwidth

Let us locate the pulsations ω_1 and ω_2 which provoke intensive efficient currents :

$$I_{eff}(\omega_1) = I_{eff}(\omega_2) = I_{eff}(\omega_0) / \sqrt{2}$$

The difference $\Delta\omega = \omega_2 - \omega_1$ is called bandwidth of the circuit (RLC). It is expressed in radians per second.

The narrower the bandwidth, the more the resonance is acute. In this case the sources of the sinusoïdal tensions which have pulsations ω comprised between ω_1 and ω_2 provoke the passage of a high intensity current in the circuit. The frequencies capable of activating the circuit are in an extremely narrow domain. We say that the circuit is selective

For a greater resistance, the curve is flatter and the resonance is blurred
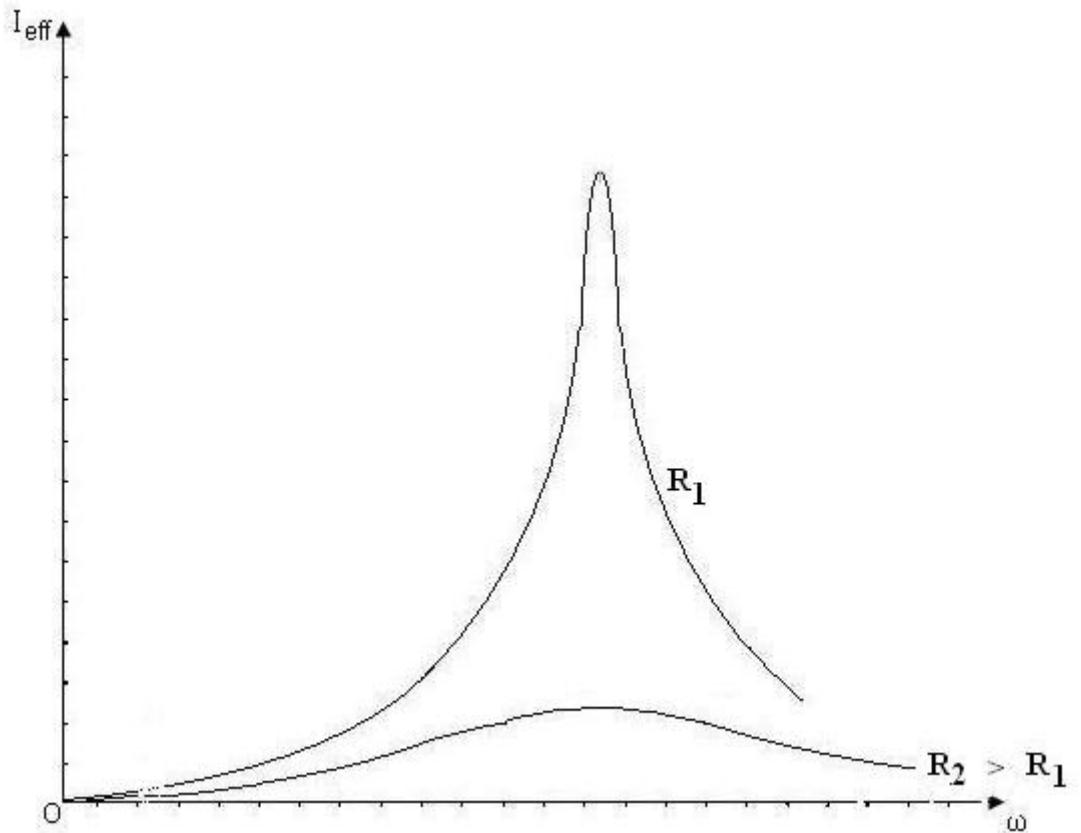
B-Quality factor

The relationship $Q = \omega_0 / \Delta\omega$ is called the quality factor.

The more acute the resonance is, the higher the quality factor.

Expressions of the bandwidth and of the quality factor

Instructions:

Demonstrate that:

- the expression of the bandwidth is $\Delta\omega = \omega_2 - \omega_1 = R / L$

- the quality factor is expressed as : $Q = \omega_0 / \Delta\omega = L\,\omega_0 / R = 1 / (CR\,\omega_0) = (1 / R)(L / C)^{1/2}$

Demonstration

A: bandwidth

Remember how the bandwidth is defined:

The bandwidth is:

$\Delta\omega = \omega_2 - \omega_1$

where ω_1 and ω_2 are the pulsations for which we have :

$$I_{eff}(\omega_1) = I_{eff}(\omega_2) = I_{eff}(\omega_0) / \sqrt{2} \qquad \textbf{(1)}$$

$I_{eff}(\omega_0)$ and ω_0 are respectively the efficient intensity and the pulsation at the reso-nance.

We must therefore determine $\omega_2 - \omega_1$. Analyze the relation (1): It contains efficient intensities evaluated in ω_0, ω_1 and ω_2 . What is the relation between I_{eff} and $Z(\omega)$?

The impedance of the circuit and the efficient intensity vary with the pulsation $\square$.

$$Z(\omega) = \sqrt{R^2 + (L\omega - \frac{1}{C\omega})^2}$$

$$I_{eff}(\omega) = \frac{U_{eff}}{Z(\omega)}$$

$\Rightarrow$ • at the resonance

$\omega = \omega_0$

$$Z(\omega_0) = R$$

$$I_{eff}(\omega_0) = \frac{U_{eff}}{Z(\omega_0)} = \frac{U_{eff}}{R} \quad \textbf{(2)}$$

$$\Rightarrow \quad \bullet \text{ for } \omega = \omega_1$$

$$I_{eff}(\omega_1) = \frac{U_{eff}}{Z(\omega_1)}$$

$$\Rightarrow \quad \bullet \text{ for } \omega = \omega_2 \qquad \textbf{(3)}$$

$$I_{eff}(\omega_2) = \frac{U_{eff}}{Z(\omega_2)}$$

Carry the relations (2) and (3) into (1)

The relation **(1)** becomes:

$$\frac{U_{eff}}{Z(\omega_1)} = \frac{U_{eff}}{Z(\omega_2)} = \frac{U_{eff}}{R\sqrt{2}} \quad \Rightarrow Z(\omega_1) = Z(\omega_2) = R\sqrt{2} \quad \textbf{(4)}$$

We must now deducce from expression (4) a relation which allows us to find ω_1 **and** ω_2

Relation **(4) allows us to write:**

$$Z^2(\omega) = 2R^2$$

$$R^2 + (L\omega - \frac{1}{C\omega})^2 = 2R^2$$

$$(L\omega - \frac{1}{C\omega})^2 = R^2$$

$$(L\omega - \frac{1}{C\omega}) = \pm R \quad \textbf{(5)}$$

What can we obtain from relation (5)?

Relation **(5)** results in two second degree equations:

$$LC\omega^2 + RC\omega - 1 = 0 \textbf{ (6)}$$

$$LC\omega^2 - RC\omega - 1 = 0 \qquad\qquad \textbf{(7)}$$

Resolve these two equations

These two equations contain a positive discriminant:

$$\Delta = R^2C^2 + 4LC > 0 \quad (\text{ note that } \sqrt{\Delta} > RC)$$

The solutions of equation **(6)** are :

$$\omega_1 = \frac{-RC + \sqrt{\Delta}}{2LC} > 0 \quad \text{ and } \quad \omega_1^0 = \frac{-RC - \sqrt{\Delta}}{2LC} < 0$$

The solutions of equation **(7)** are :

$$\omega_2 = \frac{RC + \sqrt{\Delta}}{2LC} > 0 \quad \text{ and } \omega_2^0 = \frac{RC - \sqrt{\Delta}}{2LC} < 0$$

We only keep the positive values since the pulsation is a positive number. The bandwidth is therefore:

$$\blacktriangleright \quad \Delta\omega = \omega_2 - \omega_1 = \frac{R}{L}$$

B: -Quality factor

The quality factor is:

$$Q = \frac{\omega_0}{\Delta\omega}$$

Remplace $\Delta\omega$

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{L\omega_0}{R}$$

At the resonance $X_L = X_C \Rightarrow L\omega_0 = \frac{1}{C\omega_0}$

▶ $Q = \frac{\omega_0}{\Delta\omega} = \frac{L\omega_0}{R} = \frac{1}{CR\omega_0}$

Replace ω_0

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

▶ $Q = \frac{\omega_0}{\Delta\omega} = \frac{L\omega_0}{R} = \frac{1}{CR\omega_0} = \frac{1}{R}\sqrt{\frac{L}{C}}$

Exercise 1

Let us consider an R-L-C series circuit connected to the limits of an alternative tension source $U = U_m \sin wt$. We vary the pulsation ω while keeping the amplitude U_m constant.

1- Recall the characteristics of the state of resonance of this circuit

2- Trace as a function of ω and in the same location the resistance R, the capacitive reactance X_C and the inductive reactance X_L.

3- Use the graph obtained to find the characteristics of the state of resonance.

Exercise 2

A resistance R= 200 Ω, a condenser with capacity C = 0,7µF and an inductance coil L = 1,43H are constructed in a series at the limits of a sinusoïdal tension source with a value of U_{eff} = 24V and with a variable pulsation of ω.

The values of the efficient intensity I_{eff} for different pulsations are recorded in the table below.

1 - Complete the table and comment on it

2 - Trace R, Z, X_C and X_L as a function of ω and in the same location

3 - Trace I_{eff} as a function of ω and graphically determine the pulsation at the state of resonance, the bandwidth and the quality factor Q. Find the values for R, L, C.

ω (rad.s^{-1})	I_{eff} (10^{-3} A)	Z (Ω)	R (Ω)	X_L (Ω)	X_C (Ω)
100	1,7				
300	5,6				
500	11,3				
600	15,8				
700	22,8				
800	36				
850	47,7				
900	66,8				
950	97				
1000	120				
1050	98,7				
1100	71,4				
1150	54				
1200	43,1				
1300	30,9				
1400	24,2				
1500	20,1				
1600	17,2				
2000	11,3				
2500	8,1				
3000	6,4				

Learning Activity

- Read the course material attentively and the course complement
- Give the literal expressions of the bandwidth as a function of R and L and of the quality factor as a function of R, L and C
- Discuss what you do not understand in the chat-room and ask for the other learners' explanations. A tutor supervises the chat-room and rectifies the discussion when needed. If everything is understood, the learners will begin by doing exercises. To do this, they are divided into groups for collaborative work.
- For each exercise, each group designates a delegate and finds the solution within a time-frame designated by the tutor.
- The delegates send the solutions to the course Professor under the supervision of the other members of the group by an attached e-mail document.
- Read the proposed summaries and print them if necessary

Answer Key

Solution to exercise 1

1.Characteristics of the state of resonance

At the resonance:

- The pulsation ω is equal to $\omega_0 = \dfrac{1}{\sqrt{LC}}$,
- The inductive and capacitive reactances are equal: : $X_L = X_C$
- The impedance of the circuit is Z=R
- The amplitude I_m of the current is maximal : $Z\,I_m = U_m$
- The phase difference between the current and the tension at the limits of the circuit is naught. The current is in phase with the tension

2) R, X_L and X_C as a function of ω

Let us graph R, $X_C = \dfrac{1}{C\omega}$ and $X_L = Lw$ as a function of w.

R does not depend on w. We obtain a straight line parallel to the Ow axis.

X_L is a straight line passing through the origin X_C and varies in $\dfrac{1}{\omega}$. These two curves are dissected at $w = w_0$.

The impedance of our circuit is $Z = \sqrt{R^2 + (X_L - X_C)^2}$. How do the impedance and the current I passing through the circuit vary as a function of w? To see this, let us bring our attention on $(X_L - X_C)$ when we augment w. $(X_L - X_C)$ is represented on the graph by dotted lines.

When we augment w from zero $\omega(X_L - X_C)\omega$ diminishes and is cancelled for $\omega = \omega_0$ then augments again for $\omega > \omega_0$.

Let us see the results of our observation in the table below.

	0		ω_0	
$X_L - X_C$		0		
Z		R		
$I = \dfrac{U}{Z}$		Maximum		
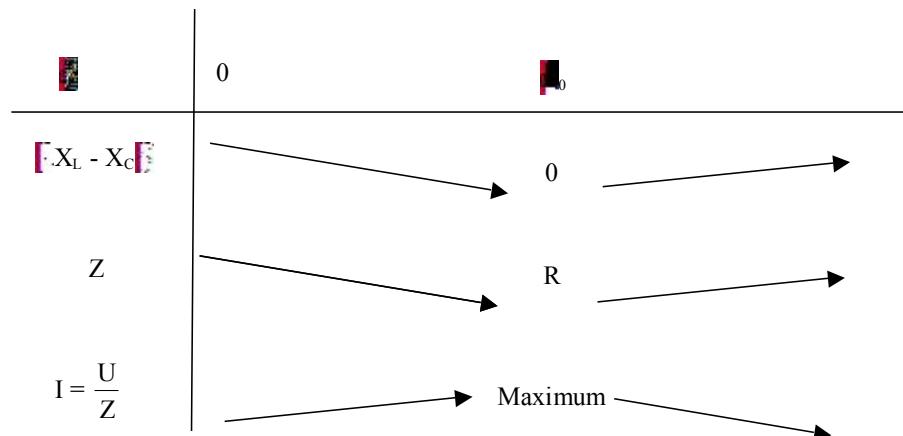
We have the resonance when the current I attains its maximum value. This value appears when the acute frequency is equal to ω_0. At this frequency :

$$X_L = X_C \Rightarrow L\omega_0 = \frac{1}{C\omega_0} \Rightarrow LC\omega_0^2 = 1 \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} .$$

But $w_0 = 2pf_0 \Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}}$.

Note also that at the resonance Z=R.

The phase difference Φ is such that:

$$\tan\Phi = \frac{X_L - X_C}{R} \text{ and } \cos\Phi = \frac{R}{Z} \Rightarrow \tan\Phi = 0 \text{ and } \cos\Phi = 1$$

It follows that $\Phi = 0 \Rightarrow$ the current and the tension are in phase

Correction of Exercise 2

1. Let us recall the expressions of the impedance Z, and the reactances X_L and X_C :

$$Z = \frac{U_{eff}}{I_{eff}} \quad ; \quad X_L = L\omega \ ; X_C = \frac{1}{C\omega}$$

$U_{eff} = 24V$; $L = 1{,}43 \ H$; $C = 0{,}7\mu F$; $R = 200\Omega$

After calculations, we obtain:

ω (rad.s^{-1})	I_{eff} (10^{-3} A)	Z (Ω)	R (Ω)	X_L (Ω)	X_C (Ω)
100	1,7	14117,6	200	143	14285,7
300	5,6	4285,7	200	429	4761,9
500	11,3	2123,9	200	715	2857,1
600	15,8	1519	200	858	2380,9
700	22,8	1052,6	200	1001	2040,8
800	36	666,7	200	1144	1785,7
850	47,7	503,1	200	1215,5	1680,7
900	66,8	359,3	200	1287	1587,3
950	97	247,4	200	1358,5	1503,8
1000	120	200	200	1430	1428,6
1050	98,7	243,2	200	1501,5	1360,5
1100	71,4	336,1	200	1573	1298,7
1150	54	444,4	200	1644,5	1242,2
1200	43,1	556,8	200	1716	1190,5
1300	30,9	776,7	200	1859	1098,9
1400	24,2	991,7	200	2002	1020,4
1500	20,1	1194	200	2145	952,4
1600	17,2	1395,3	200	2288	892,9
2000	11,3	2123,9	200	2860	714,3
2500	8,1	2963	200	3575	571,4
3000	6,4	3750	200	4290	476,2

Comments

We see from the table that

-The resistance R is independent of the pulsation ω

- For a value of $\omega = \omega_0 = 1000 \ rad.s^{-1}$

   • The efficient intensity is at its maximum: $I_{eff} (\omega_0) = 120 \ mA$

   • The impedance is minimal and is equal to R

$$Z (\omega_0) = R = 200 \ \Omega$$

- The capacitive and inductive reactances are equal

$$X_L (\omega_0) = X_C (\omega_0) \approx 1430 \ \Omega$$

We discover here the properties of the state of resonance

We also deduce from the table that :

- **for pulsations where** $\omega < \omega_0 \Rightarrow X_L < X_C$

Thus the angle of the phase difference ω between tension and current is such that :

$$\tan\phi = (X_L - X_C) / R \ \text{is} < 0 \quad \Rightarrow \text{the tension has a delay on the current}$$

The circuit has a capacitive behaviour

- **for pulsations where** $\omega > \omega_0 \Rightarrow \mathbf{X_L > X_C}$

Here $\tan \phi = (X_L - X_C) / R \ \text{est} > 0 \quad \Rightarrow$ the tension is ahead of the current

The circuit has an inductive behaviour

- **for r** $\omega = \omega \Rightarrow X_L = X_C$ and $R = Z$, in this case

$$\left. \begin{array}{l} \tan\phi = 0 \\ \cos\phi = (R / Z) = 1 \end{array} \right\} \Rightarrow \phi = 0$$

The tension and the current are in phase

2. Graphic representations of R, Z, X_L and X_C

The graphic representations of R, Z, X_L et X_C below describe these results :

At the resonance:

$\omega = \omega_0 = 1000$ rad.s-1

$Z = R = 200$ Ω

$X_L = X_C \approx 1430$ Ω

For $\omega < \omega_0 \Rightarrow X_L < X_C$

For $\omega > \omega_0 \Rightarrow X_L > X_C$
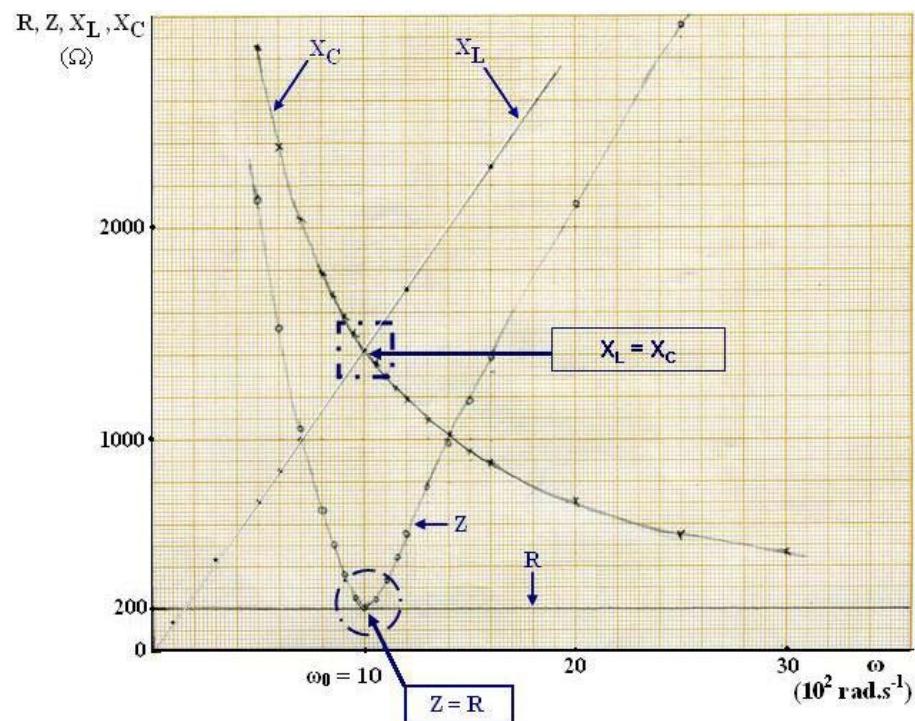
4- Efficient intensities as a function of the pulsation

We can deduce from the efficient intensity curve that :

- The bandwidth $\Delta\omega = \omega_2 - \omega_1 = 140$ rad.s^{-1}
- The resonance frequency $\omega_0 = 1000$ rad.s^{-1}

- $I_{eff}(\omega_0) = \dfrac{U_{eff}}{R} = 120 \times 10^{-3}$A

$$\Rightarrow \quad R = \frac{U_{eff}}{I_{eff}} = \frac{24V}{120 \times 10^{-3}\,A} = 200\Omega$$

The quality factor

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{1000\,\text{rad.s}^{-1}}{140\,\text{rad.s}^{-1}} = 7,14$$

- The expression of the bandwidth allows us to calculate the inductance L

$$\Delta\omega = \frac{R}{L} \Rightarrow L = \frac{R}{\Delta\omega} = \frac{200\Omega}{140\,\text{rad.s}^{-1}}$$
$$\Rightarrow L = 1,43H$$

- We use the quality factor expression to calculate the capacity C

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{L\omega_0}{R} = \frac{1}{CR\omega_0}$$

$$\Rightarrow \frac{1}{CR\omega_0} = \frac{1000}{140} \Rightarrow C = \frac{140}{1000\,R\omega_0} = 0,7\mu F$$
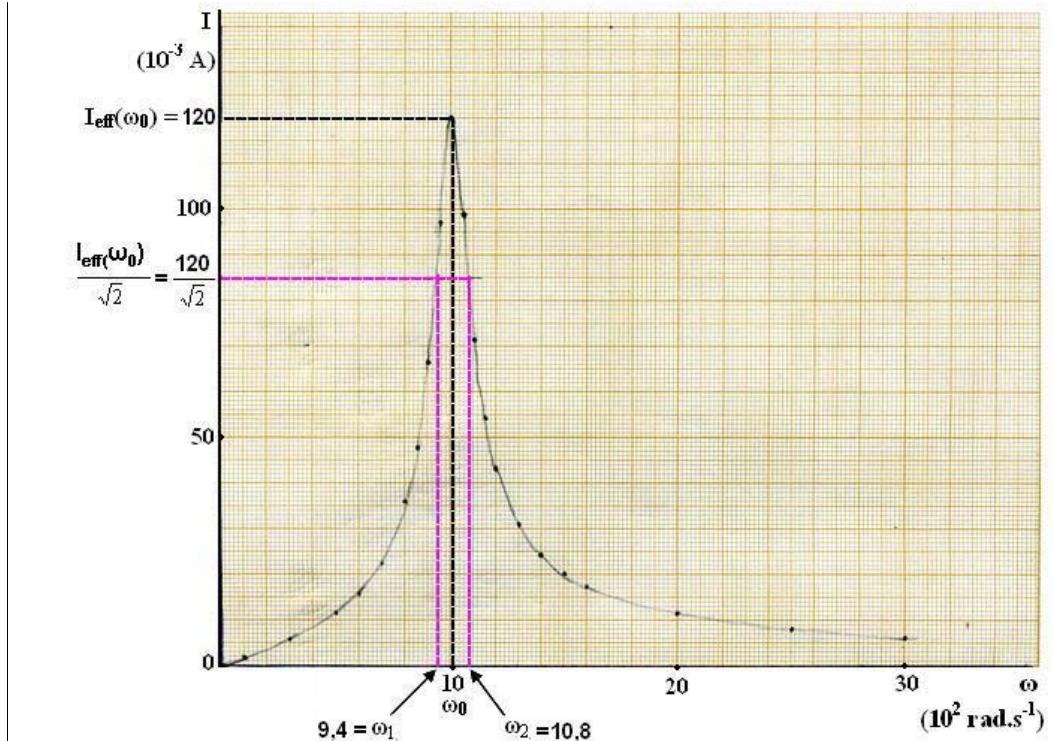
Sites to visit

Consult the following sites and put what you visualize in relation with what you have learned during the course of your learning activity.

- http://hebergement.ac-poitiers.fr/l-cc-angouleme/coulomb-exos- phy/applets/RLC_serie/RLC.htm
- http://labo.ntic.org/RLC_serie/RLC.html

These two applets allow the learner to consolidate the material examined on RLC circuits. They allow the learner to visualize:

- Tension and current oscillations
- The impedance Z, the capacitive reactances X_C and the inductive reactances X_L as a function of the frequency of the tension source
- The intensity of the current as a function of the frequency of the source
- The phase difference between the applied tension and the current which passes through it as a function of the frequency

These graphs allow you to deduce

- The properties of the state of resonance
- The bandwidth

You may vary the values of the pertinent parameters (R,L,C,F) separately and follow the effects of this variation.

- http://perso.orange.fr/olivier.granier/electro/simul/cour_rlc/circuitRLC4.htm
- http://www.fundp.ac.be/sciences/physique/didactique/elec/RLC4.php

These two links provide an interactive exercise based on the resonance frequency and the quality factor with graphic illustrations.

- http://www.fundp.ac.be/sciences/physique/didactique/elec/RLC3.php
- http://perso.orange.fr/olivier.granier/electro/simul/cour_rlc/circuitRLC3.htm

This applet allows the learner to follow the evolution of the tension amplitude of an RLC circuit at the limits of a condenser and as a function of the frequency.

- http://sitelec.free.fr/cours/rlcseries.pdf

This is a chapter on RLC circuit series. It examines in particular the following points:

- Calculations of reactances and impedances
- Tension at the limits of the circuit elements
- Temporal and vectorial representations of the tensions and of the current
- Series resonance

This is an excellent reading to complete your understanding of what you have learned until now.

- http://www.physique-eea.unicaen.fr/enseignement/deug-st/sm/dsm153/poly/dsm153-e.pdf

This link provides you with manipulation protocols for the study of the R-L coil in continuous current and in alternative current and for the study of resonance. It highlights, among other elements, the phenomenon of resonance in a simple electric circuit: resonating series circuit, over-tension, bandwidth and quality factor.

- http://www.unilim.fr/pages_perso/frederic.louradour/Oscillo_2.PDF

This site suggests manipulation protocols for the study of series and parallel RLC circuits.

Summary

What you need to remember

Resonance

When a tension $U = U_m \sin 2\pi ft$ of variable frequency f is applied to the limits of an RLC circuit series, the intensity of the current is at its maximum at the frequency f_0

$= \dfrac{1}{2\pi\sqrt{LC}}$ or when the pulsation is equal to à $\omega_0 = \dfrac{1}{\sqrt{LC}}$. We call this the state

of frequency.

At the resonance: $\qquad f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

$\qquad X_C = X_L$

$\qquad Z = R$

$\qquad I_m = \dfrac{U_m}{Z}$ is at its maximum

$\qquad I_{eff} = \dfrac{U_{eff}}{Z}$ is at its maximum

I and U are in phase

Bandwidth

The bandwidth $\Delta\omega$ characterizes the acuity of the resonance. It corresponds to the difference of the pulsations $\Delta\omega = \omega_2 - \omega_1$, pulsations for which :

$I(\omega_1) = I(\omega_2) = \dfrac{I(\omega_0)}{\sqrt{2}}$ where $I(\omega_1)$, $I(\omega_2)$ and $I(\omega_0)$ represent respectively the efficient intensities evaluated in ω_1, ω_2 and ω_0, ω_0 being the pulsation at the state of resonance.

$$\Delta\omega = \frac{R}{L}$$

The narrower the bandwidth is, the more acute the resonance is.

Quality Factor

The quality factor Q also characterizes the acuity of the resonance. It is defined as

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{L\omega_0}{R} = \frac{1}{CR\omega_0} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

The higher the quality factor is, the more acute the resonance.

Auto-evaluation

The learners take note of the difficulties encountered during the learning activity. They may review the relevant parts of the course in order to better understand their difficulties and mistakes.

Teacher's Guide

The teacher or Professor corrects the group exercises. He/She deposits the corrections in a work-space accessible to the learners. The correction will be accompanied by adequate feedback. The grades obtained by each group are attributed to each member of the group and will count for 10% of the final evaluation of the magnitude.

Learning activity 2

Name of the activity

- **Movements of charged particles in a magnetic field and its practical applications.**
- **Lenz's law, auto-induction and RL circuit**

Time Required: 30 Hours

Instructions: For this activity, if you have **at least ¾ of the points**, you have done very good work, you may continue.

If you have **less than half the points**, you must read the course material again and re-do the activities.

If you have **more than half the points and less than ¾ of the points**, you have done good work, but you must make efforts in the future.

Specific Objectives

Following this activity, you must be able to:

Specific objectives of the 1st part

- Recall the action of an electric field on a charged particle
- Recall the action of a magnetic field on a moving charged particle

- Cite different methods for measuring the relation $\dfrac{e}{m}$ for the electron

- Explain the " $\dfrac{e}{m}$ measure according to the magnetic deflection"

- Determine the relation $\dfrac{e}{m}$ by using the magnetic deflection method

- Explain the " $\dfrac{e}{m}$ measure using the rarified gas tube method"

- Determine $\dfrac{e}{m}$ by using the rarified gas tube method
  Describe the principle of operation of a mass spectrometer

- Describe the principle of operation of a speed selector

- Describe a cyclotron

- Use the Hall tension to calculate the number of charge carriers by volume unit in a conductor

- Calculate the radius of the trajectory of a charged particle penetrating a uniform magnetic field at an orthogonal speed

Specific objectives of the 2nd part

- Recall Faraday's law
- Use Faraday's law to calculate the induced e.m.f
- Recall Lenz's law
- Use Lenz's law to determine the direction of the induced current
- Recall the notion of "self inductance" or "self flux"
- Recall the phenomenon of auto-induction
- Calculate the inductance of a circuit using the $\square = L\ i$ relation
- Apply Kirchhoff's "Loop rule" or "mesh law" to establish the differential equation corresponding to an RL circuit
- Effect the resolution of the differential equations of the first order with a second member
- Explain the evolution of the current as a function of the time in an RL circuit
- Show that i=f (t) in an RL circuit
- Calculate the self inductance of a coil
- Calculate the growth rate of a current passing through an RL circuit
  Use Hall's tension to calculate the number of charge carriers per unit of volume in a conductor

Outline of the activity

This activity consists of two parts:

1) the first part concerns the study of charged particle movements in a magnetic field. We are concerned, essentially, with looking at a few practical applications of these movements. You will have to:

- Analyze different methods used to determine the (e/m) relation for electrons
- Examine:
    - the principles of operation of a mass spectrometer and of a cyclotron
    - the Hall effect which allowed us to determine the sign of charge carriers in a conductor

2) the second part studies in greater depth the electromagnetic induction pheno-
menon by:

• Examining Lenz's law which allows us to determine the direction of the
induced current

• Analyze the auto-induction phenomenon

In treating particular cases, you will have to demonstrate the origin of the induced
current and the direction of the induced current.

The study of an RL circuit completes this part.

Appropriate readings

- RASOLONDRAMANITRA, H. (2007). Champ magnétique et parti-
cules chargées en mouvement - Inductance. Madagascar. Université
d'Antananarivo. Cours inédit.

Pertinent resources

1. BARUCH, P., HULIN, M. et PETROFF J.-F. (1972). Electricité- Magné-
tisme. Cours. Hermann, Paris,

2. BEISER, A.(1979). Physique appliquée : Cours et problèmes.Serie Schaum .
Traduction française par LOBENBERG M. McGraw-Hill, Paris

3. BRAMAND, P., FAYE, P. et THOMASSIER, G. (1983). Physique- Termina-
les C et E. Collection Eurin-Gié. Hachette, Paris.

4. BREITHAUPT, J. (2000). New Understanding : Physics for Advanced Level.
Fourth Edition, Stanley Thornes Publishers Ltd,England ,

5. BRUHAT, G. (1967).Cours de Physique Générale : Electricité. Huitième
Edition Révisée (2e tirage) par G.GOUDET, Masson et Cie-, Editeurs, Paris

6. FRANCIS W. S., ZEMANSKY, M. W. and YOUNG. H. D. (1987). Univer-
sity Physics. Seventh Edition, Addison-Wesley Publishing Company, USA.

7. FRANCIS W. S., ZEMANSKY, M. W. and YOUNG. H. D. (1974). College
Physics. Fourth Edition. Addison-Wesley Publishing Company, USA.

8. HALPERN, A. (1991) Maxi Schaum Physique 2. Traduction française par
BABES,V. McGraw-Hill, Paris

9. KALACHNIKOV, S. (1980). Electricité. Traduction française. Editions Mir,
Moscou, Réédition :1983

10. NELKON, M. and PARKER, P. (1998). Advanced Level Physics. Seventh
Edition, Heinemann, (Oxford ?)

11. RESNICK, R. et HALLIDAY, D. (1979). Electricité et magnétisme, phy-
sique 2. Traduit par André Lebel et Claudine Thériault. Editions du Renou-
veau Pédagogique. Inc. Montréal (Québec) Canada

12. TIPLER, P.-A. and Company. (1999). Physics for Scientists and Engineers. Fourth Edition.   W.H. FREEMAN and company. Worth Publishers Inc., New York, USA

13. TIPLER, P. A. (1982). Physics. Second Edition. Worth Publishers Inc, New York, USA

Useful Links

http://wwwens.uqac.ca/chimie/Physique_atom/Chap_htm/CHAP_3.html

http://www.unige.ch/sciences/physique/tp/tpe/E9.htm

http://www.univ-lemans.fr/enseignements/physique/02/electri/esurm.html

http://www.sciences.univ-nantes.fr/physique/perso/gtulloue/Meca/Charges/hall.html

http://membres.lycos.fr/physicisss/labos/effet_hall.pdf

http://www.sciences.univ-nantes.fr/physique/perso/gtulloue/Meca/Charges/cyclotron.html

http://perso.orange.fr/physique.chimie/Cours_de_physique/Physique_12_PARTICULE_CH ARGEE_DANS_UN_CHAMP_MAGNETIQUE.htm

http://perso.orange.fr/daniel.robert9/Induction_electromagnetique.html

http://perso.orange.fr/daniel.robert9/Induction_electromagnetique0.html

http://www.univ-lemans.fr/enseignements/physique/02/electri/lenz.html

http://jf-noblet.chez-alice.fr/bobine/index.htm

Description of the activity

The same itinerary is suggested for both parts of this activity:

Step 1: readings on the study themes which require from your part an active implication

Step 2: a series of compulsory exercises to do

Step 3: visiting sites to compare the content which relates to the study themes and / or visualize animations
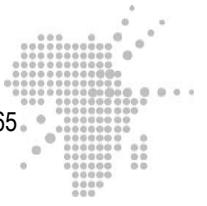
Formative Evaluation

This formative evaluation consists of the course complements to be read before doing the exercises. Each exercise represents 1% of the points.

Course complement

Part 1: A few practical applications

A) Operating principle of a mass spectrometer

The diagram below describes this principle:

We subject a beam of ions of the same charge but of different mass to an accelerating

tension U. These particles have the same speed $\vec{v}_0$ when they penetrate the region

where the field $\vec{B}$ predominates.

$\vec{B}$ is orthogonal to $\vec{v}_0$.

Under the influence of $\vec{B}$, their trajectories are circular.
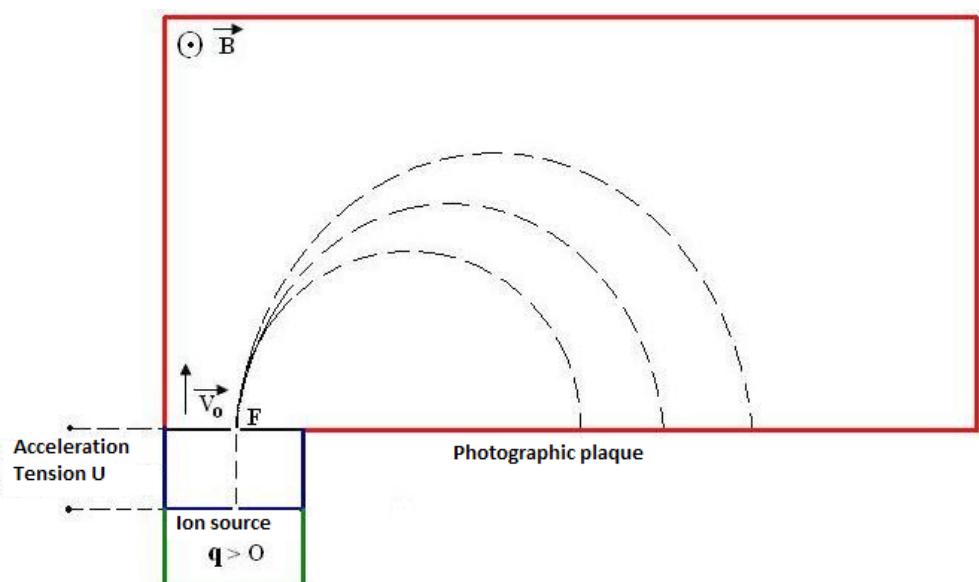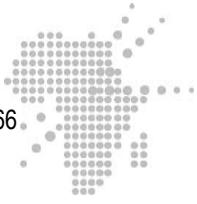
After describing a semi-circle, they are received on the photographic plaque. A charge particle q and of mass m_i travels in a semi-circle of radius $R_i = \dfrac{m_i\ v_0}{qB}$ and arrives at a point on the photographic plaque, a point situated at a distance $2R_i = D_i$ of the fissure F :

$$D_i = \frac{2m_i\ v_0}{qB} \qquad\qquad (1)$$

We obtain v_0 by using :

$$\frac{1}{2}\ m_i v_0^2 = q\ U$$

$$\Rightarrow v_0 = \left(\frac{2qU}{m_i}\right)^{\frac{1}{2}}$$

Relation (1) becomes :

$$D_i = \frac{2m_i}{qB}\left(\frac{2qU}{m_i}\right)^{\frac{1}{2}}$$

It follows that :

$$D_i^2 = \frac{4m_i^2}{q^2B^2}\frac{2qU}{m_i} = \frac{8m_iU}{qB^2}$$

$$\Rightarrow \frac{q}{m_i} = \frac{8\ U}{D_i^2B^2}$$

We deduce from this relation that we can separate the ions with the same charge and of different masses

)

Measuring $\dfrac{e}{m}$ using magnetic deflection
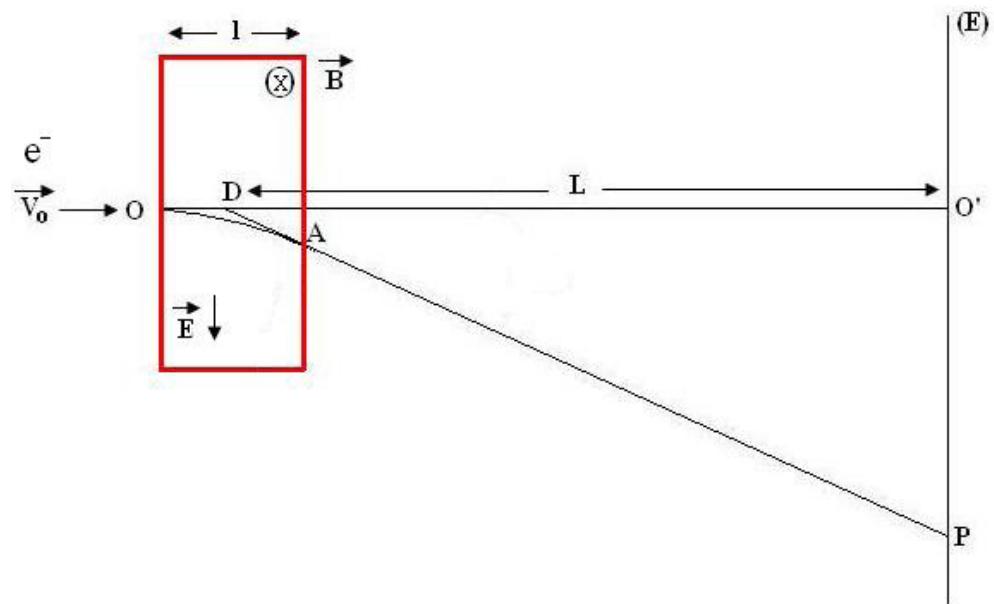
1) Simultaneous action of a field $\vec{B}$ and of a field $\vec{E}$.

Some electrons penetrate with a speed $\vec{v_0}$ in a region where the magnetic field $\vec{B}$ and the electric field $\vec{E}$ predominate. $\vec{E}$ and $\vec{B}$ are uniform and perpendicular between themselves and orthogonal to $\vec{v_0}$.

When $\vec{E} = \vec{O}$ and $\vec{B} = \vec{O}$, the beam does not deviate and hits the screen (E) in O'.

We apply the field $\vec{B}$ while maintaining $\vec{E} = \vec{O}$: the electrons undergo a magnetic deflection toward the bottom:

$$D_m = O'P = \frac{L\,l\,B}{v_0}\ \frac{e}{m} \qquad\qquad (1)$$

We then apply the field $\vec{E}$. Under the influence of $\vec{E}$, the electrons deviate toward the top. We act on the value E to return the beam in O'. Thus, the magnetic and electric forces compensate for one another and the magnitudes result as follows

$$\left|\vec{F}_e\right| = \left|\vec{F}_m\right|$$

$$\left|-e\vec{E}\right| = \left|-e\,\vec{v}_0\,\Lambda\,\vec{B}\right|$$

We obtain : $E = v_0 B \Rightarrow v_0 = \dfrac{E}{B}$

Relation (1) then becomes

$$D_m = \frac{L\,l\,B^2}{E}\ \frac{e}{m} \Rightarrow\ \frac{e}{m} = \frac{E}{L\ l\,B^2}D_m$$

L and l are the geometrical parameters of the mechanism and are assumed. E and

B are adjusted in such a maner that $\dfrac{E}{B} = v_0$. We measure D_m on the screen and we

can deduce $\dfrac{e}{m}$

b) Action of a field $\vec{B}$ alone

Under the influence of $\vec{B}$, we obtain a magnetic deflection

$$D_m = \frac{L\,l\,B}{v_0}\ \frac{e}{m}$$

The speed v_0 can be deduced from the accelerating tension U (the device contains an accelerating tension which communicates the speed v_0 to the electrons):

$$\frac{1}{2} m v_0^2 = e\, U$$

$$\Rightarrow v_0 = \left(\frac{2eU}{m} \right)^{\frac{1}{2}}$$

Whence :
$$D_m = \frac{L\ l\ B}{\sqrt{2U}} \sqrt{\frac{e}{m}} \quad \Rightarrow \frac{e}{m} = \frac{2U}{(L\ l\ B)^2} D_m^2$$

Knowing U, B and D_m (D_m is measured on the screen), we calculate $\dfrac{e}{m}$.

c) Rarified gas tube method

This method uses the circular movement in a uniform magnetic field.

The device used consists of a spherical light-bulb equipped with an electron gun and containing a very low pressure gas. The gun sends a beam of electrons whose speed is controlled by an accelerating tension U. The beam ionizes the gas in its passage, which renders the trajectory visible. The light-bulb is placed between two Helmoltz coils which create a uniform magnetic field $\vec{B}$ at the level of the trajectory of the electrons.

We orient the sphere such that the speed $\vec{v}$ of the electrons leaving the gun are normal to

$\vec{B}$ and we act appropriately on $\vec{B}$ and on the accelerating tension to obtain a circular

trajectory of radius $R = \dfrac{mv}{eB}$

We have here: $\dfrac{1}{2} mv^2 = e\, U \implies v^2 = 2\, U \left(\dfrac{e}{m}\right)$ (1)

$$e\, v\, B = m\, \dfrac{v^2}{R} \implies v = BR \left(\dfrac{e}{m}\right) \quad (2)$$

Let us raise the relation (2) to the square

$$v^2 = B^2\, R^2 \left(\dfrac{e}{m}\right)^2 \qquad (3)$$

(1) and (3) give the result :

$$B^2\, R^2 \left(\dfrac{e}{m}\right)^2 = 2\, U \left(\dfrac{e}{m}\right)$$

Thus : $\dfrac{e}{m} = \dfrac{2\,U}{B^2 R^2}$

Knowing B, U and R, we calculate $\dfrac{e}{m}$.

C) Determination of the sign and of the number of charge carriers per unit of volume in a conductor

Read over the part of the course dealing with the Hall Effect.

D) Acceleration of the charged particles

Read over the paragraph on "Cyclotrons" given in the course

Part 2

Electromagnetic induction field

Let us consider a metal stem MN placed on two parallel live rails whose extremities P and Q are connected to a galvanometer.
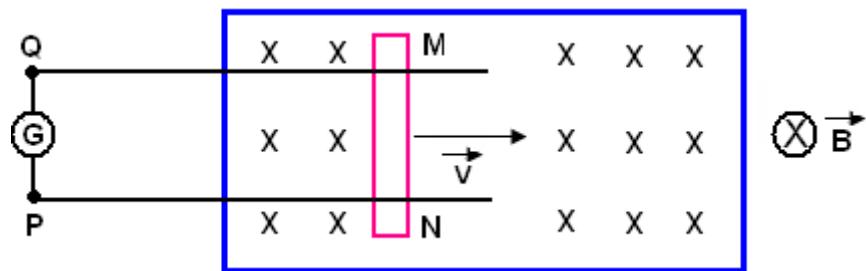
When we move the stem with a speed $\vec{F}$, the galvanometer detects a current. The appearance of this induced current is linked to the existence of a magnetic force

$\vec{F}_m = - e \ \vec{F} \wedge \vec{B}$ which acts on each free electron free of the stem . We associate this force to an electromagnetic induction field $\vec{F}_m$ such that $\vec{F}_m = - e \vec{E}_m$.

Inversely to an electrostatic field, $\vec{E}_m$ can maintain the movements of the charge carriers in a closed circuit.

What emerges from this association is that:

a)

$- e \ \vec{F} \wedge \vec{B} = - e \ \vec{E}_{m} => \vec{E}_m = \vec{F} \wedge \vec{B}$

$\vec{F}, \vec{B}$ and $\vec{E}_m$ form a direct trihedron

The direction of $\vec{E}_m$ indicates the direction of the induced current. Since $\vec{F}_m = - e$ $\vec{E}_m$ the electrons move in the opposite direction to $\vec{E}_m$.

RL Circuit – Growth rate of the current

Let us consider the circuit below.
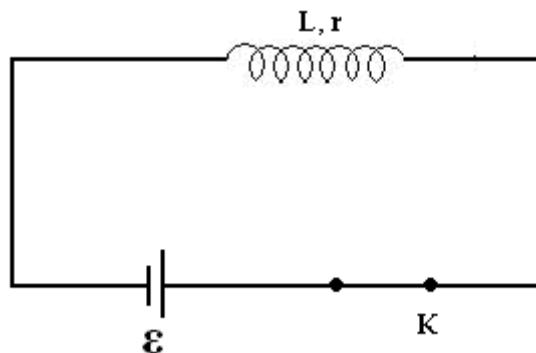
The evolution of the intensity of the current passing through the circuit is described by the differential equation:

$$\varepsilon - L\frac{di}{dt} - ri = 0 \qquad \Rightarrow L\frac{di}{dt} = \varepsilon - ri$$

$\dfrac{di}{dt}$ indicates the growth rate of the current.

At the moment $t = 0$, $i = 0$ thus we get :

$$L\frac{di}{dt} = \varepsilon \Rightarrow \frac{di}{dt} = \frac{\varepsilon}{L}.$$

This represents the initial growth of the current :

$$\left(\frac{di}{dt}\right)_{t=0} = \frac{\varepsilon}{L}$$

As the intensity of the current augments, ri augments and $L\dfrac{di}{dt} = \varepsilon - ri$ diminishes.

The growth rate of the current diminishes: the curve $i = f(t)$ progressively flattens.
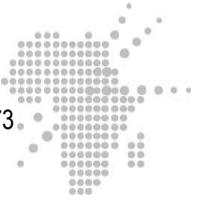
Given enough time ($t \rightarrow \infty$) the growth rate cancels :

$$L\frac{di}{dt} = 0 \Rightarrow r\,i = \varepsilon$$

The intensity of the current reaches its maximum value : $i_m = \dfrac{\varepsilon}{r}$

At the moment $t = 0$, i.e. at the origin, the curve $i = f(t)$ is tangent to the right-hand

side of the slope $\left(\dfrac{di}{dt}\right)_{t=0} = \dfrac{\varepsilon}{L}$.

When we introduce the constant of time of the circuit as defined by $t = \dfrac{L}{r}$ we notice that:

$$\frac{i_m}{\tau} = \frac{i_m}{\left(\dfrac{L}{r}\right)} = \frac{r}{L} i_m = \frac{r}{L}\left(\frac{\varepsilon}{r}\right) = \frac{\varepsilon}{L} = \left(\frac{di}{dt}\right)_{t=0}$$

At the origin, the curve $i = f(t)$ is therefore tangent to the right-hand side of the

slope $\dfrac{i_m}{\tau}$.

Thus we obtain the graphic representation as follows :
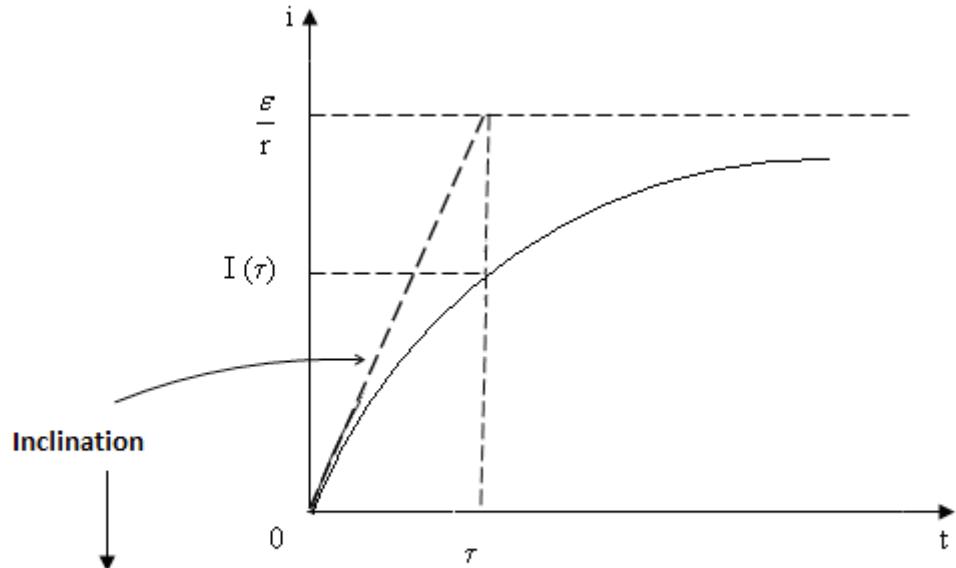
$$\frac{i_m}{\tau} = \left(\frac{di}{dt}\right)_{t=0} = \frac{\varepsilon}{L}$$

Let us calculate i (τ).

The intensity of the current is given by:

$$i\,(t) = \frac{\varepsilon}{r}\,(1\text{-}\,e^{-\frac{r}{L}t}\,) = i_m\,(1\text{ - }e^{-\frac{r}{L}t}\,)$$

At the instant t, we have :

$$i\,(t) = i_m\,(1\text{ - }e^{-\frac{r}{L}\tau}\,) = i_m\,[1\text{- }e^{-\frac{r}{L}\left(\frac{L}{r}\right)}\,]$$

$$i\,(t) = i_m\,(1\text{ - }\frac{1}{e}) = i_m\,(\frac{e-1}{e}) = 0,63 i_m$$

$$i\,(t) = \underline{(63\%)\,(i_m)}$$

After a time $t = \dfrac{L}{r}$ seconds, the intensity of the current reaches 63% of its final value i_m.

Exercise 1

Effect of a uniform magnetic field on a charged particle

Consider the graph below. A charged particle at speed q positive, with initial speed $\vec{v_0}$ and with mass m penetrates in 0 in a region of width I where predominates a uniform and constant magnetic field $\vec{B}$. The field $\vec{B}$ is perpendicular to the plan of the figure and oriented toward the front of this plan. The speed $\vec{v_0}$ is directed following 00' and is normal to $\vec{B}$.

1) Analyze the movement of the particle in the region where the field $\vec{B}$ predominates.

   A screen (E) perpendicular to 00' is placed at a distance $L \gg l$ as the figure indicates. In the absence of field $\vec{B}$, the particle describes a rectilinear trajectory and hits the screen 0'. When $\vec{B}$ is applied, this point of impact is deviated and ends up in P. The deviation 0'P is called magnetic deflection D_m. In the case of a weak deviation, express D_m as a function of L, l, B, m, v_0 and q.

2) The analysis of D_m suggests a few practical applications. Show some applications of this device and justify them.
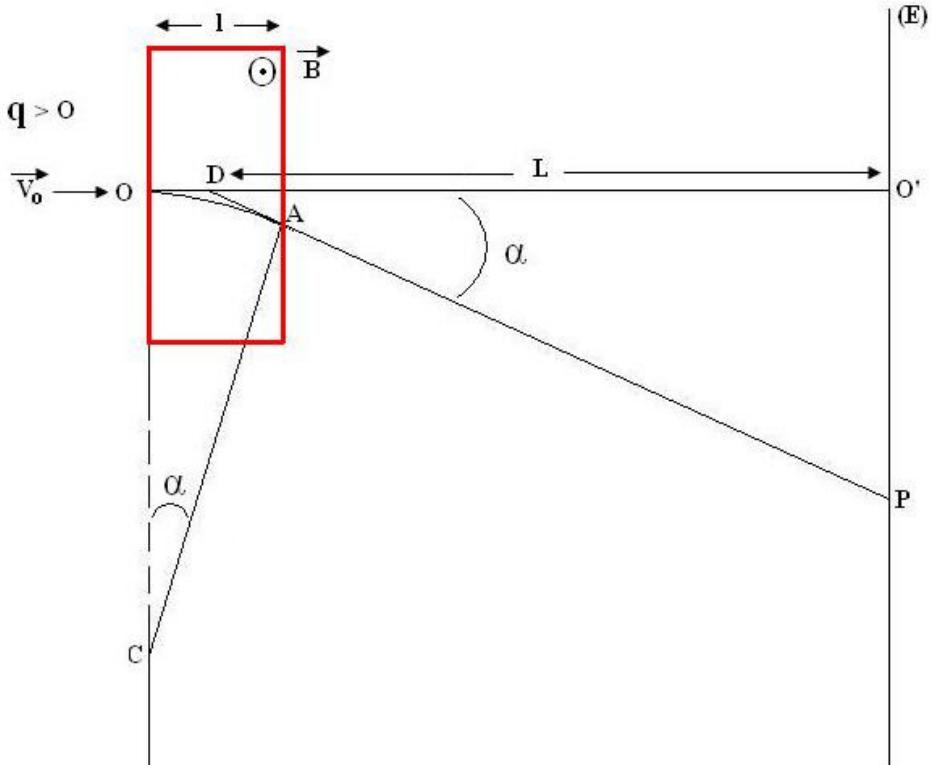
Exercise 2

In an experiment on the Hall effect, a current of intensity I=5A is passed through a metallic band with thickness a=0,1 mm and of width b=5mm. The measured Hall tension is equal to V_H = 6,25 μV and the magnetic field used is of size B=0,2 T.

1- Calculate the number of conduction electrons n per unit of volume.
2- Calculate the chain reaction speed of these electrons.

Exercise 3

A proton initially at rest is subjected to an accelerating tension of 20 kV. It then penetrates a region where a uniform magnetic field $\vec{B}$ of size equal to 0,5T predominates. The field $\vec{B}$ is orthogonal to the speed of the proton. Determine the radius of the trajectory of this proton.
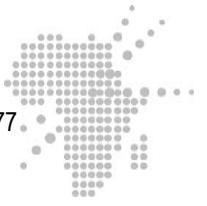
Exercise 4

Lenz's law

1) A beam of electrons penetrates a region where a uniform and constant magnetic field predominates with a speed $\vec{v}$ as the figure indicates. Field $\vec{B}$ is perpendicular to the plane of the figure and is directed toward the back of this plan.

Describe what will happen.
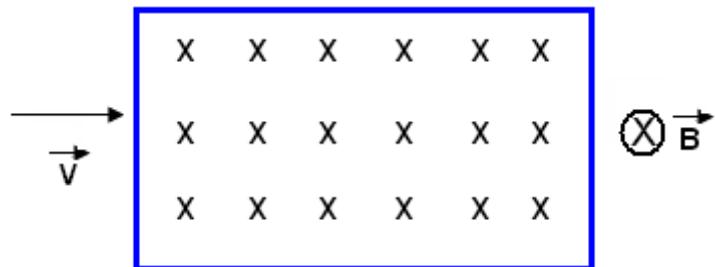
2) We replace the beam of electrons with a metallic stem MN. The stem moves toward the right-hand side with a speed $\vec{v}$ under the influence of an applied force.
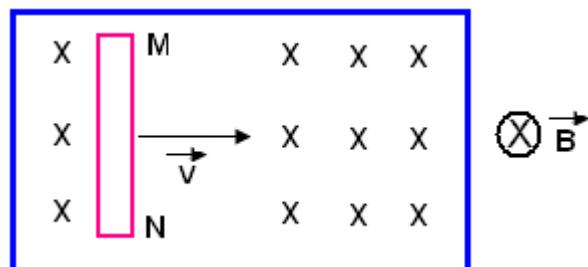
Analyze what will happen with the stem.

3) The metallic stem MN of length I is now placed on two live, horizontal and parallel rails whose extremities P and Q are connected to a galvanometer G.

We move MN toward the right with a speed $\vec{v}$. The galvanometer detects the passage of a current although the circuit constituted by the stem, the rails and the galvanometer does not have a generator.

Explain the origin of the current circulating in the circuit.

Describe the direction of this current and relate it to Lenz's law.
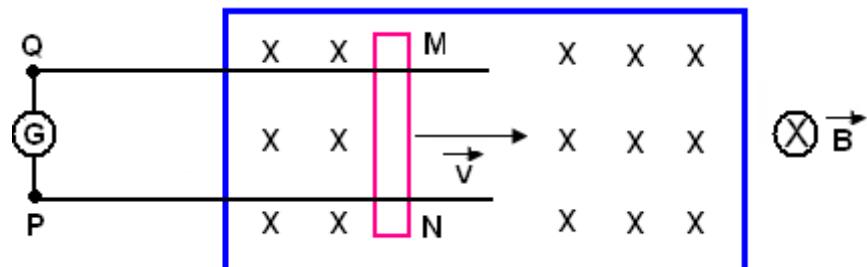
Exercise 5

A rectangular spiral is placed in a uniform magnetic field $\vec{B}$ as the figure (2-a) indicates and whose norm varies according to the figure (2-c). Check the correct answers.
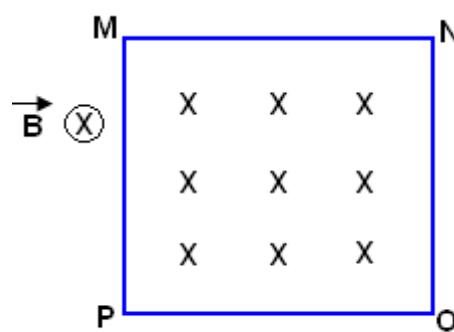
(2 - a)
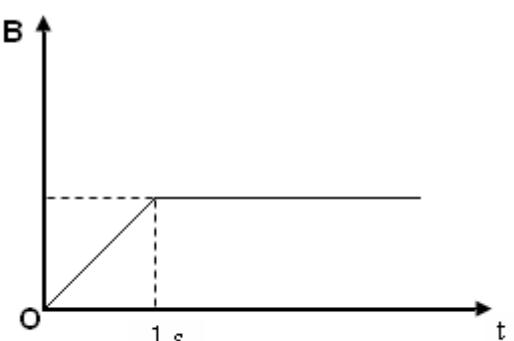
(2 - b)

5-1

☐ An induced current passes through the spiral regardless of t.

☐ An induced current passes through the spiral after a time t > 1s.

☐ As induced current passes through the spiral for   t [0, 1s]

<u>5-2</u>

☐ The induced current passes through the spiral following the direction MNOP

☐ The induced current passes through the spiral following the direction MPON

Exercise 6

Auto-induction and the RL circuit

Consider the circuit below.
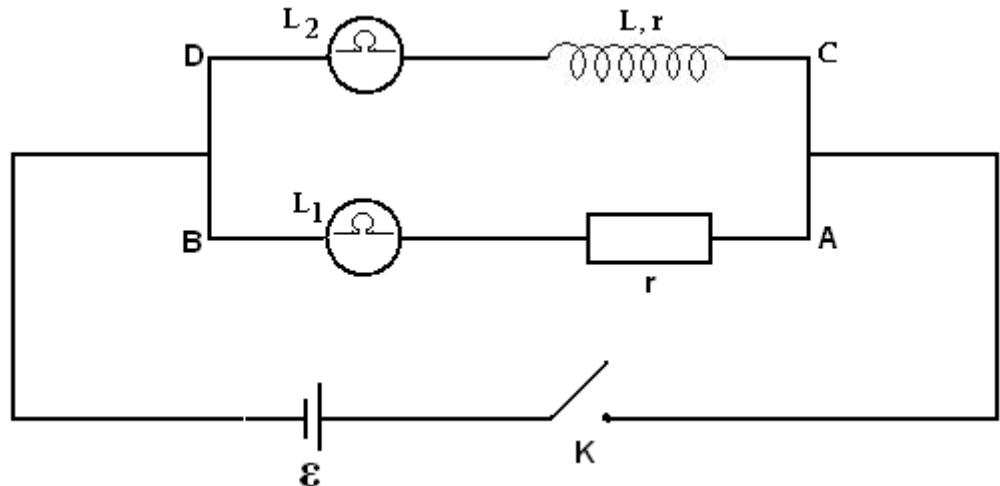
L_1 and L_2 are two identical lamps. The AB branch comprises an ohmic conductor with resistance r in series with the lamp L_1. The CD branch contains a coil in series with the lamp L_2. The ohmic conductor and the coil have the same resistance r.

When we close the switch   K, L_1 lights instantaneously while L_2 only lights up progressively.

  1- Explain why this is so.
  2- Study the currents circulating in the inductance coil and in the ohmic conductor and represent them graphically. Comment on your results.
 3- Describe what happens when we open the interrupter.

Exercise 7

Let us consider a coil fixed to a solenoid through which passes a current with intensity I. Explain why an induced f.e.m. is engendered in the coil when we vary I.

Exercise 8

Let us consider the figure below
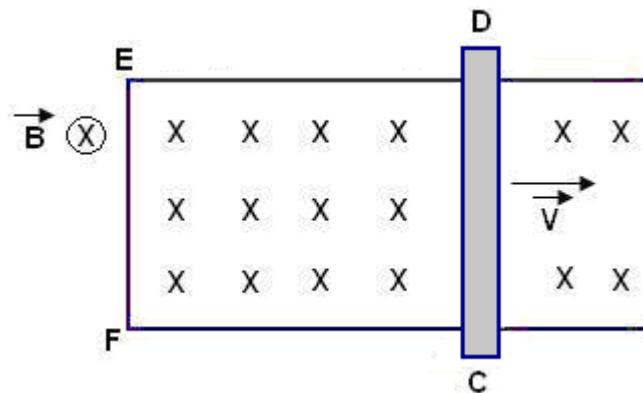
The magnetic induction field $\vec{B}$ is uniform and perpendicular to the plane. The metallic stem CD moves to the right with a speed $\vec{v}$. The induced f.e.m. in CD is 2,40V. The givens are : EF = CD = l = 0,40m and B = 1,2 T. The total resistance of the circuit is 1,2Ω.

Determine:

1-   the speed v of the stem CD

2-   the direction and intensity of the induced current

3-   the direction and the magnitude of the force which is exerted on CD

Exercise 9

Consider the figure below
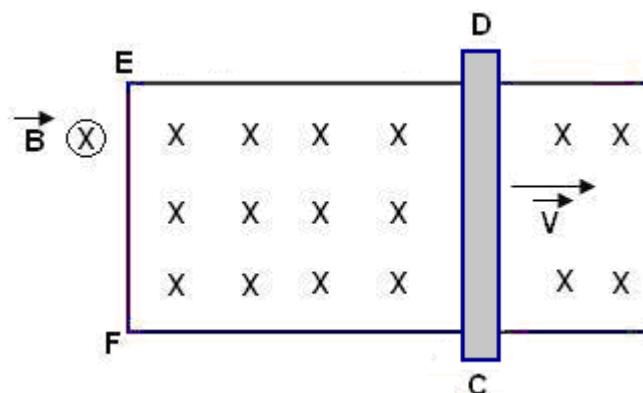
The magnetic induction field $\vec{B}$ is uniform and perpendicular to the plane of the figure. The metallic stem CD moves to the right with a speed $\vec{v}$ perpendicular to $\vec{B}$ An induced current passes through the circuit in the direction $C \rightarrow D$.

1) Analyze the appearance of this circuit from the standpoint of an observator S at rest in relation to the stem.
2) Same question but with the observatory S connected to the stem.

Exercise 10

Let us consider a solenoid with section S, comprised of N spirals and whose length I is assumed large in relation to its diameter. We vary the current passing through this solenoid from 0 to 1. Find its self-inductance.

Exercise 11

An induced 10V e.m.f. establishes itself in a coil with a current passing through it with a growth rate of 50 As⁻¹. Calculate the inductance of the coil.

Let us suppose that this coil comprises 500 spirals and that the intensity of the current passing through it is 2.5A. Determine the average flux intertwined by a spiral.

Exercise 12

An inductance coil $L = 5$ H and with resistance 10 W is connected to a 10V battery. Calculate:

- The initial rate of growth of the current
- The intensity of the current at the instant when its growth rate is equal to 1 A s⁻¹
- The final intensity of the current

Exercise 13

Let us consider a circuit series composed of an inductance coil $L = 0,2$ H whose resistance is negligible, of a resistor of 9 W , of a battery equal to 1,5 V and of internal resistance 1W. Calculate :

- The initial rate of growth of the current
- The growth rate of the current when the intensity of the current is equal to 0.1A
- The growth rate of the current when the intensity of the current is equal to 0,15A

Exercise 14

Let us consider a circuit series composed of an inductance coil L, a resistance R, of a battery with e.m.f V_0 and an interrupter K. We close the switch . Determine :

1- the intensity of the current and the incuced e.m.f. at the instant where t=0
2- the intensity of the current and the induce e.m.f. at the instant when t =

$\frac{1}{2}\tau$, τ being equal to the time constant of this circuit.

Learning Activities

- Read the appropriate material and the course complements
- Do the exercises
- Organize yourselves to do collaborative work: the learners are separated into groups under the supervision of a tutor
- Each group chooses a delegate
- The tutor organizes the work by indicating what order the exercises should be done in and the time needed to research the solutions for each one
- The delegates leave the solutions for the exercises in a work-space reserved for the learners.

Answer Key

Solution to exercise 1

1) Movement of a particle

$\vec{v_0}$ is orthogonal to $\vec{B}$.

In the region where $\vec{B}$ predominates, the force $\vec{F} = q\,\vec{v} \wedge \vec{B}$ which exerts itself on the particle is always perpendicular to $\vec{v}$. Therefore the effect of this force is always null and in consequence the speed $\vec{v}$ remains constant in size $(v = v_0)$. See below for a demonstration.

$$E_C = \frac{1}{2}m\,\vec{v}^2 \Rightarrow dE_c = m\,\vec{v}\,d\vec{v}$$

$dE_c = \vec{F}.\vec{v}\,dt = 0$ since $\vec{F}$ is perpendicular to $\vec{v}$

$\Rightarrow E_C$ is constant $\Rightarrow v$ is constant

The force $\vec{F}$ also stays constant in magnitude $(F = q\,v\,B = qv_0B)$ since v and B are constant. The particle therefore moves under the effect of the force $\vec{F}$ whose magnitude is constant and whose direction is always perpendicular to the speed vector $\vec{v}$: it then has a uniform circular movement.

$\vec{F}$ is a centripetal force :

$$qv_0B = \frac{mv_0^2}{R}$$

The radius of the trajectory is:

$$R = \frac{mv_0}{qB}$$

In the region where B predominates, the particle describes the circle arc $\overset{\frown}{CA}$ with

centre C and with radius $CO = CA = \dfrac{mv_0}{qB}$.

2) Deviation O'P

When the particle leaves the region where $\vec{B}$ predominates, its trajectory is rectilinear following the tangent A to the arc of the circle $\overset{\frown}{OA}$. This tangent dissects OO' at D.

The angle a is such that :

$$a = \dfrac{\overset{\frown}{OA}}{R}$$

$$\tan a = \dfrac{0'P}{L}$$

For a minor deviation :

$\overset{\frown}{OA}$ l and tg a $\quad \alpha = \dfrac{0'P}{L}$

Therefore :

$$\alpha = \dfrac{l}{R} = \dfrac{0'P}{L} \quad \Rightarrow \quad 0'P = \dfrac{L\,l}{R} = \dfrac{L\,l\,q\,B}{mv_0}$$

0'P is the point of impact deviation on the screen. We call this deviation : « Magnetic deviation » D_m.

$$D_m = \dfrac{L\,l}{mv_0}\,q\,B$$

3) Applications of such a construction

The magnetic deflection depends on the parameters $L, l, B, \dfrac{q}{m}$ and v_0. The parameters L and l are geometrical (they are the geometrical characteristics of the device). They are assumed given.

1st application

Let us consider a beam of particles which penetrate in O. Let us assume that all these

particles have the same speed v_0 and that they have the same relation $\dfrac{q}{m}$.
In such a case:

$$D_m = \left(\frac{L\, l}{v_0} \; \frac{q}{m} \right) B = K_1 B$$
$$\text{Constant } K_1$$

The deviation D_m is proportional to the field B. Knowing D_m, we can determine B : the device allows us to measure the magnetic fields.

2nd application

Let us suppose that:

- The relation $\dfrac{q}{m}$ is the same for all the particles constituting the beam incident

- The particles penetrate O with different initial speeds v_0

- The field $\vec{B}$ is given.

In this case, we have

$$D_m = \left(L\, l\, B\, \frac{q}{m} \right) \frac{1}{v_0} = K_2 \frac{1}{v_0}$$

The deviation D_m is proportional to $\dfrac{1}{v_0}$ since $\left(L\,l\,B\,\dfrac{q}{m}\right)\dfrac{1}{v_0} = K_2$ is a constant.

With this device, we can sort monocinetic beams. We can separate beams which penetrate O with different initial speeds.

3rd application

Let us suppose that a beam of positive homocinetic particles with the same charge but with different masses penetrate 0 and that the field $\vec{B}$ is given.

We then obtain:

$$D_m = \left(\dfrac{L\,l\,B}{v_0}\right)\dfrac{q}{m} = K_3 \dfrac{q}{m}$$

$\left(\dfrac{L\,l\,B}{v_0}\right) = K_3$ is a constant for all these particles . The deviation D_m is proportianal

to the relation $\dfrac{q}{m}$. The device allows us in this case to separate ions with the same charge but with different masses. We have here a mass spectrometer.

Solution to exercise 2

**1) number of conduction electrons n per unit of volume**

We know that the Hall tension V_H is associated with a transversal electrical field E_H such that :

$E_{H =} v\,B$ (1)   v being equal to the speed drive (read the course material again if necessary).

Let us use this equation (1) to solve V_H and n.

E_H and V_H are linked by the relation :

$E_H = \dfrac{V_H}{b}$, b being the size of the band
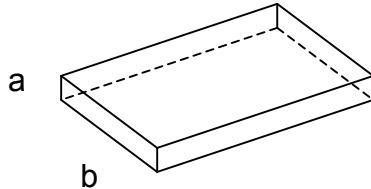
Thus we have: $\dfrac{V_H}{b} = v\,B \Rightarrow V_H = v\,B\,b$ (2)

To solve n, let us replace the speed drive v.

To do this, let us take the intensity I :

  I = j A, j being the density of the current and A the cross sectional area of the band.

We have : $j = n\,e\,v$ and $A = ab$

Whence $I = n\,e\,v\,ab \Rightarrow v = \dfrac{I}{n\,e\,(ab)}$ (3)

Equations (2) and (3) are solved as follows :

$$V_H = \frac{I\,B\,d}{n\,e\,(ab)} \Rightarrow n = \frac{I\,B}{e\,a\,V_H}$$

A.N.: $e = 1{,}6\ 10^{-19}$ C ; $B = 0{,}2$ T ; $I = 5$ A ; $V_H = 6{,}25 \times 10^{-16}$ V ; $a = 10^{-4}$ m

$$n = \frac{(5)(0{,}2)}{(1{,}6.10^{-19})(10^{-4})(6{,}25.10^{-6})}$$

$n = \underline{\mathbf{1 \times 10^{28}\ m^{-3}}}$

2) Speed Drive

Let us consider once again the density of the current

$$j = n\,e\,v$$

$$\frac{I}{A} = n\,e\,v \Rightarrow v = \frac{I}{n\,e\,A}$$

A.N. :

$I = 5\ A;\quad n = 1 \times 10^{28}\ m^{-3}\ ;\quad e = 1{,}6 \times 10^{-19}\ C;$

$$A = ab = (10^{-4}\ m)\,(5.10^{-3}m)$$

$$v = \frac{5}{(1 \times 10^{28})(1{,}6.10^{-19})(10^{-4})(5.10^{-3})}$$

v = 0,625 x 10⁻² ms⁻¹

Solution to exercise 3

In the region where the field $\vec{B}$ predominates, the proton is subjected to a force :

$$\vec{F} = q\,\vec{v} \wedge \vec{B}$$

$\vec{F}$ is here a centripetal force

$$q\,v\,B = m\frac{v^2}{R} \Rightarrow R = \frac{mv}{qB} \tag{1}$$

Let us take V_a the accelerating tension. We have :

$$qV_a = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2qV_a}{m}}$$

Relation (1) becomes :

$$R = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV_a}{m}} = \frac{1}{B}\sqrt{\frac{2V_a m}{q}}$$

A.N. : $B = 0{,}5T\ ;\quad m = 1{,}66 \times 10^{-27}\ kg\ ;\quad q = 1{,}6 \times 10^{-19}\ C\ ;\quad V_a = 20 \times 10^3\ V$

We find :

R = 4,1 cm

Solution to exercise 4

1) beam of electrons
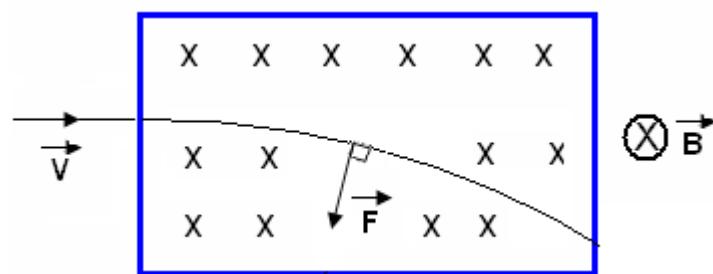
Each electron from the beam is subjected to a magnetic force $\vec{F} = -e\,\vec{v} \wedge \vec{B}$.

$\vec{F}$ is always perpendicular to $\vec{v}$ and $\vec{B}$ and has a constant size $F = e\,v\,B$. The direction of the force $\vec{F}$ is always normal to the trajectory of the beam, at the vector $\vec{v}$. The beam is then subjected to a magnetic deflection: its trajectory bends or curves.

If the force $\vec{F}$ is weak, this trajectory describes a circular arc. We obtain a circular trajectory if the force $\vec{F}$ is strong enough.

2) Metallic Stem
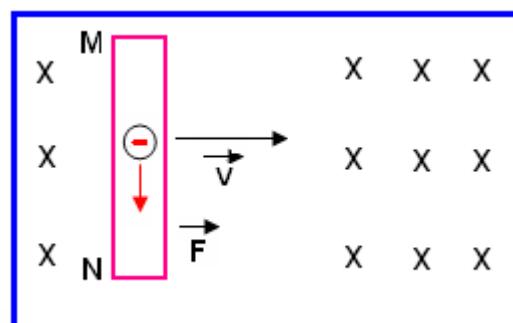
Metals contain free electrons called conduction electrons. When the stem MN moves at a speed $\vec{F}$, its free electrons are pulled with it and at the same speed. Each MN electron in MN therefore moves in a magnetic field with speed $\vec{F}$ and undergoes a force $\vec{F}_m = -e\,\vec{F} \wedge \vec{B}$.

Under the effect of this force, it moves toward N. Negative charges accumulate in N and positive charges in M (there is a lack of negative charges in M). This results in an induced e.m.f. between the extremities of the stem.

3) Origin of the induced current
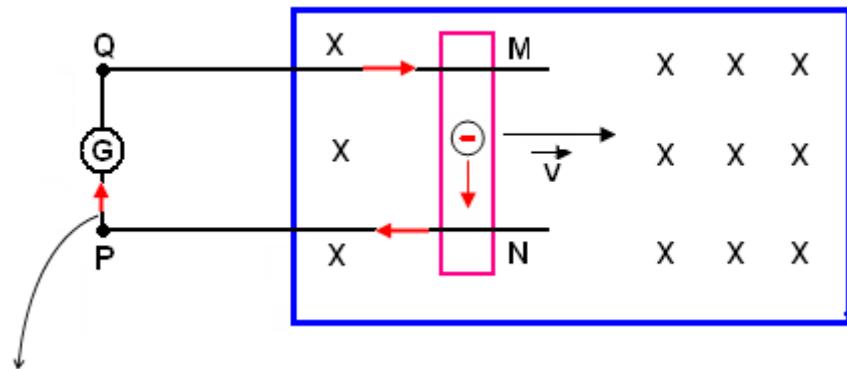
Direction of the movement of the electrons

As we established in previously (question 2), when the stem moves toward the right, each electron is subjected to a force $\vec{F}_m = -e\,\vec{F} \wedge \vec{B}$ and is pushed toward the extremity N. The circuit MNPQ being closed, a concerted movement of the free electrons takes root in this buckle resulting in the appearance of the current detected by the galvanometer.

The force $\vec{F}_m$ is at the origin of this current.

Direction of the induced current

The direction of this induced current is opposite to the direction of the movement of the electrons.
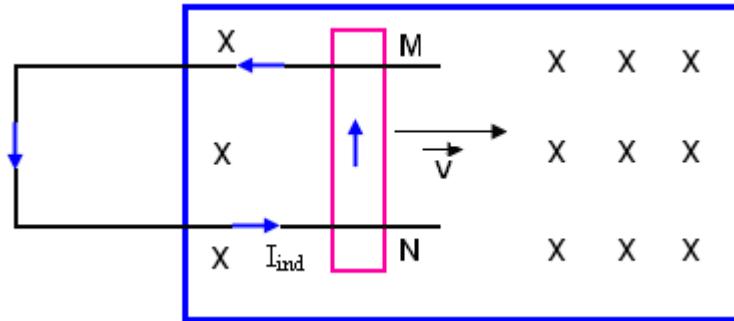
Relationship with the Lenz law

A current I_{ind} now passes through the stem MN in the direction $N \rightarrow M$ and moves in the field $\vec{B}$. A force $\vec{F} = I_{ind}\ \overrightarrow{NM} \wedge \vec{B}$ acts on this stem. This force is guided toward the left, in the opposite direction to that of MN : it opposes itself to the movement of MN toward the right.

This is none other than the Lenz law which states that : « the direction of the induced current is such that it is opposed to the cause that which produces it ».

The cause which produced the induced current is here the movement of the stem toward the right. Once the current passes through the stem, the stem is subjected to a force which guides it toward the left.

Solution to exercise 5

<u>2-1</u>

❑ An induced current passes through the spiral regardless of t. Incorrect answer

An induced current only appears when the mgnetric flux varies throughout the spiral:

i.e. it only appears when $\vec{B}$ varies . Whereas here the norm for $\vec{B}$ remains constant for t > 1s.

❑ An induced current passes through the spiral after a given time t > 1s. False

The field $\vec{B}$ does not vary anymore when t > 1s

❑ An induced current passes through the spiral for t [0, 1s] Right answer

Indeed, during this interval $\vec{B}$ the flux varies throughout the spiral.

2-2

❑ The induced current passes through the spiral in the direction MNOP False

See the explanation below

❑ The induced current passes through the spiral in the direction MPON Correct answer

The direction of the induced current is such that it creates an induced magnetic field $\vec{B}_i$ which opposes the growth of $\vec{B}$. $\vec{B}_i$ is in the opposite direction to $\vec{B}$ i.e. it goes toward the front : I_{ind} passes through the spiral in the direction MPON.

Solution to exercise 6

1- Lamp L_1 lights instantaneously, lamp L_2 lights only progressively

Observations

The explanation must therefore reside in the coil

Initially, when the interrupter is open, there is no current passing through the coil: the field inside the coil is null as well as the flux passing through it.

When we close the circuit, the current passing through the ohmic conductor reaches its equilibrium value rapidly while the current passing through the coil starts to augment. This current creates a magnetic field inside the coil. The flux passing through this has thus varied and an induced f.e.m. appears which, in conformity to Lenz' law, opposes itself to the current growth which will thus take a certain amount of time before reaching its equilibrium value.

The coil delays the increase of the current which circulates in lamp L_2 so that in consequence L_2 lights only progressively.

Comment: the flux which passes through the coil varies. This variation is due to the variation of the current which is circulating in the coil itself: the induced f.e.m. here is an *auto-induction e.m.f.* The flux which passes through the coil is a "self flux"/

The study of currents passing through coils and in the ohmic conductor proves these results.

2- Analysis of currents passing through the coil and the ohmic conductor

Let us disregard the resistances passing through lamps L_1 and L_2 (or let us assume that they are already included in the resistances r). When we close the switch K, we have the loop ($\varepsilon CD\varepsilon$) :
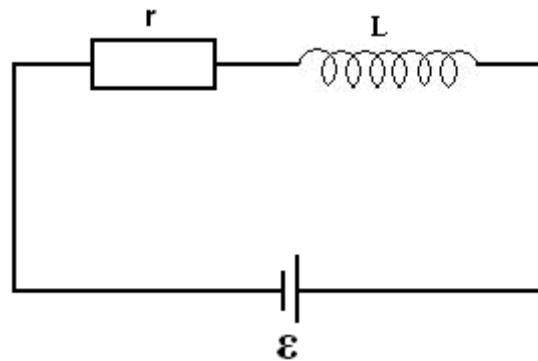
$$e - L\frac{di}{dt} - r\,i = 0$$

The solution to this differential equation of the first order and with constant coefficients is:

$$i = \frac{\varepsilon}{r}\left(1 - e^{-\frac{r}{L}t}\right)$$

The following table shows the variation of i as a function of time t.

| t	0		∞
$e^{-\frac{r}{L}t}$	1	$\searrow$	0
$\left(1 - e^{-\frac{r}{L}t}\right)$	0	$\nearrow$	1
i	0	$\nearrow$	$\dfrac{\varepsilon}{r}$
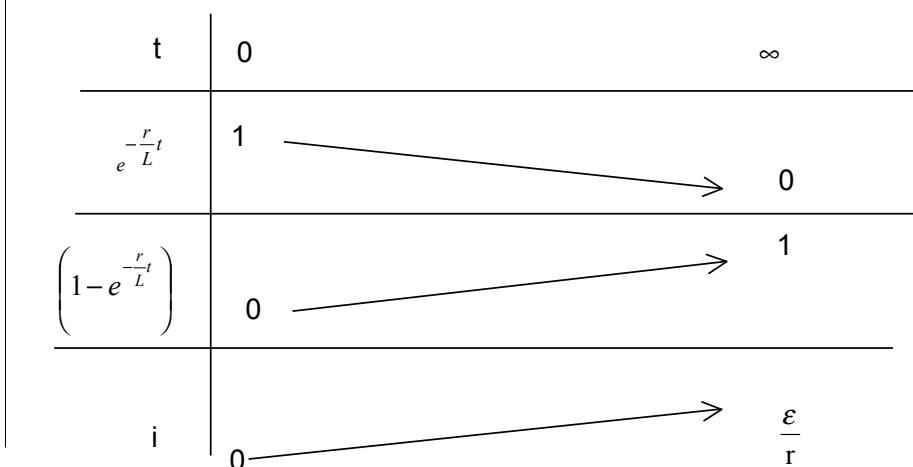
What we can see from this table is that the current increases progressively and after a certain time reaches its maximum value which corresponds to the permanent regime: this explains why the lamp L_2 lights progressively.

In the branch with lamp L_1 we only have the ohmic conductor with resistance r.

$$L = 0 \Rightarrow e^{-\frac{r}{L}t} = 0 \Rightarrow i = \frac{\varepsilon}{r}$$

The current reaches its maximum quasi instantaneously.

The graphic representation of I as a function of t indicates:
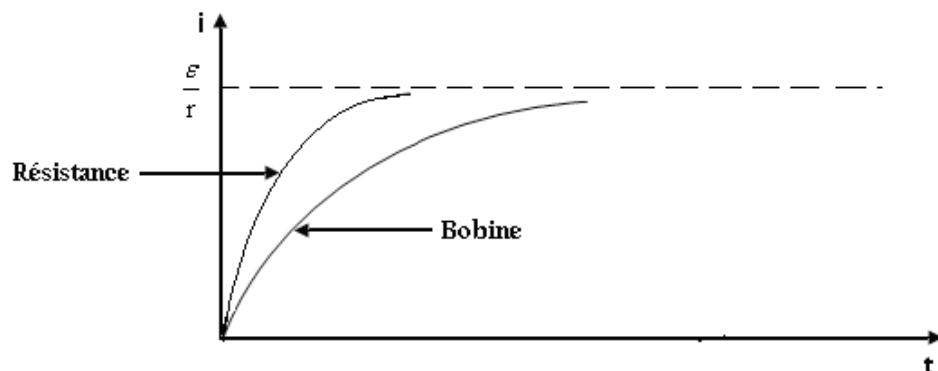
3) The interrupter is open again

When we open the interrupter K again, the intensity of the current in the CD branch diminishes immediately, but does cancel out instantaneously. A strong f.e.m. is induced in the circuit of the coil. This e.m.f is opposed to diminution of i. This current then diminishes progressively. *There is a delay in the decrease of the current and the lamp L_2 goes out progressively as opposed to L_1.*

The graphic representation of the current passing through the coil clearly describes this behaviour.

When the switch is open, the generator is put "out of circuit" and we have:

$$L\frac{di}{dt} + ri = 0 \quad \Rightarrow \quad \frac{di}{i} = -\frac{r}{L}dt$$

Whence :
$$i = C\,e^{-\frac{r}{L}t}$$

C is a constant determined by the initial condition : at the instant $t = 0$ $i = \dfrac{\varepsilon}{r} = C.$

We then obtain: $i = \dfrac{\varepsilon}{r}\, e^{-\frac{r}{L}t}$.

After a certain time, $i \rightarrow 0$
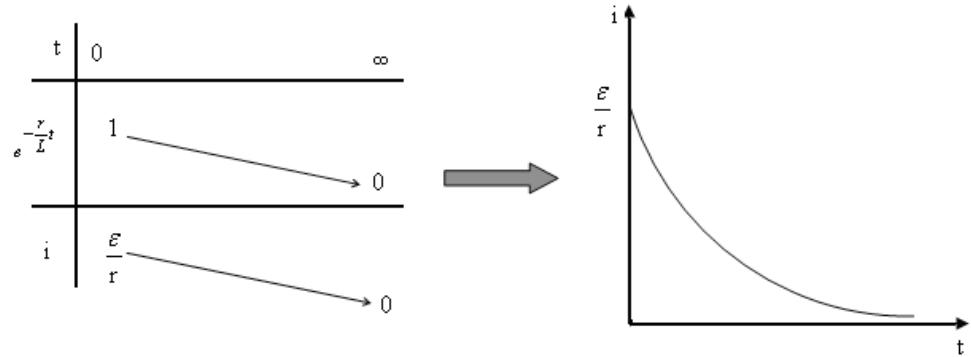
Exercise 7

The magnetic field created by the solenoid depends on I. A variation of I provokes a variation in the magnetic field passing through the coil, and therefore a variation in the magnetic flux passing through the coil whence the appearance of an induced.e.m.f

Exercise 8
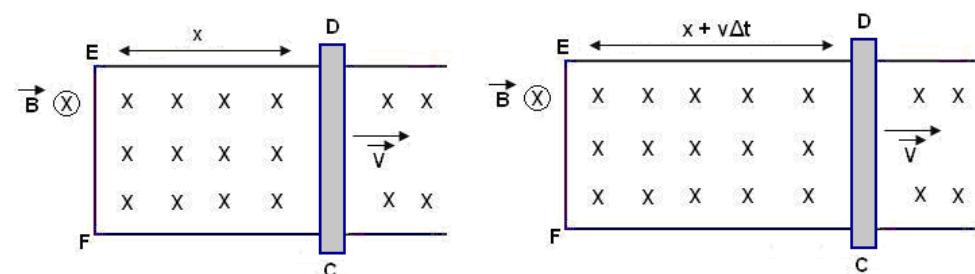
The initial flux is :

$$\Phi_i = B\, l\, x$$

During the interval of time Δt, the CD stem moved a distance of $v\Delta t$ and the inter-twined flux is :

$$\Phi_f = B\, l\, (x + v\, \Delta t)$$

The induced e.m.f. during this movement is :

$$e_{ind} = \frac{\Delta\Phi}{\Delta t} = \frac{\Phi_f - \Phi_i}{\Delta t}$$

We find :

$$e_{ind} = B\,l\,v \quad (1)$$

We deduce from the relation that :

$$v = \frac{\varepsilon_{ind}}{Bl}$$

A.N. : $e_{ind} = 2,40$ V ; CD = EF = l = 0,40 m ; B = 1,2 T

$$v = \frac{2,40}{(1,2)(0,40)} = \mathbf{5\ ms^{-1}}$$

2) Direction of the induced current

The direction of the induced current is such that it is opposed to the movement of the stem toward the right. It goes from C toward D.
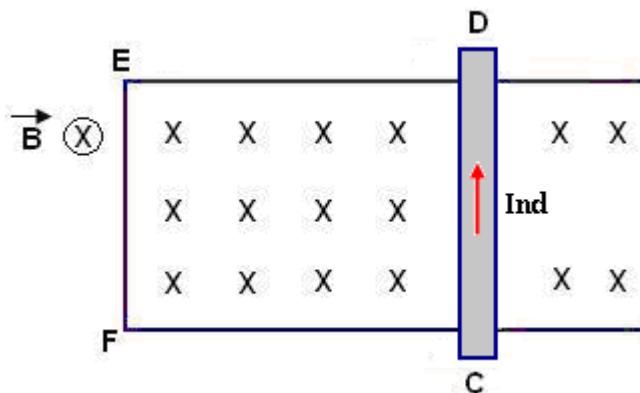

Its intensity is :

$$I_{ind} = \frac{\varepsilon_{ind}}{R}$$

A.N.: $e_{ind} = 2,40$V; R = 1,2 Ω

$$I_{ind} = \frac{2,4}{1,2} = \mathbf{2\ A}$$

3) Force applied on the stem
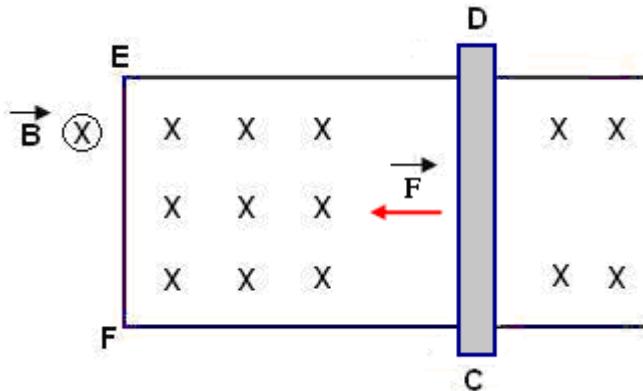
A directed current passes through the stem from C toward D. It is thus subject to a force of :

$$\vec{F} = I_{ind}\ \vec{l}\ \Lambda\ \vec{B}$$

This force is opposed to the movement of the stem toward the right. It is directed toward the left.

The vectors $\vec{l}$ and $\vec{B}$ being perpendicular between them, we have :

$$F = IBl$$

A.N.: Iind = 2 A ; l = 0,40 m; B = 1,2 T

$$F = (2)(0,4)(1,2) = \mathbf{0,96N}$$

Exercise 9

1) The observator S at rest in relation to the stem

For S, a conduction electron from the metallic stem moves with this latter toward the right at speed $\vec{v}$. This electron is subject to a force :

$$\vec{F}_m = -e\vec{v}\ \Lambda\ \vec{B}$$

This force provokes the movement of the electron toward the bottom (D $\rightarrow$ C). The conventional direction of the current is the direction opposite to the direction of the electron : the induced electron goes from C toward D.

2) **The observator S is connected to the stem**

When S is connected to the stem, it sees that it is the field $\vec{B}$ that moves. For this observation, no magnetic force is applied on the electron yet it observes that the same current appears in the metallic stem. To explain this phenomenon, it has recourse to an electromagnetic field $\vec{E}_m$ which provokes the movement of the electron toward the bottom. The electron is subjected to force:

$$\vec{F}_e = -e\vec{E}_m$$

$\vec{E}_m$ is directed from C toward D, in the direction of the induced current. The two forces $\vec{F}_m$ and $\vec{F}_e$ have the same effect on the electron :

$$\vec{F}_e = \vec{F}_m$$

$$-e\vec{E}_m = -e\vec{v} \wedge \vec{B}$$

It follows that : $\vec{E}_m = \vec{v} \wedge \vec{B}$

Exercise 10

When the current passing through the solenoid varies, it is the seat of an induced f.e.m. which can be expressed as:

$$\varepsilon = N\frac{d\Phi}{dt} \quad \text{and } \varepsilon = L\frac{dI}{dt}$$

Φ being the flux intertwined by the spiral. These two expressions result in:

$$N\frac{\Delta\Phi}{\Delta t} = L\frac{\Delta I}{\Delta t}$$

We then obtain:

$$L = N\frac{\Delta\Phi}{\Delta I}$$

When $I = 0$, the flux going through the spiral is: $\Phi_1 = 0$. Let us consider Φ_2 the flux intertwined by a spiral when the current is I. When the current varies from 0 to I, we have :

$$\Delta\Phi = \Phi_2 - \Phi_1 = \Phi_2 \quad \text{et} \quad \Delta I = I$$

It follows that :

$$L = N\frac{\Phi_2}{I} \qquad\qquad (1)$$

But $\Phi_2 = BS$, B being the field created by I and predominating in the solenoid. The length of the solénoïd being large with respect to its diameter, we have :

$$B = \mu_0 n \, I$$

n is the number of spirals per unit of length. Relation (1) then becomes :

$$L = N\frac{BS}{I} = N\frac{\mu_0 n I S}{I} = N\mu_0 nS$$

By replacing n with (N / l), we obtain :

$$L = N^2 \, \frac{\mu_0 S}{l}$$

Exercise 11

1) The inducede.m.f. in the coil is given by:

$$\varepsilon = L\frac{di}{dt} \Rightarrow L = \frac{\varepsilon}{(\dfrac{di}{dt})}$$

A.N. $\varepsilon = 10 \text{ V}$; $\dfrac{di}{dt} = 50 \text{ As}^{-1}$

$$L = \frac{10}{50} = \textbf{0,2 H}$$

2) **The coil is composed of N = 500 spirals. The flux through the coil is NΦ where Φ is the average flux intertwined by a spiral. We have:**

$$N\Phi = Li \implies \Phi = \frac{Li}{N}$$

A.N. $L = 0,2H; i = 2,5A; N = 500$

$$\Phi = \frac{Li}{N} = 10^{-3}H$$

Exercise 12

1) Initial rate of growth of the current

Kirchhoff's 'loop rule' applied to this current gives:

$$e - L\frac{di}{dt} - Ri = 0 \qquad\qquad (1)$$

At the initial instant $t = 0$, $i = 0$. We have therefore :

$$e - L\left(\frac{di}{dt}\right)_{t=0} = 0 \implies \left(\frac{di}{dt}\right)_{t=0} = \frac{\varepsilon}{L} = \frac{10V}{5H}$$

$$\left(\frac{di}{dt}\right)_{t=0} = 2 \text{ A s}^{-1}$$

2) Intensity of the current at the instant when its rate of growth is 1As⁻¹.

Relation (1) gives :

$$e - L\frac{di}{dt} - Ri = 0 \implies Ri = e - L\frac{di}{dt}$$

$$\implies i = \frac{\varepsilon}{R} - \frac{L}{R}\frac{di}{dt}$$

A.N. $\frac{di}{dt} = 1 \text{ As}^{-1}; \varepsilon = 10 \text{ V}; R = 10 \ \Omega$ et $L = 5 \text{ H}$

$$i = \frac{10}{10} - \frac{5}{10}(1)$$
$$\underline{\mathbf{i = 0,5 \ A}}$$

3) final intensity of the current

When the current reaches its final value, $\dfrac{di}{dt} = 0$. Relation (1) then becomes :

$$e = R\, i_m \implies i_m = \dfrac{\varepsilon}{R}$$

The numerical application gives us :

$$\underline{i_m = 1A}$$

Exercise 13

Initial rate of growth

Kirchhoff's 'loop rule' applied to this circuit gives us :

$$e - L\dfrac{di}{dt} - Ri = 0 \qquad\qquad (1)$$

At the initial instant: $t = 0$ and $i = 0$ whence :

$$e - L\left(\dfrac{di}{dt}\right)_{t=0} = 0 \implies \left(\dfrac{di}{dt}\right)_{t=0} = \dfrac{\varepsilon}{L}$$

A.N. $\varepsilon = 1{,}5\ V$; $L = 0{,}2\ H$

$$\left(\dfrac{di}{dt}\right)_{t=0} = \textbf{7,5 A s}^{\textbf{-1}}$$

Rate of growth when the current $i = 0{,}1A$

$$\dfrac{di}{dt} = \dfrac{\varepsilon}{L} - \dfrac{Ri}{L} \quad \text{with} \quad R = 9\ \Omega + 1\ \Omega = 10\ \Omega$$

A.N. $\varepsilon = 1{,}5\ V$; $L = 0{,}2\ H$; $R = 10\ \Omega$; $i = 0{,}1\ A$

We obtain: $\dfrac{di}{dt} = \textbf{2,5 A s}^{\textbf{-1}}$

Rate of growth when the current i = 0,15 A

We have: $\dfrac{di}{dt} = \dfrac{\varepsilon}{L} - \dfrac{Ri}{L}$ with i = 0, 15 A

A.N. ε = 1, 5 V; L = 0, 2 H; R = 10 Ω ; i = 0,15 A

We obtain : $\dfrac{di}{dt} = 0$

We note that when the intensity of the current increases, it rate of growth diminishes and cancels when the maximum intensity is reached: indeed, i = 0,15 A is the maximum intensity equal to $\dfrac{\varepsilon}{R}$

Exercise 14

The 'loop rule' applied to this circuit gives us :

$$V_0 - L\dfrac{di}{dt} - Ri = 0$$

The solution to this equation is :

$$i = \dfrac{V_0}{R}(1 - e^{-\frac{R}{L}t}) \qquad (1)$$

The induced f.e.m. is :

$$\left|\varepsilon_{ind}\right| = L\dfrac{di}{dt} \Rightarrow \left|\varepsilon_{ind}\right| = L\dfrac{V_0}{L}e^{-\frac{R}{L}t} = V_0 e^{-\frac{R}{L}t} \qquad (2)$$

1- At the instant when t=0

Equations (1) and (2) give us:

$$i = 0 \quad \text{et} \quad \left|\varepsilon_{ind}\right| = V_0$$

We can easily understand this result: indeed, at t=0, i=0, the tension at the limits of the resistance is naught and all the applied tension is then found at the limits of the inductance.

2 – at the instant $t = \dfrac{1}{2}\tau$

τ is the constant of the time : $\tau = \dfrac{L}{R}$

Equations (1) and (2) give us :

$$i = \frac{V_0}{R}(1 - e^{-\frac{1}{2}}) \text{ with } e = 2{,}718$$

$$i = \frac{V_0}{R}(1 - \frac{1}{\sqrt{e}}) = 0{,}391\frac{V_0}{R}$$

$$\left|\varepsilon\right| = V_0\, e^{-\frac{R}{L}\frac{L}{2R}} = V_0\, e^{-\frac{1}{2}} = \frac{V_0}{\sqrt{e}}$$

$$\left|\varepsilon\right| = \mathbf{0{,}606\ V_0}$$

Sites to visit

Part 1

Consult the following sites and put what you visualize in relation with what you have learned during the learning activity.

1) Specific charge of the electron (e/m)

http://wwwens.uqac.ca/chimie/Physique_atom/Chap_htm/CHAP_3.html

This site deals among other things with the determination of the specific charge of the electron. It analyzes the electric deflection of cathode rays, magnetic deflection of electron beams, the principle of a speed selector.

http://www.unige.ch/sciences/physique/tp/tpe/E9.htm

This site proposes a TP on the measurement of the specific charge of the electron.

http://www.univ-lemans.fr/enseignements/physique/02/electri/esurm.html

This is an applet on the measurement of e/m using the J.J.Thomson method and the hydrogen gas tube method. By following the instructions of the author, you can participate in its measurement.

2) Hall Effect

http://www.sciences.univ-nantes.fr/physique/perso/gtulloue/Meca/Charges/hall.html

You visualize here an animation of the Hall effect. You can choose the sign of the charges, decide on the intensity of the current, the magnetic field as well as the thickness of the sample.

http://membres.lycos.fr/physicisss/labos/effet_hall.pdf

You will see here a suggestion for an experiment which highlights the Hall effect in a semi-conductor and also tackles the determination of the number of charge carriers per unit of volume in the case of the germanium.

3) Cyclotron

http://www.sciences.univ-nantes.fr/physique/perso/gtulloue/Meca/Charges/cyclotron.html

This is an applet where you can visualize the movement of charge carriers in a cyclotron.

4) Mass Spectrograph – Magnetic deflection

http://perso.orange.fr/physique.chimie/Cours_de_physique/Physique_12_PARTI-CULE_CHARGEE_DANS_UN_CHAMP_MAGNETIQUE.htm

This site gives you one problem on the mass spectrograph and one problem on magnetic deflection as well as their solutions.

Part 2

You will find on the following sites information which reinforces and complete the material you have learned during the course of this learning activity.

http://perso.orange.fr/daniel.robert9/Induction_electromagnetique.html

This site proposes a lesson on electromagnetic inductance. It essentially examines the following points: induced f.e.m., induced current and Lenz's law, the phenomenons of of auto and mutual induction.

http://perso.orange.fr/daniel.robert9/Induction_electromagnetique0.html

We are dealing here with the energy accumulated by an inductance coil L.

http://www.univ-lemans.fr/enseignements/physique/02/electri/lenz.html

This is an applet which shows a simulation of the Lenz law

http://jf-noblet.chez-alice.fr/bobine/index.htm

This site presents simulations of R circuits in relation to what you have learned until now:

- Simulation of the establishment of a current in an RL circuit
- Rupture of a current in an RL circuit
- Inductance measurement of a coil

Summary

What you need to remember

On charged particles in an electromagnetic field

- A particle with charge q which moves with a speed $\vec{v}$ in a magnetic field $\vec{B}$ is subject to a force $\vec{F} = q\ \vec{v} \wedge \vec{B}$.

- A charged particle which penetrates a uniform magnetic field with an orthogonal speed at the field takes on a uniform circular movement

- Several experimental devices display magnetic deflection and the circular movement of charged particles in a magnetic field:

- mass spectrometers

- Devices for the measurement of $\dfrac{e}{m}$ using magnetic deflection, measurement of $\dfrac{e}{m}$ using the J.J. Thomson method, rarified gas tube method

- Cyclotrons

On the electromagnetic induction phenomena and Lenz's law

- A variation in the flux time of the magnetic field through the circuit produces in this circuit an induction no matter what the cause of this variation

- The induced. disappears as soon as the variation of the magnetic flux ceases.

- To every induced f.e.m. in a closed circuit there is a corresponding induced current

- Lenz's law: "the direction of the induced current is such that it is opposed to the cause which produces it"

- An induced f.e.m. appears in a moving metallic stem with uniform translation speed $\vec{v}$ in a field $\vec{B}$ normal $\vec{v}$. Each free electron from this conductor is subject to force $\vec{F}_m = -e\vec{v} \wedge \vec{B}$. This force provokes a concerted movement of the free electrons and is at the origin of the induced current in the closed circuit containing the stem. The force $\vec{F}_m$ is associated to an electromagnetic field $\vec{E}_m$ such that $\vec{F}_m = -e\vec{v} \wedge \vec{B} = -e\vec{E}_m$.

- An elecromagnetic field can sustain the movement of charge carriers in a closed circuit.

On auto-induction phenomena and RL circuits

- Self flux

Let us consider a circuit (C) through which passes a current i. This current creates a magnetic field therefore a magnetic field flux going through (C) itself: this flux is called "self-flux"

The self flux passing through a circuit is proportional to the intensity of the current I which passes through this circuit: the proportionality constant is called "circuit inductance".

$F = L I$, $L =$ inductance du circuit.

Auto –induction phenomenon

Any variation in the intensity of the current in a circuit (that is, variation of the self flux) generates ane.m.f. induction which causes a complementary current to circulate in this circuit whose direction is such that it is opposed to the cause which produces it.

RL Circuit

The differential equation which describes the evolution of the current in an RL circuit series connected to the limits of a continual tension source e is :

$$e - \frac{dI}{dt} - RI = 0$$

And the current passing through the circuit is :

$$I = \frac{\varepsilon}{R}(1 - e^{-\frac{Rt}{L}})$$

This current does not immediately reach its maximum value but rather increases gradually.

$(\frac{dI}{dt})$ is the rate of growth of the current

At the initial instant t = 0, $(\dfrac{dI}{dt})_{t=0} = \dfrac{\varepsilon}{L}$

When the permanent situation becomes fixed, $(\dfrac{dI}{dt}) = 0$ and the current reaches its maximum value $I_m = \dfrac{\varepsilon}{R}$

At the instant t = 0, the curve I = f (t) is tangential to the right of the curve

$$(\dfrac{dI}{dt})_{t=0} = \dfrac{\varepsilon}{L}$$

The constant of time of a circuit (RL) is defined by $t = \dfrac{L}{R}$. This constant is homogeneous with a time ; it characterizes the speed with which the permanent regime is reached. Strongly inductive circuits have a high time constant.

Auto-evaluation

Learners take note of the difficulties they have encountered during the learning activity. They may revue the relevant parts of the course in order to better understand the origin of their difficulties and mistakes.

Teacher's guide

The teacher or Professor will correct the group exercises. He/She will leave the corrections in a work-space that is accessible to the learners. The corrections will be accompanied by adequate feed-back. The grades obtained for each group are attributed to the separate members of the group and will count for 20% of the final evaluation of the magnitude.

Learning Activity 3

Name of the Activity

Magnetization and molecular currents: magnetic vectors $\vec{B}$, $\vec{H}$ and $\vec{M}$; paramagnetic, diamagnetic and ferromagnetic substances.

Learning time: 30 Hours

Instructions: For this activity, if you have **at least ¾ of the points,** you have done very good work, and can continue.

If you have **less than half the points**, you must re-read the course material and redo the activity.

If you have **more than half the points but less than ¾ of the points**, you have done good work, but need to make efforts in the future.

Specific Objectives

- Following this activity, the learner should be able to:
- Remember the origin of molecular currents
- Calculate the magnetic moment of a loop of current

- Remember the relation connecting the field $\vec{B}$, the magnetic excitation $\vec{H}$ and the magnetization $\vec{M}$.
- Calculate the magnetization M of a magnetic substance
- Remember Curie's law for paramagnetics
- Calculate the permeability of a material
- Use the coercitive field to determine the current necessary to demagnetize a magnetized material
- Calculate Bohr's magneton

Outline of the activity

This activity will examine the magnetic properties of matter: paramagnetism, diamgnetism, and ferromagnetism. The emphasis will be on their microscopic interpretation. You will discover that:

The magnetization of magnetic substances is linked to the existence of molecular currents in these substances. You will analyze the origin of these currents and their behaviour in the presence of a magnetic field discharger

When we place magnetic substances in a field $\vec{B}$, their elementary magnetic dipoles create their own fields which modify the initial field. Two other vectors, in addition to $\vec{B}$, are introduced to describe the phenomenom : the magnetic excitation $\vec{H}$ and magnetization $\vec{M}$. You will learn the relationship which exists between these vectors and how they are determined.

Appropriate Readings

- RASOLONDRAMANITRA, H. (2007). Les propriétés magnétiques de la matière. Madagascar. Université d'Antananarivo. Cours inédit.

Pertinent Resources

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13. TIPLER, P. A. (1982). Physics. Second Edition. Worth Publishers Inc, New York, USA

Useful Links

http://www.thyssenfrance.com/documents/mut%20devt%20text%20doc%20tech%20aim_MAGNETISME%20-%20QUELQUES%20RAPPELS.pdf
http://perso.univ-lr.fr/vmathe/16_annexeA.pdf
http://semainescience.u-strasbg.fr/magnetisme/fondement.html
http://www.grasp.ulg.ac.be/cours/2cm/elec5.pdf
http://www.sciences.univ-nantes.fr/physique/perso/cortial/bibliohtml/weisli_j.html
http://www.sciences.univ-nantes.fr/physique/perso/cortial/bibliohtml/praimh.html

Activity Description

In this learning process, you are invited to progress through four learning steps:

Step 1: First, you will have some readings to do on the theme being studied

Step 2: Following the readings which are meant to familiarize you with some overall concepts, some course complements will develop certain important points. Some adequate exercises incorporated in these complements will help you to gain a good understanding of these concepts.

Step 3: In order to put into perspective everything you have learned, you will be guided toward certain websites which will contain animations relative to the themes you have studied and which will enhance your understanding of the material.

Step 4: A series of exercises will be suggested to allow you to test, revue, and reinforce your knowledge.

The following plan outlines this progression:

↓

Readings

↓

Course complement

↓

Site visit

↓

Formative evaluation
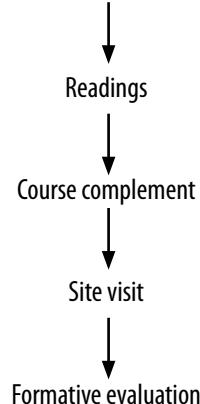
Formative evaluation

This formative evaluation consists of course complements to be read before doing the exercises. Each exercise represents 1% of the points.

Course complement 1

**Magnetization and molecular currents – Magnetic field created by a magnetic dipole**

Magnetic dipole in a magnetic field

1-1: Molecular current

Various substances, when placed in a magnetic field become themselves sources of magnetic fields. We call this magnetization. This magnetization is connected to the existence of microscopic electric currents which circulate inside the atoms.

Indeed, each atom comprises electrons which, in the classical model, gravitate around the atom's centre. When these electrons move, they represent small loops of electric currents which we call molecular currents. They also produce their own magnetic field.

A molecular current, which is a closed current, has a magnetic moment $\vec{m}$ such that $\vec{m} = i\,S\,\vec{n}$ where i is the intensity of the current, S the surface swept by the current and $\vec{n}$ a unitary vector normal to this surface. The direction of $\vec{n}$ is given by the law of the bottle-opener.
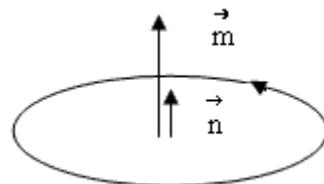

A molecular current constitutes a magnetic dipole

1-2: Magnetic field created by a magnetic dipole

Let us suppose that, in its movement around the centre, an electron follows a circular orbit (classical model). Let us designate the centre of this orbit O and its radius A. Let us determine the magnetic field created at point O by the molecular current thus constituted. In order to do this, we place the circular orbit in plane x o y with point of reference O x y z as the figure below illustrates.
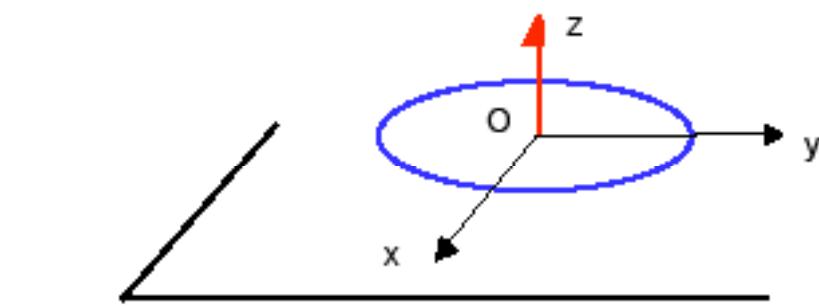
Magnetic field at point O
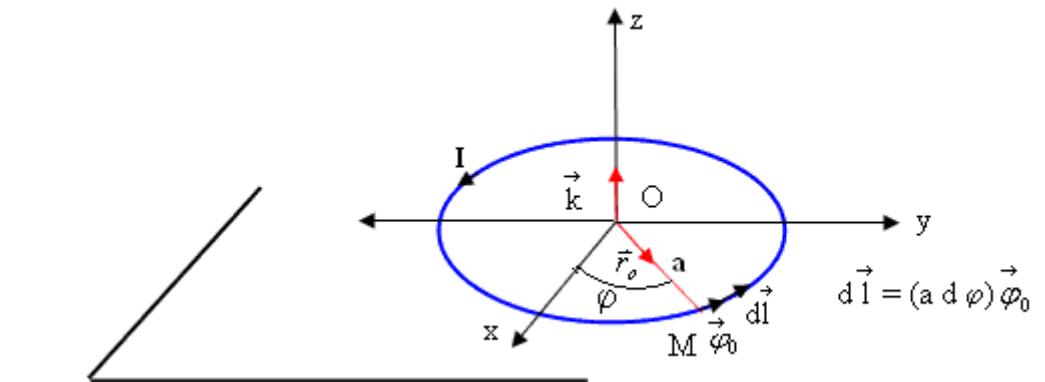
$\vec{\varphi}_0$ is a unitary vector tangent to the circular orbit. An element of length $d\vec{l}$ from the trajectory can be expressed as :

$d\vec{l} = (a\,dj)\,\vec{\varphi}_0$, a being the radius.

The current element $I\,d\vec{l}$ creates at point O a field $d\vec{B}$ such that :

$$d\vec{B} = -\frac{\mu_0}{4\pi}\frac{I\,d\vec{l} \wedge \vec{r}_0}{a^2}$$

(1) (Note that in this expression $\vec{r}_0$ is guided from O toward M and not from M toward O, whence the (-) sign).

We have : $I\,d\vec{l} \wedge \vec{r}_0 = (I\,a\,d\varphi)(\vec{\varphi}_0 \wedge \vec{r}_0)$

But $\vec{\varphi}_0 \wedge \vec{r}_0 = -\vec{k}$, $\vec{k}$ being the unitary vector carried by oz. Indeed, the unitary

vectors $\vec{r}_0$ and $\vec{\varphi}_0$ are expressed as:

$\vec{r}_0 = (\cos j)\ \vec{i}\ + (\cos j)\ \vec{j}$

$\vec{\varphi}_0 = (-\sin j)\ \vec{i}\ + (\cos j)\ \vec{j}$

$\vec{i}$ and $\vec{j}$ are the unitary vectors on Ox and Oy

$\Rightarrow \vec{\varphi}_0 \wedge \vec{r}_0 = -(\vec{i}\ \wedge\ \vec{j}) = -\vec{k}$

Relation (1) can now be written :

$$d\vec{B} = \frac{\mu_0 I\ a\ d\varphi}{4\pi\ a^2}\ \vec{k}$$

When we integrate this last expression, we obtain the total field at point O :

$$\vec{B} = \frac{\mu_0 I}{4\pi\ a}\int_0^{2\pi}d\varphi\ \vec{k}\ = \frac{\mu_0 I}{4\pi\ a}2\pi\ \vec{k}\ = \frac{\mu_0 I}{2\ a}\vec{k} \qquad (2)$$

If we make the dipolar moment $\vec{m}$ occur, we have :

$\vec{m} = I\vec{S} = (I\ \pi\ a^2)\vec{k}$ since $\vec{m}$ here goes in the direction of $\vec{k}$.

$$\Rightarrow I\vec{k}\ =\ =\ \frac{\vec{m}}{\pi\ a^2} \qquad (3)$$

Expressions (2) and (3) give us :

$$\vec{B} = \frac{\mu_0}{2\ a}\left(\frac{\vec{m}}{\pi\ a^2}\right)$$

$$\Rightarrow \vec{B}\ = \frac{\mu_0}{2\ \pi\ a^3}\vec{m}\ .$$

It follows that $\vec{B}$ and $\vec{m}$ have the same direction and course.
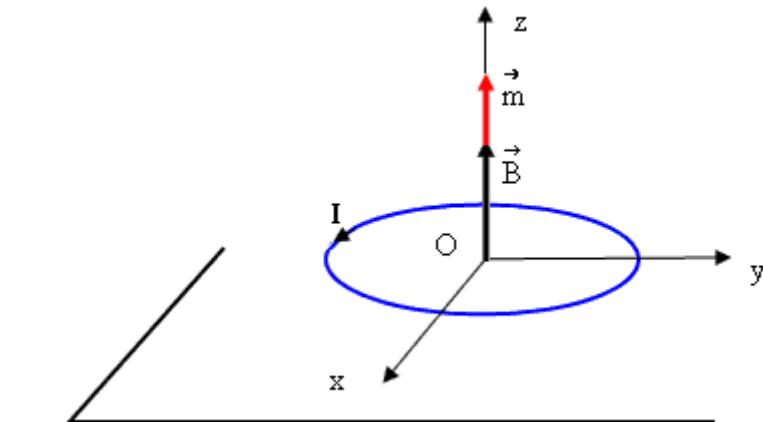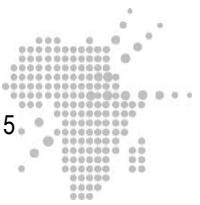
1-3 : Dipole in a magnetic field $\vec{B}_{ex}$

* When a magnetic dipole with moment $\vec{m}$ is placed in a uniform magnetic field $\vec{B}_{ex}$, it is subjected to a moment couple $\vec{\Gamma}$ given by :

$$\vec{\Gamma} = \vec{m} \wedge \vec{B}_{ex}.$$

Note : This expression is absolutely analogous to the one giving the moment couple applied to an electric dipole placed in an electric field ($\vec{\Gamma} = \vec{P} \wedge \vec{E}$)

This couple tends to make the dipole turn or revolve.

* This interaction "Field – dipole" is described by the potential energy:

$$U = - \vec{m} \wedge \vec{B}_{ex}$$

U is minimal $\vec{B}_{ex} // \vec{m}$

U is maximal if $\vec{B}_{ex}$ and $\vec{m}$ are antiparallel.

The most stable position is the one corresponding to $\vec{m} // \vec{B}_{ex}$
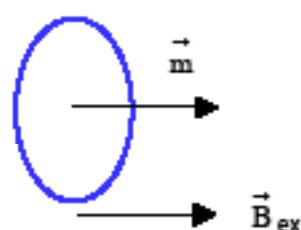
1-4: Magnetization and molecular current

In a non-magnetized substance, i.e. in the absence of an exterior field $\vec{B}_{ex}$, the molecular currents or magnetic dipoles are oriented in a disordered way and the resulting field which they produce is null.

When a substance becomes magnetized, its molecular currents become partially or totally ordered and the resulting field is no longer null.

The figure below describes this model of molecular currents in a uniformly magnetized material.
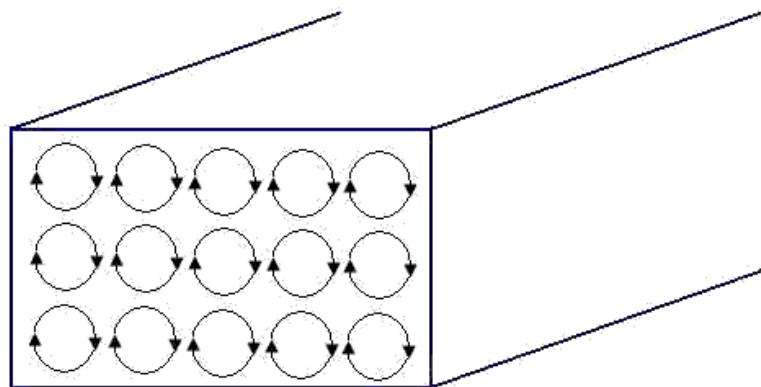
Note that the adjacent circuit portions of the current are opposed and cancel each other out.

Ces courants se neutralisent
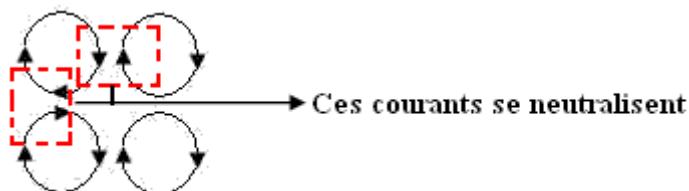
At all points of the interior of the substance, the current is then null and there remains an equivalent surface current. The surface current is called 'amperian current".

An equivalent surface current therefore appears in a magnetized substance. <u>This surface current creates an induced field $\vec{B}_i$ which adds itself vectorially to the magnetizing field $\vec{B}_{ex}$.</u>

We can note here the analogy to surface charges connected in the polarized dielectrics. The difference resides in the fact that the electric field resulting from polarization charges are opposed (in direction) to the field created by free excitatory charges.

Complement to course 2

Relationship between vector $\vec{B}$, the magnetized intensity vector $\vec{H}$ and the magnetizing vector $\vec{M}$ - Ampere's law for a magnitized area - Magnetic susceptibility.

Recall that when material is magnetized, a surface current appears. This current induces a field $\vec{B}_{ind}$ which is added to the magnetizing field, also called discharging field. The total field in the material is then:

$$\vec{B} = \vec{B}_{ex} + \vec{B}_{ind}$$

Let us consider a long solenoid consisting of n spirals per unit of length and through which passes a current I_C. Let us wrap the spirals of this solenoid on a magnetic material in the form of a cylindrical stem with section S and let us determine the magnetization of length I of the stem and the total field in this system.

In the absence of the material, the field on the inside of the solenoid and further from its extremities is uniform and parallel with the axis of the solenoid. This field is:

$$B_0 = \mu_0 \, n \, I_C$$

where $n \, I_C$ is the current per unit of length with I_C the current which passes through each spiral.

When the material is inserted, it is magnetized and a surface current I appears and creates a field $\vec{B}_{ind}$.

Let us take a portion of length I of the cylindrical stem.

Its volume and its magnetic moment are respectively V=S I and m=I S, S being its section.

The magnetization M (the magnetic moment per unit of volume) from the stem is then:

$$M = \frac{m}{V} = \frac{I\,S}{I\,S} = \frac{I}{I}$$

Note : The magnetization is therefore expressed in Am^{-1}.

Note that the magnetization M is equal to the surface current per unit of length.

M and $n \, I_C$ play the same role : current per unit of length. It follows that the field

B_{ind} which results from this magnetization has an expression analogous to $B_0 = \mu_0 n I_C$. We have :

$B_{ind} = \mu_0 M$

The total field in the system is then :

$B = B_0 + B_{ind}$ if the material is paramagnetic

$B = B_0 - B_{ind}$ if the material is diamagnetic

If the stem is paramagnetic, the surface current I and the excitatory current I_C have the same direction. In this case $\vec{B}_0$ and $\vec{B}_{ind}$ are guided in the same direction and we have $B = B_0 + B_{ind}$

If the stem is diamagnetic, I and I_C are in opposite directions as are $\vec{B}_0$ and $\vec{B}_{ind}$ and we obtain $B = B_0 - B_{ind}$

Using vectorial notation, we get :

$$\vec{B} = \vec{B}_0 + \vec{B}_{ind}$$

$$\vec{B} = \vec{B}_0 + \mu_0$$

$\vec{B}_0$ is here the excitatory field

2-1 : Relationship between vector $\vec{B}$ magnetic intensity $\vec{H}$ and magnetization $\vec{M}$

We define the magnetic intensity

$$* \vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

The three vectors, $\vec{H}$, $\vec{B}$ et $\vec{M}$ are connected by this relation. The magnetic intensity is expressed in Am^{-1}. We can find this relation using Ampere's law for a magnetized area.

Recalling Ampere's law

Ampere's law says that the circulation of $\vec{B}$ along a closed curve (C) is equal to the algebraic sum of the currents intertwined by (C) and multiplied by μ_0.

$$C = \oint_{(C)} \vec{B}.\vec{dl} = \mu_0 \Sigma I$$
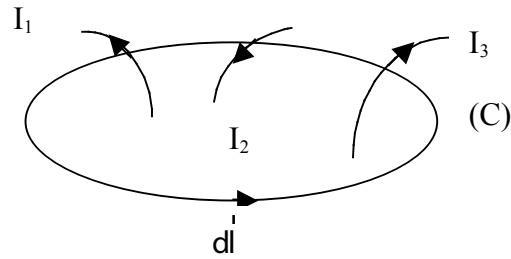
$$\Sigma I = I_1 - I_2 + I_3$$

This law allows us to calculate the field $\vec{B}$ produced by currents I. By applying the Stokes theorem, we obtain :

$$C = \oint_{(C)} \vec{B}.\vec{dl} = \int_{(S)} (\vec{\nabla} \wedge \vec{B}).\vec{dS}$$

(S)
is an open surface limited by (C).

When we state $I = \int_s \vec{J}.\vec{dS}$, $\vec{J}$ being the density of the current, we have :

$$\int_{(S)} (\vec{\nabla} \wedge \vec{B}).\vec{dS} = \mu_0 \int_s \vec{J}.\vec{dS}$$

From this, we draw:

$$\vec{\nabla} \wedge \vec{B} = \mu_0 \vec{J} \quad (1)$$

This is Ampere's law in its differential form.

2-2: Ampere's law and magnetized medium

The induced field $\vec{B}_{ind.}$ which appears in a magnetized medium is produced by an equivalent current (amperian current) density $\vec{J}_a$ defined by :

$$\vec{J}_a = \vec{\nabla} \wedge \vec{M} \quad (2)$$

We can understand expression (2) by applying the relation (1) to $\vec{B}_{ind}$:

$$\vec{\nabla} \wedge \vec{B}_{ind.} = \mu_0 \vec{J}_a$$

But $\qquad\qquad \vec{B}_{ind.} = \mu_0 \vec{M} \Rightarrow \vec{\nabla} \wedge (\mu_0 \vec{M}) = \mu_0 \vec{J}_a$

$$\Rightarrow \vec{\nabla} \wedge \vec{M} = \vec{J}_a$$

In a magnetized medium, the field $\vec{B}$ exists essentially due to :

- the excitatory current with density $\vec{j}_{ex}$
- the equivalent current with density $\vec{J}_a$

Ampere's law can therefore be expressed as :

$$\vec{\nabla} \wedge \vec{B} = \mu_0 \vec{j} = \mu_0 (\vec{j}_{ex} + \vec{j}_a)$$

When we develop this relation, we obtain :

$$\vec{\nabla} \wedge \vec{B} = \mu_0 \vec{j}_{ex} + \mu_0 \vec{j}_a = \mu_0 \vec{j}_{ex} + \mu_0 (\vec{\nabla} \wedge \vec{M})$$

$$\Rightarrow \vec{\nabla} \wedge (\vec{B} - \mu_0 \vec{M}) = \mu_0 \vec{j}_{ex}$$

$$\Rightarrow \vec{\nabla} \wedge (\frac{\vec{B}}{\mu_0} - \vec{M}) = \vec{j}_{ex}$$

We define : $\vec{H} = \dfrac{\vec{B}}{\mu_0} - \vec{M}$

Ampere's law becomes

$$\vec{\nabla} \wedge \vec{H} = \vec{j}_{ex}$$

2-3 : Magnetic susceptibility

In isotropic and linear magnetic materials, the magnetization $\vec{M}$ and the magnetic intensity $\vec{H}$ are proportional.

$$\vec{M} = X_m \vec{H}$$

We call X_m the magnetic susceptibility

Magnetic susceptibility is positive for paramagnetic substances and negative for diamagnetic substances.

For the diamagnetics therefore, the magnetization vector is opposed to the magnetic intensity vector.

Note: X_m is a number

The table below gives the susceptibility values of a few materials at 20 degrees C

Material	X_m
Aluminum	2.3×10^{-5}
Bismuth	$-1{,}66 \times 10^{-5}$
Copper	$-0{,}98 \times 10^{-5}$
Magnesium	$1{,}2 \times 10^{-5}$
Sodium	$-0{,}24 \times 10^{-5}$

Permeability

So: $\vec{H} = \dfrac{\vec{B}}{\mu_0} - \vec{M}$ et $\vec{M} = X_m \vec{H}$

We deduce from this that:

$$\vec{B} = \mu_0 (\vec{H} + \vec{M}) = \mu_0 (\vec{H} + X_m \vec{H}) = \mu_0 (1 + X_m) \vec{H}$$

We state $\mu = \mu_0(1 + X_m)$ whence $\vec{B} = \mu \vec{H}$

μ is the permeability of the material. We can also write :

$\mu = \mu_0 \mu_r$ and $\mu_r = (1 + X_m)$

μ_r is called relative permeability

In the void or in a non magnetic material, we have :

$$X_m = 0 \implies \mu_r = \frac{\mu}{\mu_0} = 1$$

For paramagnetic materials

$$X_m > 0 \implies \mu_r > 1$$

For diamagnetic materials

$$X_m < 0 \implies \mu_r < 1$$

For ferromagnetic materials :

$\vec{M}$ is no longer proportional to $\vec{H}$

$\vec{B}$ is no longer proportional to $\vec{H}$

We have a hysteresis phenomenon

[See hysterisis phenomenon in the « appropriate readings »]

Exercise 1

A long solenoid, without an iron centre, consists of 12 spirals per cm. an intensity current of I=0,50A passes through it.

1- Determine the magnetic intensity H and the field B at the centre of the solenoid.

2- When we introduce an iron centre in the solenoid, field B has the value 1,36 T.

     - calculate once again the magnetic intensity H

     - find the magnetization M, and the permeability of the centre.

Exercise 2

An iron torus, with an average circumference equal to 0,5m, with relative permeability μ_r = 5000 has a winding of 500 spirals through which passes a current I=0,15A. Determine the magnetization M and the average magnetic moment per iron atom.

Given :

-    the density of the iron ρ =7850 kg/ m³
-    the molar mass M = 55,8 x 10^{-3}kg/ mol
-    the number of Avogadro N_A = 6,02 x 10^{23} atoms/ mol

Exercise 3

Let us consider a toroidal coil with average circumference I=0,8m, consisting of N=1200 spirals through which passes a current with intensity I=1,5A

1 – Calculate B and H supposing the centre is empty

2-   We now suppose that this same coil has a bismuth centre with magnetic susceptibility X_m = -1,66 10^5

   - Calculate the permeability of this material and compare with the permeability when the coil is empty.
   - Calculate B and H. Draw a conclusion

3 – We replace the bismuth with a magnesium centre with susceptibility X_m = 1,2 10^{-5}.

Repeat question 2

We will take π = 3,1415926

Exercise 4

A magnetic rod has a coercive field of 4 x 10^3 Am^{-1}. We insert this rod in a solenoid with a 12 cm length and comprising 60 spirals. What current must pass through the solenoid to demagnetize the rod?

Exercise 5

The table below gives the magnetic susceptibility of iron ammonium alum with respect to temperature.

T, °C	X_m
- 258,15	129×10^{-4}
- 173	$19,4 \times 10^{-4}$
- 73	$9,7 \times 10^{-4}$
27	$6,5 \times 10^{-4}$

Find and graph $\dfrac{1}{X_m}$ as a function of the temperature expressed in Kelvin degrees.

Is Curie's law confirmed? If yes, determine the Curie constant.

Exercise 6

Suppose the magnetic dipole moment of each iron atom is one Bohr magneton, determine the maximum magnetization and the magnetic field which results from it.

Given:

- The density of the iron r = 7850 kg/m³
- The molar mass M = 55,8 10^{-3}kg/mol
- The number of Avogadro N_A = 6,02 x 10^{23} atomes /mol
- A Bohr magneton μ_B = 9,27 x 10^{-24} Am²

Exercise 7

According to the classical model, an electron in an atom follows a circular orbit around its center.

1- Calculate the magnetic moment associated to this movement if its kinetic orbital moment is equal to 1,05 x 10^{34} Js.

2- An electron has an intrinsic magnetic moment equal to 0,928 x 10^{23}A.m². We place it in a magnetic field $\vec{B}$ of size 1,2T. Calculate the magnetic energy potential difference ΔU between the two following spin directions : parallel to $\vec{B}$ and anti-parallel to $\vec{B}$.

3- What must the absolute temperature be for $\Delta U = \dfrac{1}{2}$ kT.

Given is the Boltzmann constant k = 1,38 10^{-23} J/K.

Exercise 8

Consider an electron in an atom which follows a circular orbit around its centre in the presence of a uniform and perpendicular field $\vec{B}$ to the plan of the orbit. Suppose the electric force which is exerted on this electron is N time greater than the magnetic force. Determine the two possible angular speeds of the electron. Numerical application: N=100 and B= 0,427 T.

Exercise 9

The Bohr magneton is $\mu_B = \dfrac{eh}{2m_e} = \dfrac{e}{2m_e}\left(\dfrac{h}{2\pi}\right)$ where $h = \dfrac{h}{2\pi}$, h being the Planck constant : $h = 6,63 \times 10^{34}$ J.s ; m_e is the electron mass

1- Calculate μ_B

2- Show that μ_B can also be expressed in $\dfrac{J}{T}$

Learning Activity

-   Read the course complements

-   Use the chat room under the supervision of a tutor to familiarize yourself with the contents of the course complement

-   Do the exercises

-   Organize yourselves into groups for collaborative work : the learners go into groups under the supervision of a tutor

-   Each group chooses a delegate.

-   The tutor organizes the work by indicating the order in which the exercises should be done and the length of time required for each exercise

-   The delegates leave the exercise solutions in a work-space reserved for the learners.

Answer Key

Solution to exercise 1:

1- Calculating H and B

Let us recall that $\vec{H} = \dfrac{\vec{B}}{\mu_0} - \vec{M} \Rightarrow \vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}$

In the absence of the iron centre (i.e. in the void): $\vec{M} = \vec{O} \Rightarrow \vec{B} = \mu_0 \vec{H}$

$\vec{B}$ and $\vec{H}$ are parallel at the solenoid axis. Let us designate B_0 the field B in the absence of the iron centre. We have at a point removed from the solenoid extremities:

$B_0 = \mu_0 \, n \, I$ and $H = n \, I$

A.N.: $\mu_0 = 4 \, p \times 10^{-7} \, \dfrac{Tm}{A}$; $I = 0{,}50A$; $n = 1200 \ m^{-1}$

$B_0 = (4 \, p \times 10^{-7}) \, (1200) \, (0{,}50)$

$B_0 = 7{,}54 \ 10^{-4} \ T$ $H = (1200) \, (0{,}50)$

$H = 600 \ Am^{-1}$

Magnetic intensity H

The magnetic intensity does not change :

$$H = n\, I = 600 \text{ Am}^{-1}$$

Permeability of the centre

In the presence of the iron centre we have:

$$B = \mu\, H \implies \mu = \frac{B}{H}$$

$\underline{A.N.}$: $B = 1{,}36$ T ; $H = 600$ Am^{-1}

$$\mu = \frac{1{,}36}{600}$$

$$\mu = 0{,}00226 \ \frac{\text{Tm}}{\text{A}}$$

Permeability relative to the centre

In the absence of the iron centre : $B_0 = \mu_0\, H$

In the presence of the iron centre : $B = \mu\, H$

We can draw the conclusion :

$$\frac{B}{B_0} = \frac{\mu}{\mu_0} = \mu_r$$

$\underline{A.N.}$: $B = 1{,}36$ T ; $B_0 = 7{,}54 \ 10^{-4}$ T

$$\mu_r = \frac{1{,}36}{7{,}54 \text{ x } 10^{-4}}$$
$$\mu_r = 1800$$

or

$\underline{A.N.}$: $\mu = 0{,}00226 \dfrac{\text{Tm}}{\text{A}}$; $\mu_0 = 4p \text{ x } 10^{-7}\dfrac{\text{Tm}}{\text{A}}$

$$\mu_r = \frac{\mu}{\mu_0} = \frac{0{,}00226}{4 \pi \text{ x } 10^{-7}}$$
$$\mu_r = 1800$$

Magnetization

Let us take $B = \mu_0 H + \mu_0 M$

But $\mu_0 H = B_0 \Rightarrow B = B_0 + \mu_0 M$

It follows that :

$$M = \frac{B - B_0}{\mu_0}$$

<u>A. N.</u> : $B = 1{,}36 T$; $B_0 = 7{,}54 \ 10^{-4} \ T$; $\mu_0 = 4p \ x \ 10^{-7} \ \dfrac{Tm}{A}$

$$M = \frac{1{,}36 \ - 7{,}54 \ x \ 10^{-4}}{4 \ \pi \ x \ 10^{-7}}$$

$M = 1{,}08 \ x \ 10^6 \ A \ m^{-1}$

Solution to exercise 2

Magnetization

Let us take as our starting point the relation :

$B = \mu_0 H + \mu_0 M \Rightarrow B = B_0 + \mu_0 M$ (1)   with $B_0 = \mu_0 M$

B_0 is the field in the absence of the iron centre.

We deduce from relation (1) :

$$M = \frac{B - B_0}{\mu_0}$$

a) Let us first calculate B_0 and B

* $B_0 = \mu_0 H = \mu_0 n I$ with $n = \dfrac{N}{l}$ the number of spirals per unit of length, N and

l being respectively the total number of spirals and I the average circumference of

the torus.

* $B = \mu H = \mu n I = \mu_0 \mu_r n I = \mu_r B_0$

μ_r is the relative permeability

A.N. : N = 500 ; l = 0,5m ; μ_r = 5000 ; I = 0,15 A ; μ_0 = 4p x $10^{-7}\dfrac{Tm}{A}$

$$\Rightarrow B_0 = (4p \times 10^{-7}) \left(\frac{500}{0,5}\right) (0,15)$$

$$B_0 = 1,88\ 10^{-4}\ T$$

$$\Rightarrow B = (5000)\ (1,88 \times 10^{-4})$$

$$B = 9,42 \times 10^{-1}\ T$$

b) Let us now calculate M

$$M = \frac{B-B_0}{\mu_0} = \frac{9,42 \times 10^{-1} - (1,88)(10^{-4})}{4\pi \times 10^{-7}}$$

$$M = 7,5 \times 10^5\ Am^{-1}$$

Average magnetic moment per atom

M is the magnetic moment per unit of volume. The average magnetic moment per atom is then:

$$m = \frac{M}{n_a}$$, n_a being the number of iron atoms per unit of volume.

c) Let us calculate n_a

We have $n_a = (N_A)\ (\rho / M)$

r is the density of the iron, M its molar mass and N_A le number of Avogadro.

A.N.: N_A = 6,02 x 10^{23} atoms / mol; M = 55,8 Kg/mol; r = 7850 kg/m³

$$n_a = (6,02 \times 10^{23})\ \frac{7850}{55,8}$$

$$n_a = 8,47\ 10^{28}\ atoms\ /m^3$$

d) Let us now calculate the average magnetic moment per atom m

$$m = \frac{M}{n_a}$$ with M = 7,5 10^5 Am^{-1} and n_a = 8,47 10^{28} atoms / m³

$$m = 8,85\ 10^{-24}\ A.m^2$$

Solution to exercise 3

1) B and H corresponding to an empty space

$B_0 = \mu_0\, n\, I$ with $n = \dfrac{N}{l}$

$H = n\, I$

<u>A.N.</u>: $\mu_0 = 4\pi 10^{-7}\,\dfrac{Tm}{A}$; $N = 1200$; $l = 0,8m$; $I = 1,5A$

$* \; B_0 = 4(3,\,1415926)10^{-7} \left(\dfrac{1200}{0,8}\right)(1,5)$

$B_0 = 2,827433 \times 10^{-3}$ T

$* \; H = \left(\dfrac{1200}{0,8}\right)(1,5) \;\Rightarrow\; H = 2250\ Am^{-1}$

2) Permeability of bismuth

$\mu = \mu_0\mu_r$, μ_r being the relative permeability.

But $\mu_r = (1 + X_m) \Rightarrow \mu = \mu_0(1 + X_m)$

<u>A.N.</u>: $\mu_0 = 4\pi 10^{-7}\,\dfrac{Tm}{A}$; $Xm = -1,66 \times 10^{-5}$

$\mu = 4(3,\,1415926)\,10^{-7}\,(1 - 1,66 \times 10^{-5})$

$\mu = 1,256616 \times 10^{-6}\,\dfrac{Tm}{A}$

For the empty space we have:

$\mu_0 = 4(3,1415926)\,10^{-7}\,\dfrac{Tm}{A}$

$\mu_0 = 1,256637 \times 10^{-6}\,\dfrac{Tm}{A}$

Bismuth is a diamagnetic material ($X_m < 0$) : μ is slightly inferior to μ_0.

B and H

H is always equal to I

$H = 2250$ Am^{-1} (its value does not change)

$B = \mu H$

$$B = (1{,}256616 \times 10^{-6} \frac{Tm}{A}) (2250 \frac{A}{m})$$

$B = 2{,}827386 \ 10^{-3}$ T

B is slightly inferior to B_0

3) Permeability of magnesium

$\mu = \mu_0 (1 + X_m)$ with $X_m = 1{,}2 \ 10^{-5}$

$$\mu = (4 (3{,}1415926)10^{-7} \frac{Tm}{A} \ (1+ 1{,}210^{-5})$$

$$\mu = 1{,}256652 \times 10^{-6} \ \frac{Tm}{A}$$

Magnesium is paramagnetic ($X_m > 0$) : μ is slightly superior to μ_0

B and H

* H is always equal to 2250 Am^{-1}

* $B = \mu H$

$$B = (1{,}256652 \times 10^{-6} \frac{Tm}{A}) (2250 \frac{A}{m})$$
$$B = 2{,}827467 \ T$$

B is slightly superior to B_0.

Solution to exercise 4

The coercive field is needed to demagnetize the rod. The current passing through the solenoid must be such that it produces a magnetic intensity equal to this value.

$H_{coercive}$ = n I, n being the number of spirals on the solenoid per unit of length.

We have then :

$$I = \frac{H_{coercitif}}{n} = \frac{H_{coercitif}}{\left(\dfrac{N}{1}\right)}$$

N is the number of spirals and I the length of the solenoid

<u>A.N.:</u> $H_{coercive}$ = 4 x 10^3 Am^{-1}; N = 60; l = 0,12m

$$I = \frac{4 \times 10^3 \times 0{,}12}{60}$$
$$I = 8 \text{ A}$$

Solution to exercise 5

We have the table of values below:

T ; K	$\dfrac{1}{X_m}$
15	77,52
100,15	515,46
200,15	1030,93
300,15	1538,46

(O°C = 273,15 K)

$\dfrac{1}{X_m}$ = f (T) (see graph below) is a straight line passing through the origin.
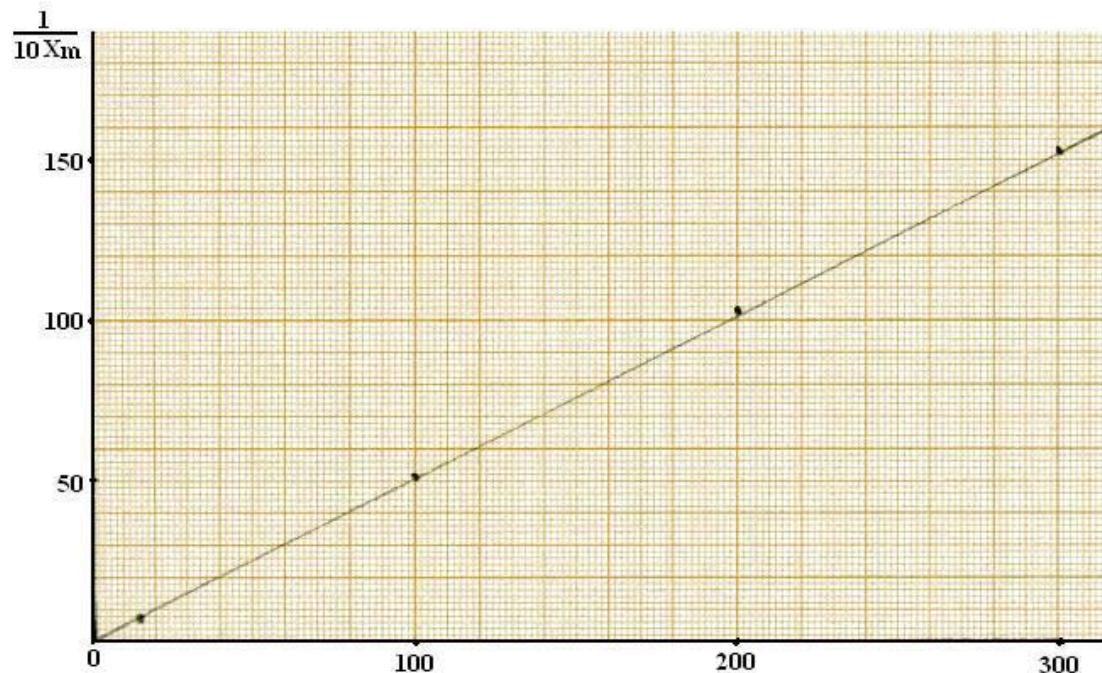
Let us take Curie's law for the paramagnetics. This law says that:

$M = A \dfrac{B}{T}$ where A is a constant. An increase in B tends to increase the alignment of the magnetic dipoles of the paramagnetic material. An increase in the temperature T provokes a greater opposition to this alignment.

This law can also be stated:

$X_m = \dfrac{C}{T}$ where C is called the Curie constant.

$$X_m = \frac{C}{T} \Rightarrow \frac{1}{X_m} = \frac{T}{C}$$

This relation shows that $\dfrac{1}{X_m}$ is proportional to T. If we graph $\dfrac{1}{X_m}$ as a function of the temperature T we must obtain a straight line passing through the origin and

whose slope is $\dfrac{1}{C}$

The slop obtained with the experimental right is

$$\dfrac{1}{C} = \dfrac{1538,46 - 515,46}{(300,15 - 100,15)K} = 5,115 \text{ K}^{-1}$$

$$\Rightarrow C = \dfrac{1}{5,115} K$$
$$C = 0,195 \text{ K}$$

Solution to exercise 6

Maximum magnetization

The maximum value M_s of magnetization is reached when all the magnetic dipoles are aligned. In this case we have:

$M_s = n\, m$

where n is the number of atoms per unit of volume and m the magnetic moment of each atom.

Let us calculate n :

We have :

$n = N_A\, \rho /\, M_{mo}$

r is the density of the iron, M its molar mass and N_A the number of Avogadro.

<u>A.N.</u>: $\rho = 7850 \text{kg/m}^3$; $M_{mol} = 55,8\ 10^{-3} \text{kg/mol}$; $N_A = 6,02 \times 10^{23}$ atoms / mol

$$n = (6,02 \times 10^{23})\ \dfrac{7850}{55,8 \times 10^{-3}}$$

$n = 8,47\ 10^{28}$ atomes /m^3

Knowing n, we can now calculate M_s

$M_s = n\, m$

<u>A.N.</u> : $n = 8,47\ 10^{28}$ atoms/m^3 ; $m = \mu_B = 9,27 \times 10^{-24} \text{ Am}^2$

$M_s = (8,47\ 10^{28})\ (9,27 \times 10^{-24})$

$M_s = 7,85 \times 10^5 \text{ Am}^{-1}$

Magnetic field

The magnetic field resulting from this is :

$B = \mu_0 M_s$

<u>A.N.</u>: $\mu_0 = 4\,\pi \text{ x } 10^{-7}\dfrac{Tm}{A}$

$B = (4\pi \text{ x } 10^{-7})\,(7,85 \text{ x } 10^5)$

$B = 0,986$ T

Solution to exercise 7

1- Magnetic moment

Let us suppose that the electron follows a circular orbit with radius r and speed v. The magnetic moment associated to this movement is:

$m_l = I\,S$ (1) with $S = \pi r^2$ and $I = \dfrac{e}{T}$ where T is the revolution time

We have : $vT = 2\pi r \Rightarrow \dfrac{1}{T} = \dfrac{v}{2\,\pi\,r} \Rightarrow I = \dfrac{e}{T} = \dfrac{ev}{2\,\pi\,r}$

Relation (1) becomes :

$m_l = I\,S = \left(\dfrac{ev}{2\,\pi\,r}\right)(\pi\,r^2) \Rightarrow m_l = \dfrac{e}{2}\,vr$

$\Rightarrow m_l = \dfrac{e}{2\,m}\,(mvr)$

<u>A.N.</u> : $e = 1{,}6\ 10^{-19}$ C ; $m = 9{,}11 \text{ x } 10^{-31}$ kg ; $mvr = 1{,}05 \text{ x } 10^{-34}$ J.s

$m_l = \dfrac{1{,}6 \text{ x } 10^{-19}}{2(9{,}11 \text{ x } 10^{-31})}(1{,}05 \text{ x } 10^{-34})$

$m_l = 9{,}22\ 10^{-24}\ Am^2$

2- **Energy potential difference**

Let us consider $\vec{m}_s$ the intrinsic magnetic moment. When $\vec{m}_s$ and $\vec{B}$ are parallel, the potential energy is :

$$U_1 = -\vec{m}_s . \vec{B} = -m_s B$$

When $\vec{m}_s$ and $\vec{B}$ are anti-parallel, this energy is:

$$U_2 = -\vec{m}_s . \vec{B} = m_s B$$

It follows that :

$$\Delta U = U_2 - U_1 = 2m_s B$$

A.N. : $m_s = 0{,}928 \times 10^{-23} A.m^2$; $B = 1{,}2$ T

$$\Delta U = 2 (0{,}928 \times 10^{-23}) (1{,}2)$$

$$\Delta U = 2{,}23 \; 10^{-23} \text{ J}.$$

3- **Temperature at which** $DU = \dfrac{1}{2} kT$

$$\Delta U = \dfrac{1}{2} kT \Rightarrow T = \dfrac{2\Delta U}{k}$$

A.N. : $k = 1{,}38 \; 10^{-23} \dfrac{J}{K}$

$$T = \dfrac{2(2{,}23 \times 10^{-23})}{1{,}38 10^{-23}}$$

$$T = 3{,}23 \text{ K}$$

Solution to exercise 8

Let us consider both possibilities, that is, the electron moving in a clock-wise direction and in the anti-clockwise direction.

Let us suppose that $\vec{B}$ is guided as the figure shows
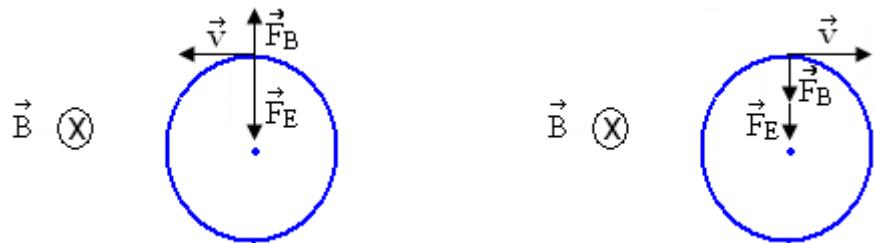
We have here :

$F_E \pm F_B = m w^2 r$ with r the radius of the trajectory and w the angular speed.

According to the hypothesis : $F_E = N F_B \Rightarrow (N \pm 1) F_B = m w^2 r = m \dfrac{v^2}{r}$

But $F_B = e v B$, whence:

$(N \pm 1) e v B = m \dfrac{v^2}{r}$

$\Rightarrow v = (N \pm 1) \dfrac{e B r}{m}$

$\omega = (N \pm 1) \dfrac{e B}{m}$

<u>A.N.</u> : $B = 0{,}427$ T ; $N = 100$; $e = 1{,}6 \ 10^{-19}$ C ; $m = 9{,}11 \ 10^{-31}$ kg

* $\omega_1 = (100 - 1) \left(\dfrac{(1{,}6 \ 10^{-19})(0{,}427)}{9{,}11 \times 10^{-31}} \right.$

$\omega_1 = 7{,}42 \ 10^{12}$ rads^{-1}

* $\omega_2 = (100 + 1) \left(\dfrac{(1{,}6 \ 10^{-19})(0{,}427)}{9{,}11 \times 10^{-31}} \right.$

$\omega_2 = 7{,}57 \ 10^{12}$ rads^{-1}

Solution to exercise 9

1) $\mu_B = \dfrac{e}{2m_e}\left(\dfrac{h}{2\pi}\right)$

• $\dfrac{h}{2\pi} = \dfrac{6{,}63\text{x }10^{-34}\,\text{J.s}}{2\pi} = 1{,}056 \text{ x}10^{-34}\text{ J.s}$

• $e = 1{,}6\text{x}10^{-19}$ C ; $m_e = 9{,}11$ x10^{-31}kg

• $\mu_B = \dfrac{(1{,}6\text{x } 10^{-19}\text{ C})}{2\,(9{,}11\text{ x } 10^{-31}\text{ kg})} \text{ x } (1{,}056 \text{ x}10^{-34}\text{ J.s})$

$\mu_B = 9{,}27 \text{ x}10^{-24}\,\dfrac{\text{C J s}}{\text{kg}}$

We can transform $\dfrac{\text{C J s}}{\text{kg}}$. We have :

$$C = As$$

$J = Nm = (\text{kg m s}^{-2})\,(m) = \text{kg m}^2\text{ s}^{-2}$

Whence $\dfrac{\text{C J s}}{\text{kg}} = \dfrac{(A\text{ s})(\text{kg m}^2\text{s}^{-2})(s)}{\text{kg}} = Am^2$

We can see that the magnetic moment is indeed expressed in Am² (see m = I S)

$\mu_B = 9{,}27 \text{ x}10^{-24}$ Am²

2) The Bohr magneton can also be expressed in $\dfrac{J}{T}$. We saw that :
$$J = \text{kg m}^2\text{ s}^{-2} \qquad (1)$$

Let us transform the Tesla. In order to do this, let us take as our starting point F = q v B (force which is applied on a particle moving with speed v in a field B).

$\Rightarrow B = \dfrac{F}{qv}$ In the international system B is in Tesla, F in Newton, q in Coulomb and v en mètre in meters per second. We thus have :

$$T = \dfrac{N}{Cms^{-1}}$$

$$\left. \begin{array}{l} N = (kgms^{-2}) \\ (2) \\ \\ C = (As) \end{array} \right\} \Rightarrow T \dfrac{(kg\ m\ s^{-2})}{(A\ s)\ m\ s^{-1}} = \dfrac{kg}{A\ s^{2}}$$

(1) and (2) result in : $\dfrac{J}{T} = \dfrac{kg\ m^{2}s^{-2}}{kg/A\ s^{2}} = Am^{2}$

We therefore also have $\mu_{B} = 9,27x\ 10^{-24}\ \dfrac{J}{T}$

Site visit

You will find in the following sites information which reinforces and completes what you have learned during this learning activity.

http://www.thyssenfrance.com/documents/mut%20devt%20text%20doc%20 0tech%20aim_MAGNETISME%20-%20QUELQUES%20RAPPELS.pdf

This site revues some material on magnetism

http://perso.univ-lr.fr/vmathe/16_annexeA.pdf

This link develops a few basic notions concerning the magnetic properties of matter. It revues in particular: The fundamental magnetic sizes B, H and M   Paramagnetism, diagmagnetism, and ferromagnetism   The hysteresis phenomenon

http://semainescience.u-strasbg.fr/magnetisme/fondement.html

This site looks at the fundamental theories of magnetism

http://www.grasp.ulg.ac.be/cours/2cm/elec5.pdf

This is a document on magnetic mediums. It describes magnetization, magnetic susceptibility and the relation connecting vectors $\vec{B}$, $\vec{H}$ and $\vec{M}$.
http://www.sciences.univ-nantes.fr/physique/perso/cortial/bibliohtml/weis-li_j.html

An animation of the Weiss fields is proposed here. You can visualize the movement of surfaces in these fields. Both reversible and irreversible movement is considered.
http://www.sciences.univ-nantes.fr/physique/perso/cortial/bibliohtml/prai-mh.html

First magnetization slope, nonvolatile magnetization and coercive fields are considered here.

Summary

What you need to remember

The three vectors $\vec{B}$, $\vec{H}$ and $\vec{M}$

The magnetic phenomena in empty space can be described with a single vector, the vector $\vec{B}$

To describe magnetic phenomena in a material environment, we introduce two other vectors: the magnetic intensity (or the magnetic excitability) $\vec{H}$ and magnetization $\vec{M}$. The vectors $\vec{B}$, $\vec{H}$ and $\vec{M}$ are connected by the relation :

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} \Rightarrow \vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}$$

$\vec{B}$ comprises two contributions :

- a contribution $\vec{H}$ that is independent of the microscopic properties of the medium or environment

- a contribution $\vec{M}$ that is dependent on the microscopic properties of the medium or environment

In the international system, $\vec{H}$ and $\vec{M}$ are expressed in A/m or J/Tm³.

Magnetic susceptibility and permeabilities

For an isotropic and linear environment

- $\vec{M} = X_m \vec{H}$; X_m is called « magnetic susceptibility » It is a size without dimension.

- $\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M} = \mu_0 \vec{H}(1+X_m) = \mu \vec{H}$ with $\mu = \mu_0(1+X_m) = \mu\mu_r$

μ_r is called« relative permeability ». This is also a size without dimension.

Molecular currents

Molecular currents are microscopic electric currents which exist in all substances. They have a double origin connected to the movement of the electrons in the constitutive atoms of these atoms: The orbital movement of the electrons and the rotation of the electrons themselves. These current loops constitute small magnetic dipoles and possess magnetic moments.

Paramagnetism, diamagnetism and ferromagnetism

There are three classes of magnetic substances: paramagnetics, diamagnetics and ferromagnetics.

Paramagnetics are constituted of atoms which possess permanent magnetic moments. In the absence of magnetic fields, these elementary magnetic moments are not oriented following any order, (they randomly oriented) and their resultant is naught. On the other hand, if they are placed in a magnetic field exciter, they tend to align themselves with the field. The thermal agitation tends to destroy this alignment even more so as the temperature rises. Paramagnetics are also characterized by a positive magnetic susceptibility.

The atoms constitutive of diamagnetics do not have magnetic moments. When we apply a magnetic field on them, magnetic moments opposed to this field are induced on these atoms. The magnetic susceptibility of the diamagnetics is negative.

The field $\vec{B}$ produced by a current is modified by the presence of a magnetic material.

Usually this variation is slight. A slight increase in $\vec{B}$ appears in a paramagnetic and a slight decrease in the case of a diamagnetic. On the other hand, in ferromagnetic substances, i.e. in substances susceptible to high magnetization, there is a strong

increase in $\vec{B}$. A ferromagnetic is also constituted of atoms having permanent magnetic moments. It contains a large number of magnetic domains called magnetic domains or Weiss domains. The form of these domains is modified in the presence of exterior magnetic.

Auto evaluation

The learners take note of the difficulties encountered during the course of the learning activity. They will be able to revue the relevant parts of the course in order to better understand the difficulties and mistakes.

Teacher's Guide

The teacher or Professor will correct the group exercises. He/She will leave the corrections in a work space accessible to the learners. The correction will be joined with adequate feedback. The grades obtained by each group are attributed to the members of the group and will count for 20% of the final evaluation of the magnitude.

Learning Activity 4

Name of the activity: Polarization –Poynting Energy Vector, radiation pressure

Learning Time

Instructions: For this activity, if you have **at least ¾ of the points**, you have done very good work and may continue.

If you have **less than half the points**, you must re-read the course material and re-do the activity.

If you have **more than half the points and less than ¾ of the points**, you have done good work, but you must make efforts in the future.

Specific Objectives

The learner must be able to

Examine the evolution of an electromagnetic wave

Determine the state of polarization of an electromagnetic wave

Determine the Poyning vector associated to a plane electromagnetic wave

Determine the intensity of an electromagnetic wave

Determine the radiation pressure of an electromagnetic wave

Summary of the activity

This activity will focus on the examination of the four fundamental equations which underlie electromagnetism. It will also put emphasis on the polarization of an electromagnetic wave and the energy it transports during the course of the propagation. You will learn to:

- distinguish the characteristics of a plane wave in particular the relation which links the electric field, the magnetic field and the propagation vector associated to this wave
- analyze the state of polarization of an electromagnetic wave
- examine the Poynting vector and its relation with the energy carried by a wave

Appropriate Readings

- RASOLONDRAMANITRA, H. (2007). Les équations de Maxwell ; Les ondes électromagnétiques. Madagascar. Université d'Antananarivo. Cours inédit.

Appropriate Resources

1. BEISER, A.(1979). Physique appliquée : Cours et problèmes.Serie Schaum . Traduction française par LOBENBERG M. McGraw-Hill, Paris
2. FRANCIS, W. S., ZEMANSKY, M. W. and YOUNG. H. D. (1987). University Physics. Seventh Edition, Addison-Wesley Publishing Company, USA.
3. FRANCIS, W. S., ZEMANSKY, M. W. and YOUNG, H. D. (1974). College Physics. Fourth Edition. Addison-Wesley Publishing Company, USA.
4. HALPERN, A. (1991) Maxi Schaum Physique 2. Traduction française par BABES,V. McGraw-Hill, Paris
5. NELKON, M. and PARKER, P. (1998). Advanced Level Physics. Seventh Edition, Heinemann, (Oxford ?)
6. RESNICK, R. et HALLIDAY, D. (1979). Electricité et magnétisme, physique 2. Traduit par André Lebel et Claudine Thériault. Editions du Renouveau Pédagogique. Inc. Montréal (Québec) Canada
7. TIPLER, P.-A. and Company. (1999). Physics for Scientists and Engineers. Fourth Edition.   W.H. FREEMAN and company. Worth Publishers Inc., New York, USA
8. TIPLER, P. A. (1982). Physics. Second Edition. Worth Publishers Inc, New York, USA

Useful Links

http://www.univ-lemans.fr/enseignements/physique/02/electri/oem1.html
http://www.walter-fendt.de/ph11f/emwave_f.htm
http://www.sciences.univ-nantes.fr/physique/perso/cortial/bibliohtml/2pcirc_ j.html
http://www.sciences.univ-nantes.fr/physique/perso/cortial/bibliohtml/polond_ j.html

Description of the activity

Your itinerary in this activity consists of three steps:

Step 1: readings on the study themes will be assigned to you

Step 2: in this phase, you will have to solve a series of exercises to help you evaluate the material you have learned

Step 3: you are then guided toward sites which examine your study themes and which will allow you to visualize certain animations

Formative Evaluation

It is compulsory for the exercises to be done collaboratively. The grade for the group is the grade assigned to each member of the group.

Each exercise counts for 10% of the points.

Exercise 1

Let us consider the following waves:

$$\vec{E}_1 = \vec{i} \; E \sin(kz-\omega t) + \vec{j} \; E \cos(kz-\omega t)$$

$$\vec{E}_2 = \vec{i} \; E \cos(kz-\omega t) + \vec{j} \; E \cos(kz-\omega t + \frac{\pi}{4})$$

$$\vec{E}_3 = \vec{i} \; E \sin(kz-\omega t) - \vec{j} \; E \sin(kz-\omega t)$$

a) Study the evolutions of $\vec{E}_1(0,t)$, $\vec{E}_2(0,t)$ and $\vec{E}_3(0,t)$ in the course of time

b) Determine the state of polarization of $\vec{E}_1$, of $\vec{E}_2$ and of $\vec{E}_3$

Exercise 2

Consider an electromagnetic wave whose electric field is defined by :

$$\vec{E}(z,t) = E_0 \left[\vec{i} \; \cos(kz-\omega t) - \vec{j} \; \sin(kz-\omega t) \right]$$

Determine the Poynting vector $\vec{P}$ and the energy density U associated with this wave.

Given: $\vec{a} \wedge (\vec{b} \wedge \vec{c}) = (\vec{a} \cdot \vec{c}) \vec{b} - (\vec{a} \cdot \vec{b}) \vec{c}$.

Exercise 3

Consider, in complex notation, an electric field $\vec{E} = \vec{e}_x \; j \omega \; A_0 \; e^{j(kz-\omega t)}$ and a

magnetic field $\vec{B} = \vec{e}_y \; j \, k \, A_0 \; e^{j(kz-\omega t)}$, $\vec{e}_x$ and $\vec{e}_y$ representing respectively the unitary vectors following the x axis and the y axis. Verify that $\vec{E}$ and $\vec{B}$ respectively represent the electric and magneitc fields of an elecromagnetic wave plane whose propagation axis and polarization need to be specified.

Exercise 4

Recall the meaning of each of the Maxwell equations

Exercise 5

Check the right answer.

Let $\vec{E} = E_x \, (\, z\, , \, t\,) \; \vec{i}$ the electric field of an elecromagnetic wave plane. It is:

❑ polarized along the x axis
❑ polarized along the y axis
❑ polarized along the z axis

Exercise 6

Check the right answer

Let $\vec{E} = E_x \, (\, z\, , \, t\,) \; \vec{i}$ the electric field of an electromagnetic wave plane. The magnetic field associated with this wave is :

❑ $\vec{B} = B_x \, (\, z\, , \, t\,) \; \vec{i}$

❑ $\vec{B} = B_y \, (\, z\, , \, t\,) \; \vec{j}$

❑ $\vec{B} = B_z \, (\, z\, , \, t\,) \; \vec{k}$

Exercise 7

Check the right answer

1. The Maxwell equations only apply to electric and magnetic fields which do not vary in the course of time.

    ❑ true
    ❑ False

2. Electromagnetic waves are longitudinal waves

    ❑ True
    ❑ False

3. The energy densities stored in the electric field and in the magnetic fields of an electromagnetic wave are equal.

    ❑ True
    ❑ False

Exercise 8

Consider an electromagnetic wave described by $\vec{E} = \vec{E_0} \cos(kx - \omega t)$ in empty space. Establish the expression of the intensity of this wave. Calculate the intensity knowing that the amplitude of the field $\vec{E}$ is $100 \dfrac{V}{m}$. The permittivity of the empty space is $\varepsilon_0 = 8,85 \times 10^{-12} \ C^2 \ N^{-1} \ m^{-2}$.

Exercise 9

A radio station r has an average power radiant equal to 50kW.

1 – Determine the intensity of the wave at 50kW from the station and its pressure. In order to do this, we assume that the radiation is uniform on a sphere of centre R.

2- Determine, at this distance, the amplitudes of the fields $\vec{E}$ and $\vec{B}$ associated with this wave.

Exercise 10

Consider a circular polarized wave whose axis of propagation follows Oz. This wave is such that $\vec{E}(0,0) = \dfrac{E_0}{2}(\vec{e}_x + \sqrt{3}\,\vec{e}_y)$, $\vec{e}_x$ and $\vec{e}_y$ representing respectively the unitary vectors following the x axis and the y axis. Find its expression $\vec{E}(z,t)$

Learning Activity

- The learners must do all the appropriate reading prior to doing the exercises.
- The teacher organizes them in groups to do collaborative work.
- They discuss in chat rooms the different points that they may not have understood under the supervision of the teacher or tutor.
- When the teacher or tutor thinks that the learners have a satisfactory level of understanding, then they will be able to start solving the exercises.
- All the groups find the answers to the same exercises at the same time under the supervision of the tutor who will fix a time for the exercises to be completed.
- Each group chooses a delegate who will put the names of all the group members on the assigned exercises before sending them by e-mail attachment to the course Professor.

Answer Key

Exercise 1

a) **the evolutions of $\vec{E}_1(0,t)$, $\vec{E}_2(0,t)$ and $\vec{E}_3(0,t)$ during the course of time**

Evolution of $\vec{E}_1(0,t)$ and polarsization of $\vec{E}_1$

We can write :

$$\vec{E}_1 = \vec{i}\,E\sin(kz-\omega t) + \vec{j}\,E\cos(kz-\omega t) = \vec{i}\,E_{1x} + \vec{j}\,E_{1y}$$

with $\quad E_{1x} = E\sin(kz-\omega t)$ and $E_{1y} = E\cos(kz-\omega t)$

For $z = 0$, and by replacing ω with $2\pi f$, f being the wave frequency, we have :

$E_{1x}(0,t) = -E \sin \omega t = -E \sin 2\pi \, \text{f} t$

$E_{1y}(0,t) = E \cos \omega t = E \cos 2\pi \, \text{f} t$

Let us now examine the evolutions of $E_{1x}(0,t)$, of $E_{1y}(0,t)$ and of $\vec{E_1}(0,t)$ as a function

Of time and let assign their values in the table below. The x axis and the y axis are illustrated below:
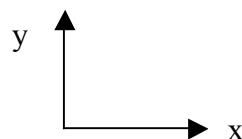
T	0	$\dfrac{1}{8\,\text{f}}$	$\dfrac{2}{8\,\text{f}}$	$\dfrac{3}{8\,\text{f}}$	$\dfrac{4}{8\,\text{f}}$	$\dfrac{5}{8\,\text{f}}$
$E_{1x}(0,t)$	0	$-E \sin\dfrac{\pi}{4}$	$-E$	$-E \sin\dfrac{3\pi}{4}$	0	$-E \sin\dfrac{5\pi}{4}$
$E_{1y}(0,t)$	E	$E \cos\dfrac{\pi}{4}$	0	$E \cos\dfrac{3\pi}{4}$	$-E$	$E \cos\dfrac{5\pi}{4}$
$\vec{E_1}(0,t)$	↑	↖	←	↙	↓	↗

$\vec{E_1}$ turns counter clock-wise and its amplitude is constant. Its extremety describes a circle. This wave is thus polarized in a circular manner to the left. We can verify this by taking:

$E_{1x}^2 + E_{1y}^2 = E^2$. This is the equation of a circle with radius E.

Evolution of $\vec{E_2}(0,t)$ and polarization of $\vec{E_2}$

$\vec{E_2} = \vec{i}\, E \cos(kz-\omega t) + \vec{j}\, E \cos(kz-\omega t + \dfrac{\pi}{4}) = \vec{i}\, E_{2x} + \vec{j}\, E_{2y}$

with $\quad E_{2x} = E \cos(kz - \omega t)$ et $E_{2y} = E \cos(kz - \omega t + \dfrac{\pi}{4})$

For z = 0, and when we replace ω with $2\pi f$, we have :

$E_{2x}(0,t) = E \cos\omega t = E \cos 2\pi ft$

$E_{2y}(0,t) = E \cos(\dfrac{\pi}{4} - \omega t) = E \cos(\dfrac{\pi}{4} - 2\pi ft)$

The evolutions of $E_{2x}(0,t)$, of $E_{2y}(0,t)$ and of $\vec{E_2}(0,t)$ as a function of time are described below :
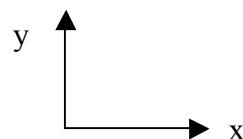
T	0	$\dfrac{1}{8f}$	$\dfrac{2}{8f}$	$\dfrac{3}{8f}$	$\dfrac{4}{8f}$
$E_{2x}(0,t)$	E	$E \cos\dfrac{\pi}{4}$	0	$E \cos\dfrac{3\pi}{4}$	$-E$
$E_{2y}(0,t)$	$E \cos\dfrac{\pi}{4}$	E	$E \cos\dfrac{\pi}{4}$	0	$E \cos\dfrac{3\pi}{4}$
$\vec{E_2}(0,t)$	↗	↑	↑	←	↙

Exercise 2

Poynting Vector

$$\vec{E}(z,t) = E_0 \left[\vec{e}_x \cos(kz-\omega t) - \vec{e}_y \sin(kz-\omega t) \right]$$

Let us recall the expression of the Poynting vector :

$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \wedge \vec{B})$$

We have here a plane wave which moves in the direction z. Its propagation vector $\vec{k}$ is thus expressed :

$$\vec{k} = k\,\vec{e}_z$$

We then have:

$$\vec{B} = \frac{\vec{k}}{\omega} \wedge \vec{E} = \frac{k}{\omega}(\vec{e}_z \wedge \vec{E})$$

But $k = \dfrac{\omega}{c} \Rightarrow \vec{B} = \dfrac{1}{c}(\vec{e}_z \wedge \vec{E})$

The Poynting vector is thus expressed :

$$\vec{S} = \frac{1}{\mu_0 c}(\vec{E} \wedge (\vec{e}_z \wedge \vec{E}))$$

When we apply the relation $\vec{a} \wedge (\vec{b} \wedge \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$, we obtain

$$\vec{S} = \frac{1}{\mu_0 c}\vec{E}^2\,\vec{e}_z$$

But $\vec{E}^2 = E_0^2 \implies \vec{S} = \dfrac{E_0^2}{\mu_0\,c}\,\vec{e}_z$

We see that the Poynting vector has the same direction as the wave propagation.

Energy Density

The energy density transported by an electromagnetic wave is given by :

$$U = \dfrac{1}{2}\,\varepsilon_0\,E^2 + \dfrac{1}{2\,\mu_0}\,B^2$$

Where $E^2 = E_0^2$

$$\vec{B} = \dfrac{1}{c}\,(\vec{e}_z \wedge \vec{E}) \implies \left|\vec{B}\right| = \dfrac{1}{c}\,\left|\vec{e}_z\right|\,\left|\vec{E}\right|\,\sin(\vec{e}_z,\vec{E})$$

But $\sin(\vec{e}_z,\vec{E}) = \sin\dfrac{\pi}{2}$ where we have $\implies \left|\vec{B}\right| = \dfrac{1}{c}\,\left|\vec{E}\right| \implies B = \dfrac{E}{c}$ et

$B^2 = \dfrac{1}{c^2}\,E^2$

The energy density thus becomes :

$$U = \dfrac{1}{2}\,\varepsilon_0\,E^2 + \dfrac{1}{2\,\mu_0}\,B^2 = (\,\dfrac{1}{2}\,\varepsilon_0 + \dfrac{1}{2\,\mu_0\,c^2}\,)\,E_0^2$$

But $c^2 = \dfrac{1}{\mu_0\,\varepsilon_0} \implies \dfrac{1}{c^2} = \mu_0\,\varepsilon_0$. By replacing c^2 we obtain :

$$U = (\,\dfrac{1}{2}\,\varepsilon_0 + \dfrac{\mu_0\,\varepsilon_0}{2\,\mu_0}\,)\,E_0^2 = \varepsilon_0\,E_0^2$$

Exercise 3

If $\vec{E}$ and $\vec{B}$ are respectively the electric and magnetic field of a plane electromagnetic wave, then they satisfy the relation:

$$\vec{B} = \frac{1}{\omega} \left(\vec{k} \wedge \vec{E} \right)$$

Calculate $\left(\vec{k} \wedge \vec{E} \right)$. Here $\vec{E}$ and $\vec{B}$ propagate in the direction $z \Rightarrow \vec{k} = \vec{e}_z \, k$. We obtain:

$$\left(\vec{k} \wedge \vec{E} \right) = \vec{e}_z \, k \wedge \vec{e}_x \, j \, \omega \, A_0 \, e^{j(kz - \omega t)} = \vec{e}_z \wedge \vec{e}_x \, j \, k \, \omega \, A_0 \, e^{j(kz - \omega t)}$$

But $\vec{e}_z \wedge \vec{e}_x = \vec{e}_y$, where we have :

$$\left(\vec{k} \wedge \vec{E} \right) = \vec{e}_y \, j \, k \, \omega \, A_0 \, e^{j(kz - \omega t)}$$

$$\frac{\vec{k} \wedge \vec{E}}{\omega} = \vec{e}_y \, j \, k \, A_0 \, e^{j(kz - \omega t)}$$

It is the field $\vec{B}$

Exercise 4

The Maxwell equations under integral form are:

$$\oint_S \vec{E} \cdot \vec{dS} = \frac{1}{\varepsilon_0} \int_{(V)} \rho \, dV \qquad (1)$$

(V) is a volume enclosed by (S) and density r of the charges. This is Gauss's law: the flow of an electric field through a closed surface S is connected to the algebraic sum of all the charges located within S.

$$\oint_S \vec{B} \cdot \vec{dS} = 0 \qquad (2)$$

This is equivalent to Gauss' law in magnetism. It reflects the fact that there is no isolated magnetic pole.

$$\oint_{C} \vec{E} \cdot \vec{dl} = - \frac{d}{dt} \int_{S} \vec{B} \cdot \vec{dS} \qquad (3)$$

S is a surface bounded by the curve C. This is Faraday's law.

$$\oint_{C} \vec{B} \cdot \vec{dl} = \mu_{0} \ I + \varepsilon_{0} \ \frac{d}{dt} \int_{S} \vec{E} \cdot \vec{dS} \qquad (4)$$

This is Ampere's law (generalised).

Exercise 5

❑ polarized along the x-axis : Correct answer. Actually $\vec{E}$ does not have components following y and z

❑ polarized along the y-axis : False. Re-analyze the expression of $\vec{E}$

❑ polatized along the z-axis : Incorrect. The z-axis is the direction of propagation of the electromagnetic wave.

Exercise 6

❑ $\vec{B} = B_{x} \ (z , t) \ \vec{i}$: False. $\vec{E}$ and $\vec{B}$ are not polarized following the same direction.

❑ $\vec{B} = B_{y} \ (z , t) \ \vec{j}$: Correct answer. In face $\vec{E}$ and $\vec{B}$ are perpendicular to each other, and perpendicular to the direction of propagation. Here, the direction of propagation is the z-axis. $\vec{E}$ is polarized along the x-axis, thus $\vec{B}$ must be polarized along the y-axis.

❑ $\vec{B} = B_{z} \ (z , t) \ \vec{k}$: False. For a plane electromagnetic wave, $\vec{B}$ is perpendicular to the direction of wave propagation, which is, in this case, the z-axis.

Exercise 7

1- False. In fact, under integral form, the Maxwell equations are :

$$\oint_S \vec{E} \cdot \vec{dS} = \frac{1}{\varepsilon_0} \int_{(V)} \rho \, dV \qquad (1)$$

$$\oint_S \vec{B} \cdot \vec{dS} = 0 \qquad (2)$$

$$\oint_C \vec{E} \cdot \vec{dl} = - \frac{d}{dt} \int_S \vec{B} \cdot \vec{dS} \qquad (3)$$

$$\oint_C \vec{B} \cdot \vec{dl} = \mu_0 \, I + \varepsilon_0 \, \frac{d}{dt} \int_S \vec{E} \cdot \vec{dS} \qquad (4)$$

Equations (3) and (4) show that these equations apply to variable fields: a magnetic field varies over time creating an electric field and vice versa.

2- False. In an electromagnetic wave, the electric field vectors $\vec{E}$, field magnitude $\vec{B}$ and the wave vector $\vec{k}$ that define the direction of wave propagation are

linked by $\vec{B} = \frac{1}{\omega} (\vec{k} \wedge \vec{E})$ and $\vec{E}$ is orthogonal to $\vec{B}$. The vectors $\vec{k}$, $\vec{E}$ and $\vec{B}$ form a direct trihedral.

$\vec{E}$ and $\vec{B}$ are perpendicular to $\vec{k}$ $\Rightarrow$ an electromagnetic wave is a transverse wave.

3- True. In fact, the energy density of electric and magnetic energy density are respectively:

$$u_E = \frac{1}{2} \, \varepsilon_0 \, E^2 \qquad u_B = \frac{1}{2 \mu_0} \, B^2$$

Due to the relation $E = B \, c$, we have $u_B = \frac{1}{2 \mu_0} \frac{E^2}{c^2}$. But $c^2 = \frac{1}{\varepsilon_0 \, \mu_0}$. It follows that :

$$u_B = \frac{1}{2\,\mu_0}\,E^2\,\varepsilon_0\,\mu_0 = \frac{1}{2}\,\varepsilon_0\,E^2 = u_E$$

Exercise 8

Let I be the intensity: $I = \,< S >$ where $< S >$ is the average value of the normal of the Poynting vector. Find the Poynting vector $\vec{S}$:

$$\vec{S} = \frac{\vec{E} \wedge \vec{B}}{\mu_0}\quad \text{with}\ \vec{B} = \frac{\vec{k} \wedge \vec{E}}{\omega}.$$

Here we have a wave that propagates in the direction of the x-axis. It follows that $\vec{k} = k\,\vec{e}_x,\ \ \vec{e}_x$ being the unit vector along the x-axis. We have:

$$\vec{B} = \frac{k\,\vec{e}_x \wedge \vec{E}}{\omega} = \frac{\vec{e}_x \wedge \vec{E}}{c}$$

$$\vec{B} = \frac{1}{c}\,\vec{e}_x\ \wedge\ \vec{E}_0\ \cos(kx - \omega t) = \vec{B}_0\ \cos(kx - \omega t)\ \ \text{with}\ \vec{B}_0 = \frac{\vec{e}_x\ \wedge\ \vec{E}_0}{c}.$$

$\vec{e}_x,\ \vec{E}_0\ \text{et}\ \vec{B}_0$ form a direct trihedral $\Rightarrow\ B_0 = \dfrac{E_0}{c}$.

The Poynting vector is thus written as:

$$\vec{S} = \frac{\vec{E} \wedge \vec{B}}{\mu_0} = \frac{\vec{E}_0 \wedge \vec{B}_0}{\mu_0}\ \cos^2(kx - \omega t)$$

$$\left|\vec{S}\right| = S = \frac{1}{\mu_0}\,\left|\vec{E}_0\ \wedge\ \vec{B}_0\right|\cos^2(kx - \omega t) = \frac{1}{\mu_0}\,E_0\,B_0\,\cos^2(kx - \omega t)$$

$$S = \frac{1}{\mu_0 \, c} \, E_0^2 \, \cos^2(kx - \omega t)$$

The intensity of the wave is :

$$I = <S> = \frac{1}{\mu_0 \, c} \, E_0^2 <\cos^2(kx - \omega t)> = \frac{E_0^2}{2 \, \mu_0 \, c} = \frac{c}{2} \, \varepsilon_0 \, E_0^2 \text{ since the average}$$

value $<\cos^2(kx - \omega t)> = \frac{1}{2}$.

Numerical application:

$$\varepsilon_0 = 8,85 \text{ x } 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2} \text{ , } E_0 = 100 \, \frac{V}{m} \text{ and } c = 3 \text{ x } 10^8 \text{ m s}^{-1}$$

$$I = \frac{c}{2} \, \varepsilon_0 \, E_0^2 = \frac{1}{2} \, (3 \text{ x } 10^8) \, (8,85 \text{ x } 10^{-12}) \, (100^2)$$

$$I = 13,3 \, \frac{W}{m^2}$$

Exercise 9

Wave intensity

The wave intensity is $I = <S>$ where $<S>$ is the average value of the norm of the Poynting vector. S is an energy per unit time and per unit area. Power is energy per unit time. We have:

$$I = \frac{P}{A} \text{ , P being the radiating power, A the area of the sphere of radius R = 50km}$$
. the area of the sphere is $A = 4 \, \pi \, R^2$. We obtain:

$$I = \frac{50 \text{ x } 10^3}{4 \, \pi (50 \text{ x } 10^3)^2}$$

$$I = 1,59 \text{ x } 10^{-6} \text{ Wm}^{-2}$$

Radiation pressure

Let P_r be this pressure

$$P_r = \frac{I}{c} \quad \Rightarrow \quad P_r = \frac{1{,}59 \times 10^{-6}}{3 \times 10^{8}}$$

$$P_r = 0{,}53 \times 10^{-14} \ \text{Pa}$$

Amplitudes of $\vec{E}$ and $\vec{B}$

By taking the results of the previous exercise, we have :

$I = \dfrac{c}{2} \ \varepsilon_0 \ E_0^2$, E_0 being the amplitude of the field $\vec{E}$. We obtain:

$$E_0 = \sqrt{\frac{2\,I}{c\,\varepsilon_0}}$$

Numerical application :

$\varepsilon_0 = 8{,}85 \times 10^{-12} \ C^2 \ N^{-1} \ m^{-2}$; $I = 1{,}59 \times 10^{-6} \ Wm^{-2}$; $c = 3 \times 10^{8} \ m \ s^{-1}$

$$E_0 = \sqrt{\frac{2 \times 1{,}59 \times 10^{-6}}{3 \times 10^{8} \times 8{,}85 \times 10^{-12}}} = 3{,}5 \times 10^{-2} \ \frac{V}{m}$$

Since $B_0 = \dfrac{E_0}{c}$ the amplitude of the field $\vec{B}$ is:

$$B_0 = \frac{E_0}{c} = \frac{3{,}5 \times 10^{-2}}{3 \times 10^{8}} = 1{,}17 \times 10^{-10} \ \text{T}$$

Exercise 10

We have Here a circular polarization, thus generally $\vec{E}\ (z,t)$ has the expression :

$$\vec{E}\ (z,t) = E_0 \left[\vec{e}_x \ \cos(kz - \omega t + \varphi) \ \pm \ \vec{e}_y \ \sin(kz - \omega t + \varphi) \right]$$

We must determine the value of the phase φ . We have $z = 0$ and $t = 0$:

$$\vec{E}\ (0,0) = E_0 \left[\vec{e}_x \ \cos(\varphi) \ \pm \ \vec{e}_y \ \sin(\varphi) \right] \ (1)$$

But:

$$\vec{E}\ (0,0) = \frac{E_0}{2} \ (\vec{e}_x \ + \sqrt{3} \ \vec{e}_y) \ (2)$$

We deduce relations (1) and (2) :

$$E_0 \ \cos\varphi \ = \ \frac{E_0}{2} \ \text{and} \ E_0 \ \sin\varphi \ = \ \frac{E_0 \ \sqrt{3}}{2} \ \Rightarrow \quad \cos\varphi = \frac{1}{2} \ \text{and} \ \sin\varphi = \frac{\sqrt{3}}{2}$$

This is verified for $\varphi \ = \ \dfrac{\pi}{3}$ and thus the expression for $\vec{E}\ (z,t)$ is :

$$\vec{E}\ (z,t) = E_0 \left[\vec{e}_x \ \cos(kz - \omega t + \frac{\pi}{3}) \ \pm \ \vec{e}_y \ \sin(kz - \omega t + \frac{\pi}{3}) \right]$$

Sites to visit

You will find information in the following links to supplement and reinforce what you learned during this activity.

http://www.walter-fendt.de/ph11f/emwave_f.htm

 http://www.univ-lemans.fr/enseignements/physique/02/electri/oem1.html

Both sites offer an animation of the propagation of an electromagnetic plane wave.

http://www.sciences.univ-nantes.fr/physique/perso/cortial/bibliohtml/2pcirc_j.html

This link shows the superposition of two circular polarizations left and right.

http://www.sciences.univ-nantes.fr/physique/perso/cortial/bibliohtml/po-lond_j.html

This is an animation of an elliptical polarization.

Self evaluation

The learners will record the difficulties encountered and mistakes made during the search of solutions to exercises in order to avoid them later. They will review parts of the course that are not well understood and preparing the summative evaluation.

Teacher's guide

The professor will correct group productions and place the corrected production in a workspace accessible to learners. The correction will be accompanied by appropriate feedback explaining all the mistakes. The scores for each group are assigned to all group members and will count for 20% of the final evaluation of the module.

XI. List of all key concepts (Glossary)

Hall Effect

When a metal carrying a current is introduced into a magnetic field perpendicular to the direction of this current, it appears in the metal as an electric field. This electric field is perpendicular both to the current direction and magnetic field. We call the Hall Effect the emergence of such a field.

Electromagnetic induction

A variation in the time of magnetic flux Φ through a circuit has the effect of producing an emf which circulates in it induces a current. This phenomenon is called electromagnetic induction. This phenomenon occurs whenever the magnetic flux through a circuit varies, whatever the cause of this variation.

Self induction

Any circuit carrying a current of intensity I is crossed by a magnetic flux Φ generated by the current itself. Φ is proportional to I. Any variation of I, then Φ has the effect of producing an emf induction that circulates a complementary current in this circuit. It is this phenomenon that gives the name of «self induction».

Mutual induction of two circuits

Consider two circuits C1 and C2 driven by currents of intensity I1 and I2. If the intensity varies I1 in the circuit (C1), the magnetic flux through C2 varies and appears in (C2) an emf induction called emf mutual inductance.

Paramagnetism

It is a property of substances whose constituent atoms each have a permanent magnetic elementary moment $\vec{\mu}$. In the absence of magnetic field exciter $\vec{B}_{ex}$, the magnetic moments are randomly oriented and their resultant moment is zero. In the presence of an exciter $\vec{B}_{ex}$, the magnetic dipoles $\vec{\mu}$ tend to align with $\vec{B}_{ex}$. The thermal motion prevents perfect alignment but the magnetic moment is nonzero. The substances that pose such a property are called paramagnetic bodies.

Ferromagnetism

It is a property of substances likely to take strong magnetizations. As the paramagnetic, the ferromagnetic consist of atoms with permanent magnetic moments, but these magnetic moments can be aligned even in the absence of a field exciter.

Diamagnetism

It is a property of substances whose constituent atoms have no magnetic moment. When we apply an excitatory field $\vec{B}_{ex}$, orbital magnetic moments are induced which opposes the direction of $\vec{B}_{ex}$.

Magnetization

The magnetization characteristic of the magnetic state of a magnetized body. It is the magnetic moment per unit volume. It expresses Am^{-1} in the international system of units. It uses a vector $\vec{M}$ called a magnetization vector to describe this condition.

$$\vec{M} = \frac{\sum \vec{\mu}}{V},\ \sum \vec{\mu}$$ being the magnetic moment and V the volume of the magnetized body.

Hysteresis

When magnetizing a ferromagnetic body originally magnetized, its magnetization increases with the applied field B_{app} or field magnetization and eventually reach a limiting value called intensity of saturation magnetization. We speak here of initial magnetization of the body. If we then decrease B_{app}, the magnetization decreases. The goal for each value of B_{app} is that it retains a value greater than it had during the initial magnetization. This is the phenomenon of magnetic hysteresis. The variation of the magnetization has a delay on the field variation of magnetization. "Hysteresis" is a Greek noun which means delay.

Remanent magnetization

If after a ferromagnetic body was magnetized, it has a gradually decreasing field magnetization, the body maintains a certain magnetization even when the applied field B_{app} becomes zero: it is the remanent magnetization.

Coercive field

To nullify the residual magnetization, we must apply a magnetic field Bc in the opposite direction to the field of initial magnetization. Bc is called the coercive field.

Curie point

The magnetization of a ferromagnet decreases with increasing temperature and vanishes at a certain temperature Tc called the «Curie point».

Maxwell equations

Equations, four in number, which underlie and govern the electromagnetic behavior of electromagnetic waves.

Poynting vector

The transport of energy in an electromagnetic field can be described / defined using a vector flow of electromagnetic energy called Poynting vector. The flow of the vector through a surface is the amount of electromagnetic energy through the surface per unit time; Its direction indicates the direction of energy propagation.

XII. Compiled list of required readings

Reading #1

Complete reference : RASOLONDRAMANITRA, H. (2007). Circuits à courant alternatif. Madagascar. Université d'Antananarivo. Cours inédit.

Summary : This chapter analyzes the oscillations of voltage and current in circuits containing various combinations of resistance, inductance and capacity. It studies the impedances of these circuits, the phenomenon of resonance in RLC series circuit, and the AC power.

Rationale : This reading prepares the learner for the learning of activity 1. He or she will solidify their knowledge in this reading:

-   To calculate the phase shift between voltage and current, the impedance of a circuit
-   To establish the complex impedance of a circuit
-   To establish and calculate the instantaneous current that runs through a circuit and the voltage instant at its terminals

Reading #2

Complete reference : RASOLONDRAMANITRA, H. (2007). Champ magnétique et particules chargées en mouvement. Madagascar. Université d'Antananarivo. Cours inédit.

Summary : This chapter describes and interprets the experience of Thomson, who has contributed to the discovery of the electron, the Hall effect determined the sign of charge carriers in a conductor. He also studied particle accelerators such as the cyclotron and synchrotron.

Rationale:

The points made here will help the learner to determine the relationship $\frac{e}{m}$ by the method of magnetic deflection, to better understand how a mass spectrometer to determine the number of charge carriers per unit volume in a conductor, to calculate the radius of the trajectory of a charged particle penetrating a magnetic field and the velocity of a charged particle at the exit of a cyclotron.

Reading #3

Complete reference : RASOLONDRAMANITRA, H.(2007).Inductance.Madagascar.Université d'Antananarivo. Cours inédit.

Summary : This chapter:

- Outlines and explains the law of Lenz
- Explains the phenomenon of self-induction, mutual inductance
- Defines the inductance of a device and shows how one can calculate it
- Considers some practical applications of the phenomenon of electromagnetic induction
- Shows that energy is stored in a magnetic field

Rationale: This reading helps the learner to apply Lenz's law to determine the direction of induced current in a circuit, to calculate the inductance of a circuit and the coeffHereent of mutual induction of two circuits, to consolidate its gains on the phenomenon of electromagnetic induction.

Reading #4

Complete reference : RASOLONDRAMANITRA, H. (2007). Les propriétés magnétiques de la matière. Madagascar. Université d'Antananarivo. Cours inédit.

Summary : This chapter begins with a reminder of the magnetic field and electric lines and a discussion of Gauss's law applied to magnetism. The magnetization of magnetic materials is then discussed. This study is followed by the description and interpretation of properties of paramagnetic, diamagnetic and ferromagnetic materials.

Rationale: This reading allows the learner to capture the key points for understanding the magnetization processes and understanding the phenomena of paramagnetism, of diamagnetism and ferromagnetism.

Reading #5

Complete reference : RASOLONDRAMANITRA, H. (2007).Les équations de Mawxell.Madagascar. Université d'Antananarivo. Cours inédit.

Summary : This chapter shows what varying a magnetic field creates an electric field. He believes after the opposite effect, i.e. that a variable electric field induces a magnetic field. He introduced the displacement current and the generalized Ampere's law. Maxwell's equations are then presented and analyzed.

Rationale: The learner discovers and learns through the reading that Maxwell's equations summarize the laws of electrHerety and magnetism, and they constitute the fundamental equations of electromagnetism.

Reading #6

Complete reference : RASOLONDRAMANITRA, H. (2007).Les ondes électromagnétiques.Madagascar. Université d'Antananarivo. Cours inédit.

Summary This section establishes the equations of propagation (or wave equations) fields and associated with an electromagnetic wave. It also deals with the energy carried by an electromagnetic wave and brought by the Poynting vector. The polarization of an electromagnetic wave is also discussed.

Rationale: This reading gives the learner the ability to determine:

- The state of polarization of an electromagnetic wave
- The Poynting vector associated with an electromagnetic wave
- The energy density associated with an electromagnetic wave
- The intensity of a wave
- The radiation pressure

XIII. Useful Links

Useful link # 1

Titre : http://membres.lycos.fr/physHeresss/labos/effet_hall.pdf

URL : http:www.

Screen capture:

Description: This is a proposed experiment on the Hall effect in semiconductors. It addresses determining the number of charge carriers per unit volume.

Rationale: Reading of this link will enable the learner to verify, and reinforce their knowledge of Unit 2 of the module.

Useful link # 2

Title : http://jf-noblet.chez-alice.fr/bobine/index.htm

URL : http:www.

Screen capture:
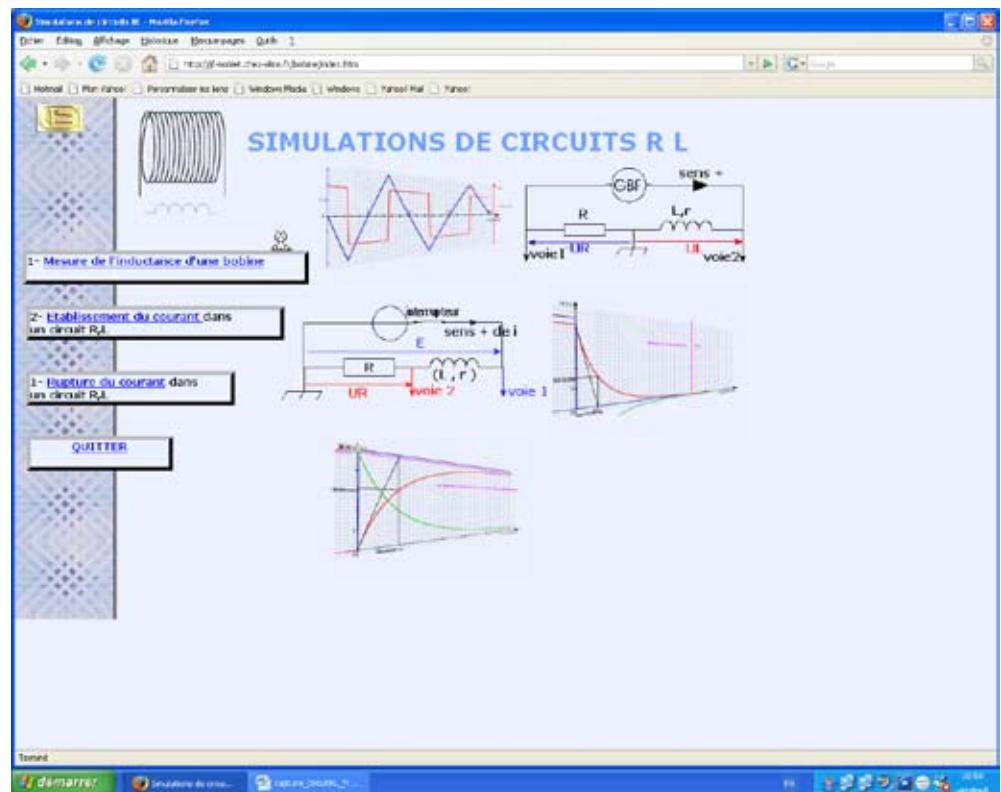
Description: This site presents simulations of the establishment and breaking of current in an RL circuit simulation and a measurement of the inductance of a coil.

Rationale: These simulations are compared with what the learner (e) gained in Unit 2 of the module.

Useful link # 3

Title : http://perso.orange.fr/daniel.robert9/Tension_alternative0.html

URL : http:www.

Screen capture:
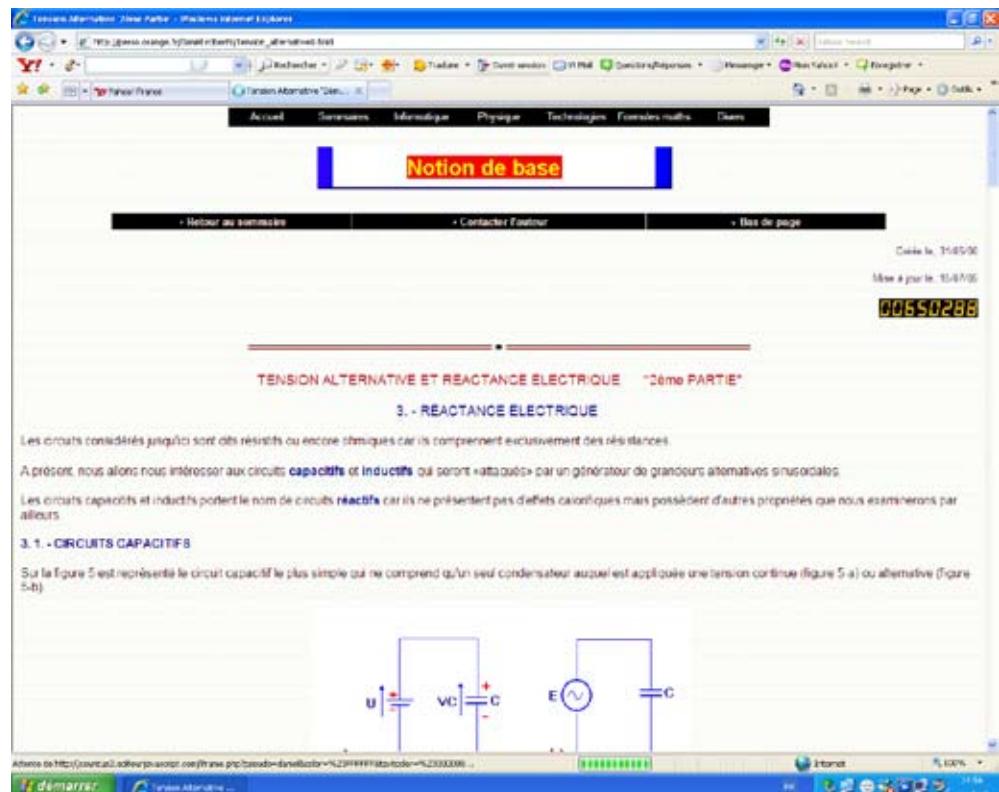
Description: This site explores the phase shift between voltage and current in a circuit capacitance and capacitive reactance.

Rationale: Reading of the link is suggested but not mandatory. It can help the learner understand Unit 1 of the module.

Useful link # 4

Title : http://perso.orange.fr/daniel.robert9/Tension_alternative1.html

URL : http:www.

Screen capture:

Description: This site explores the phase shift between voltage and current in an inductive circuit and inductive reactance.

Rationale: Reading of the link is suggested but not mandatory. It can help the learner understand Unit 1 of the module.

Useful link # 5

Title : http://sitelec.free.fr/cours/rlcseries.pdf

URL : http:www.

Screen capture:
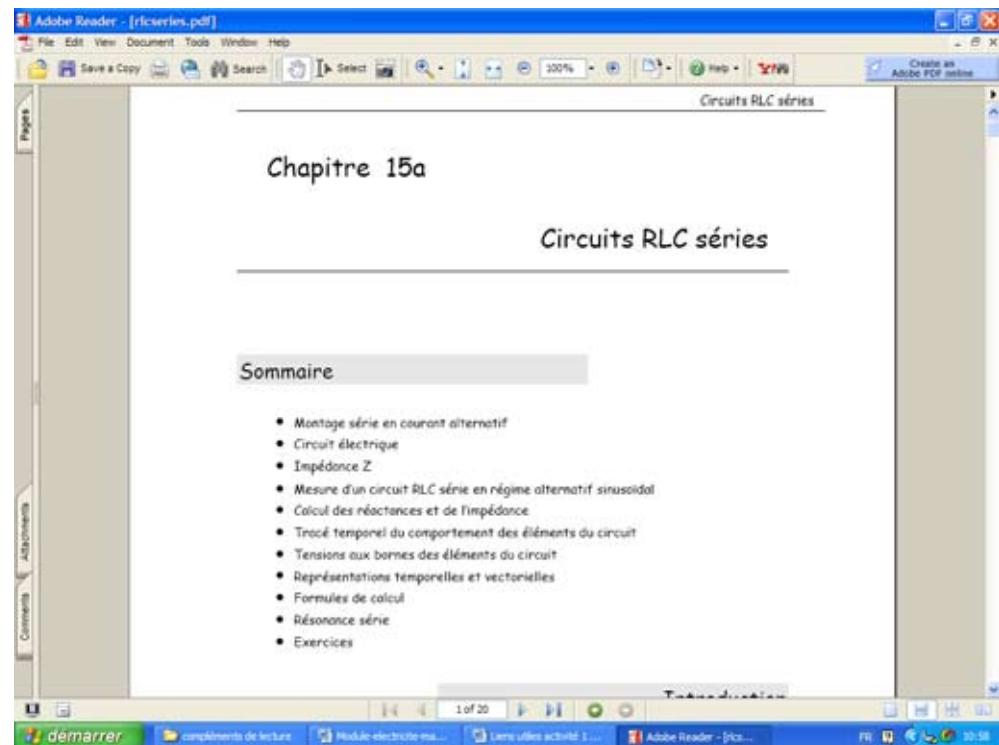
Description: This site analyzes a series RLC circuit connected to an alternative source and studies its behavior when the source frequency varies. It examines in detail the phenomenon of resonance, the impedance, the capacitive and inductive reactance, the phase shift between oscillations of current and voltage cutter terminals of different elements

Rationale: Reading of the link is suggested but not mandatory. It can help the learner understand learning activity 1-2 of Unit 1 of the module.

Useful link # 6

Title : http://www.walter-fendt.de/ph11f/accircuit_f.htm

URL : http:www.

Screen capture:
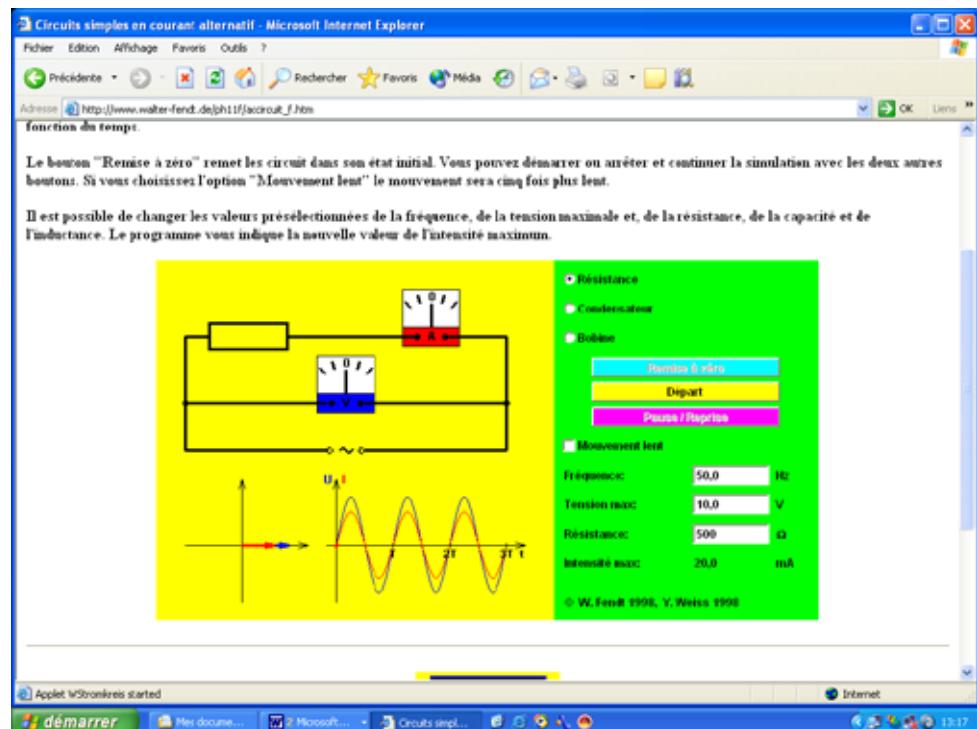
Description: This is an animation of diagrams and Fresnel oscillations of voltage and current corresponding to a circuit with only resistance, inductance and capacity. The learner may act on the parameters R, L, C and voltage.

Rationale: Reading of the link is suggested but not mandatory. It can help the learner understand learning activity 1-1 of Unit 1 of the module.

Useful link # 7

Title : http://www.univ-lemans.fr/enseignements/physique/02/electri/rlcsinus. html

URL : http:www.

Screen capture:
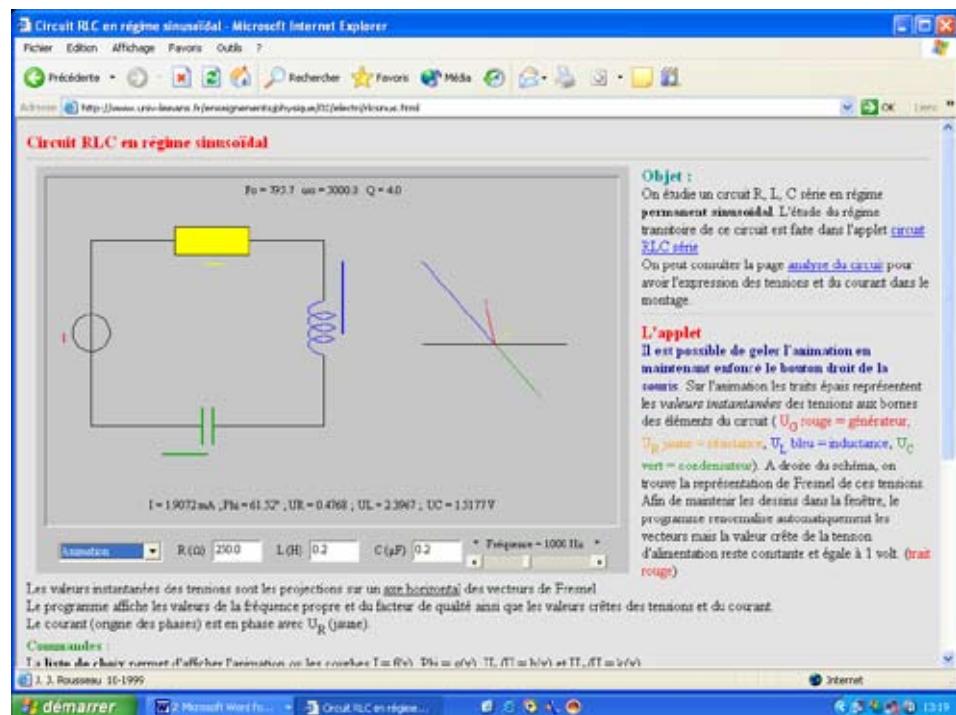
Description: It is a study of series RLC circuits. The site shows curves of current intensity and phase, an animation of the oscillations of voltage and current.

Rationale: Reading of the link is suggested but not mandatory. It can help the learner understand learning activity 1-1 of Unit 1 of the module.

Useful link # 8

Titles : http://perso.orange.fr/olivier.granier/electro/simul/cour_rlc/circuitRLC3. htm

http://www.fundp.ac.be/sciences/physique/didactique/elec/RLC3.php

URL : http:www.

Screen capture:
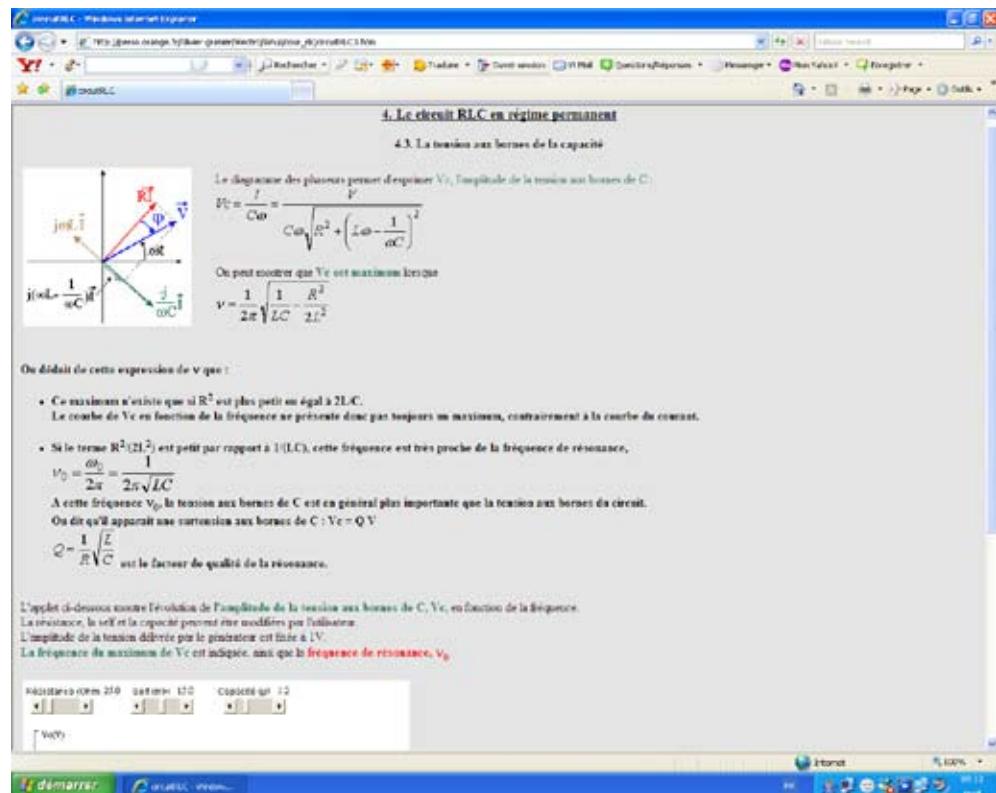
Description: These links offer the same activity. This is followed in an RLC circuit changing the amplitude of the cutters tensiwe terminals of a capacitor depending on frequency.

Rationale: Reading of the link is suggested but not mandatory. It can help the learner understand learning activity 1-2 of Unit 1 of the module.

Useful link # 9

Title :

http://perso.orange.fr/olivier.granier/electro/simul/cour_rlc/circuitRLC4.htm

http://www.fundp.ac.be/sciences/physique/didactique/elec/RLC4.php

URL : http:www.

Screen capture:
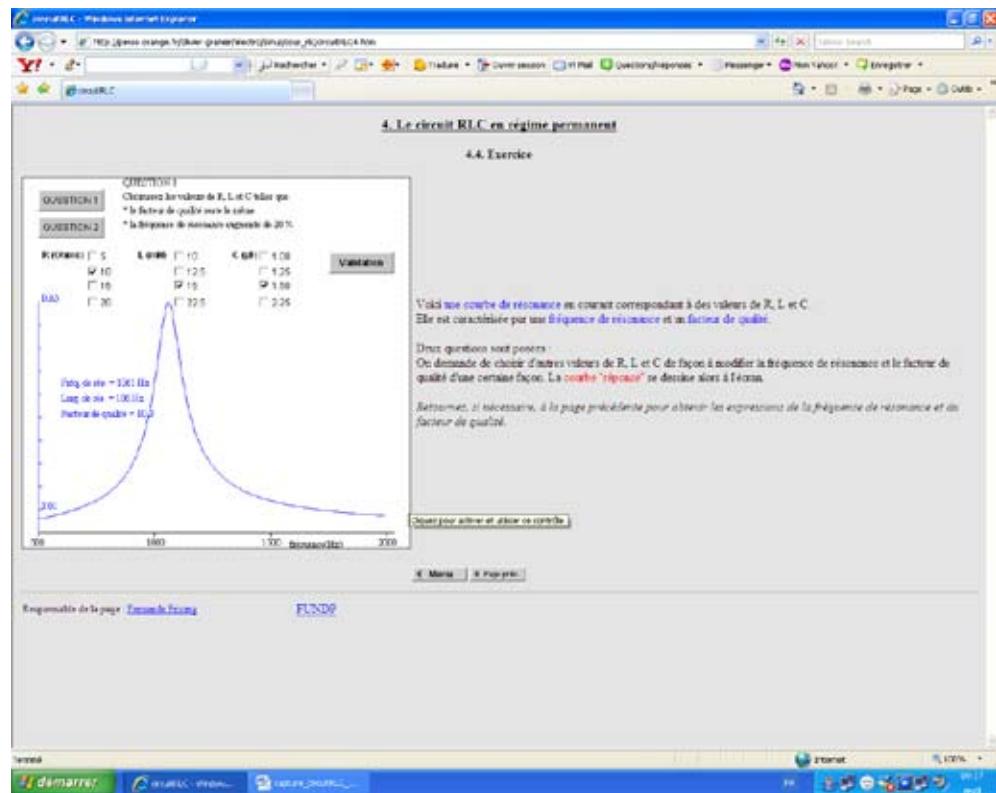
Description: These two links provide the same interactive exercise for a RLC circuit and based on the resonant frequency and quality factor with a graphic illustration.

Rationale: Reading of the link is suggested but not mandatory. It can help the learner understand learning activity 1-2 of Unit 1 of the module.

Useful link # 10

Title : http://labo.ntic.org/RLC_serie/RLC.html

http://hebergement.ac-poitiers.fr/l-cc-angouleme/coulomb-exos- phy/applets/RLC_serie/RLC.htm

URL : http:www.

Screen capture:
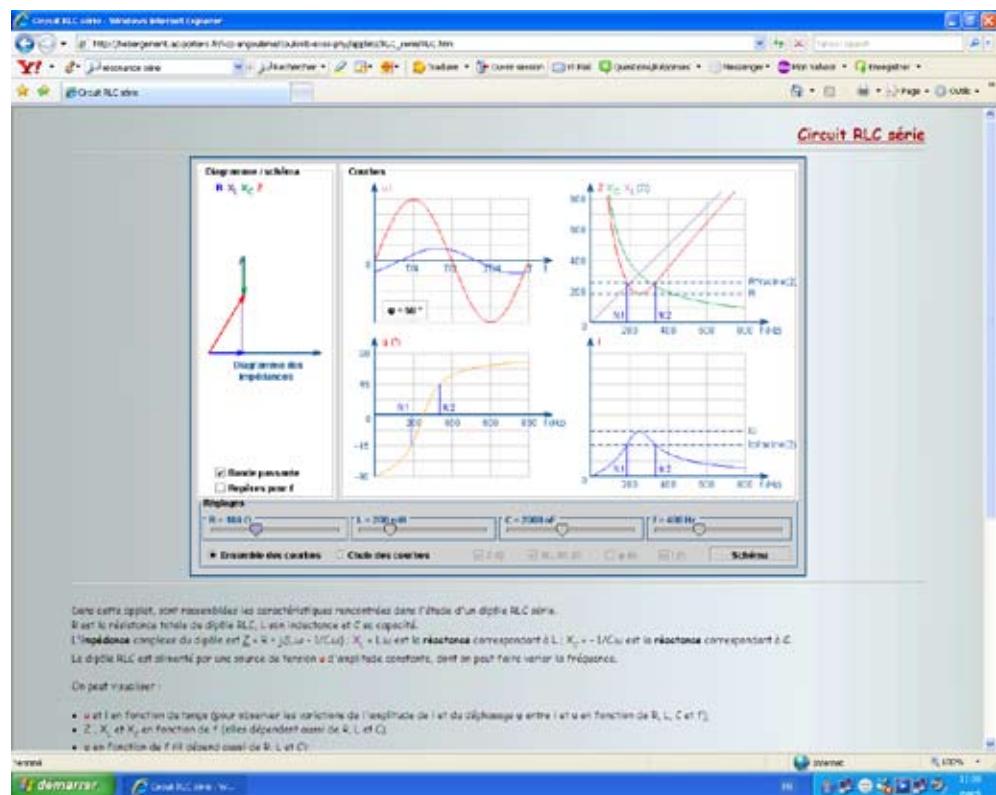
Description: Both applets offer a study of the characteristics of series RLC circuit (Z, XL, XC, I, oscillations of voltage and current, phase shift between voltage and current, bandwidth) depending on the source frequency.

Rationale: Reading of the link is suggested but not mandatory. It can help the learner understand learning activity 1-2 of Unit 1 of the module.

Useful link # 11

Title : **http://www.physique-eea.unicaen.fr/enseignement/deug-st/sm/dsm153/poly/dsm153-e.pdf**

URL : http:www.

Screen capture:

Description: This site offers protocols for handling the study of a DC coil and AC. It studies the phenomenon of resonance, bandwidth and quality factor.

Rationale: A visit to this site introduces the learner to experimental approaches and can help to strengthen understanding on the issues in learning activity 1-2 of Unit 1 of this module.

Useful link # 12

Title : http://www.unilim.fr/pages_perso/frederic.louradour/Oscillo_2.PDF

URL : http:www.

Screen capture:
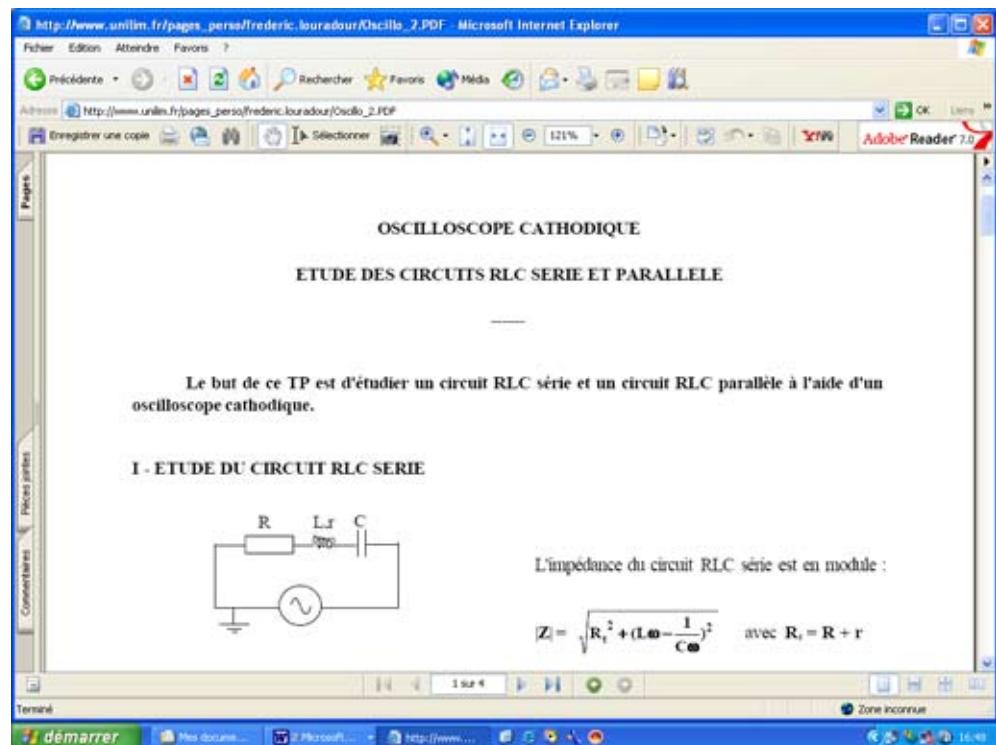
Description: This site gives TP protocols for the study of RLC circuits in series and parallel.

Rationale: A visit to this site introduces the learner to experimental approaches and can help to strengthen understanding on the issues in learning activity 1-2 of Unit 1 of this module.

Useful link # 13

Title : http://www.thyssenfrance.com/documents/mut%20devt%20text%20doc%20tech%20aim_MAGNETISME%20-%20QUELQUES%20RAPPELS.pdf

URL : http:www.

Screen capture:

Description: This site develops reminders for the learning of magnetism.

Rationale: This reading is suggested for Unit 3 of this module.

Useful link # 14

Title : http://perso.univ-lr.fr/vmathe/16_annexeA.pdf

URL : http:www.

Screen capture:
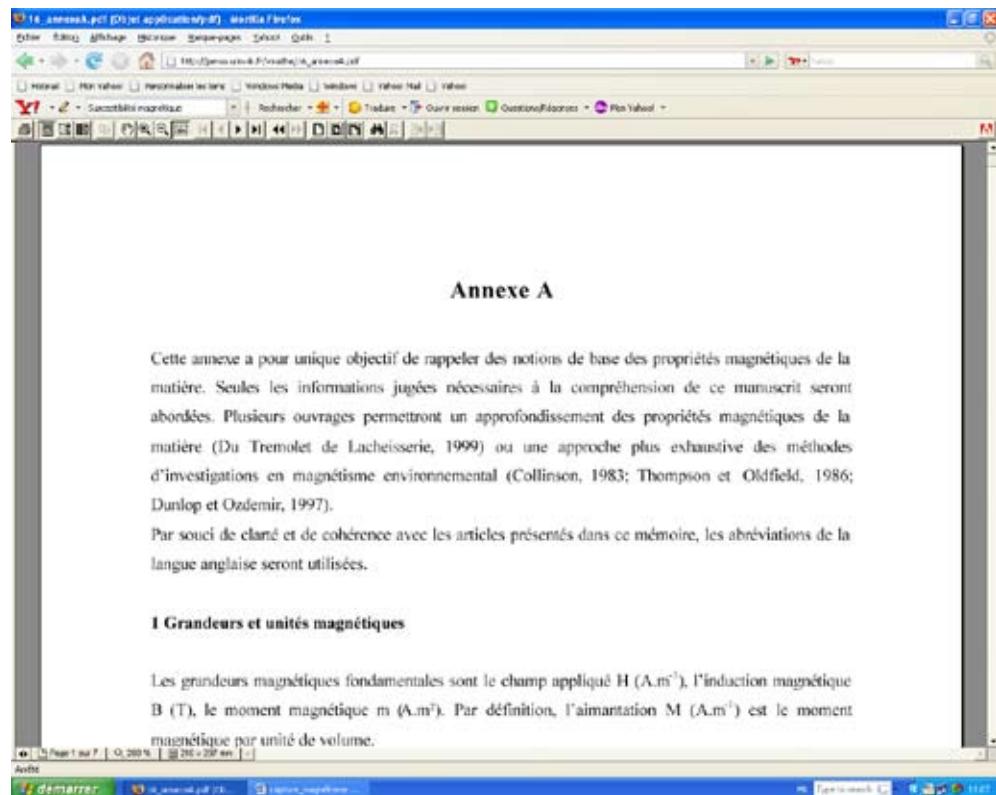
Description: This link provides some reminders about the magnetic properties of matter, especially on the magnetic core B, H and M, on paramagnetism, diamagnetism and ferromagnetism, and the hysteresis phenomenon.

Rationale: A visit to this site introduces the learner to experimental approaches and can help to strengthen understanding on the issues in learning activity 3 of Unit 3 of this module.

Useful link # 15

Title : http://semainescience.u-strasbg.fr/magnetisme/fondement.html

URL : http:www.

Screen capture:

Description: This site discusses the theoretical foundations of magnetism. It focuses on the microscopic level interpretations of Haveu magnetic properties of matter.

Rationale: A visit to this site introduces the learner to experimental approaches and can help to strengthen understanding on the issues in learning activity 3 of Unit 3 of this module.

Useful link # 16

Title http://www.sciences.univnantes.fr/physique/perso/cortial/bibliohtml/
weisli_j.html

URL : http:www.

Screen capture:
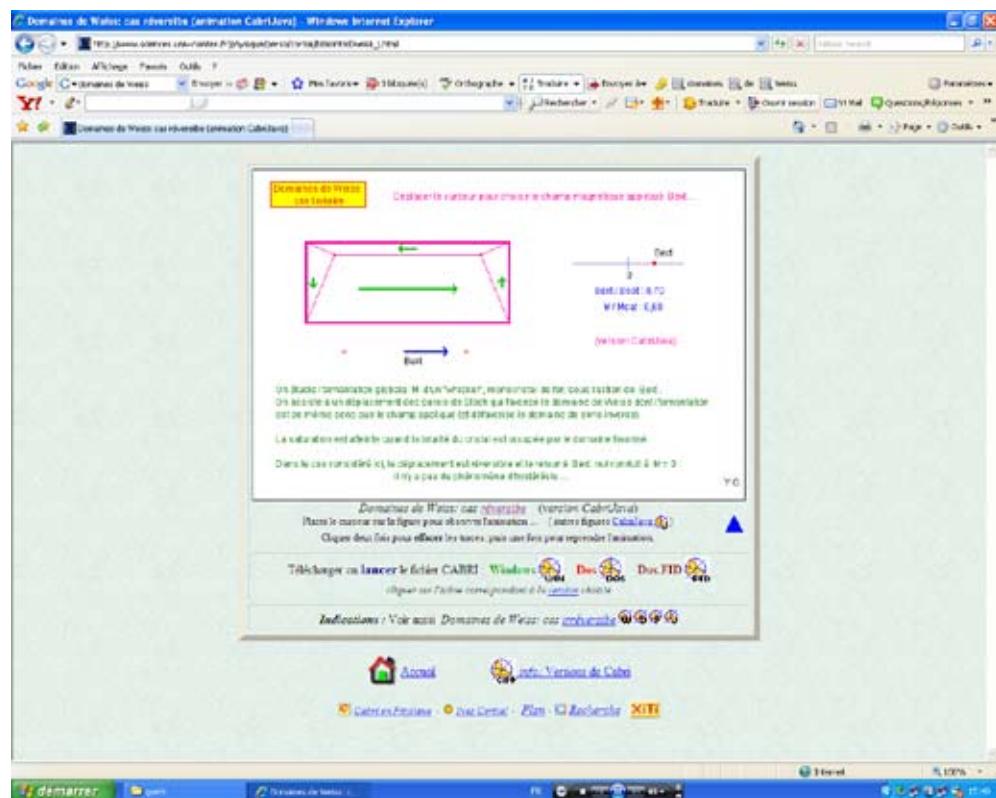
Description: This site features an animation of Weiss domains. One can visualize the
movement of the walls of these areas. Cases of reversible and irreversible displace-
ment are considered.

Rationale: This link demonstrates through animation the points made in «ferromag-
netism» of Unit 3 of this module.

Useful link # 17

Title :

http://www.sciences.univ-nantes.fr/physique/perso/cortial/bibliohtml/praimh.
html

URL : http:www.

Screen capture:
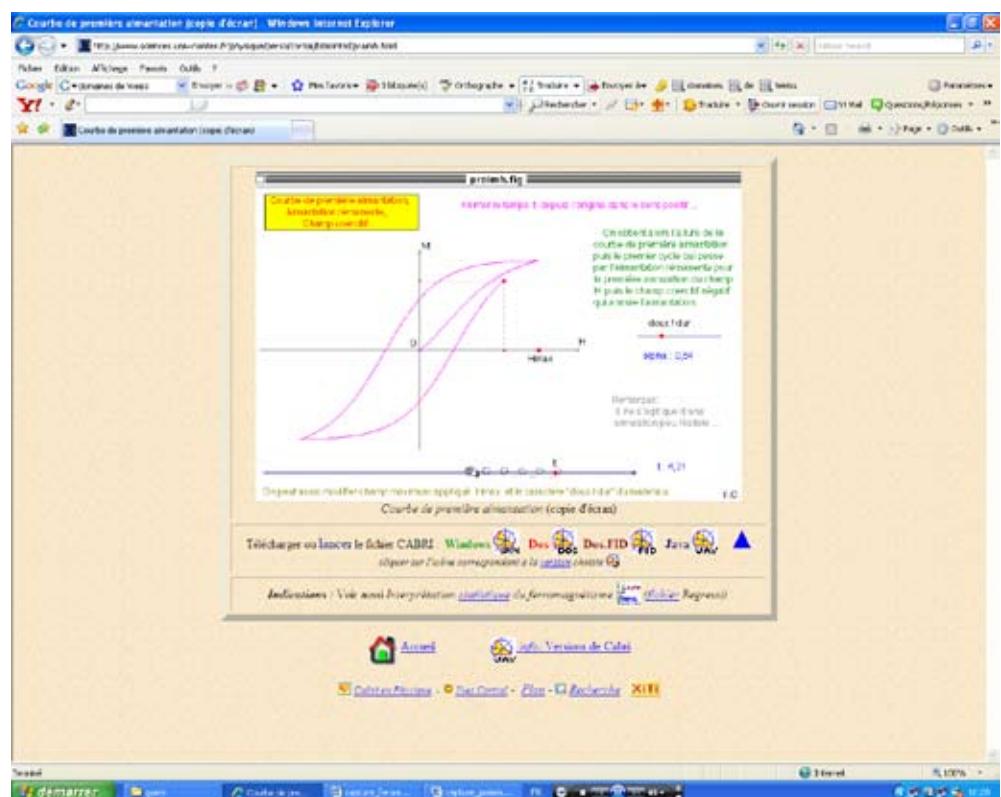
Description: The initial magnetization curve, the remanent magnetization and co-ercivity are discussed here.

Rationale: A visit to this site is suggested. The learner may create a link that displays the points made in the sections' initial magnetization curve and» hysteresis « of Unit 3 of this module.

Useful link # 18

Title : http://www.grasp.ulg.ac.be/cours/2cm/elec5.pdf

URL : http:www.

Screen capture:
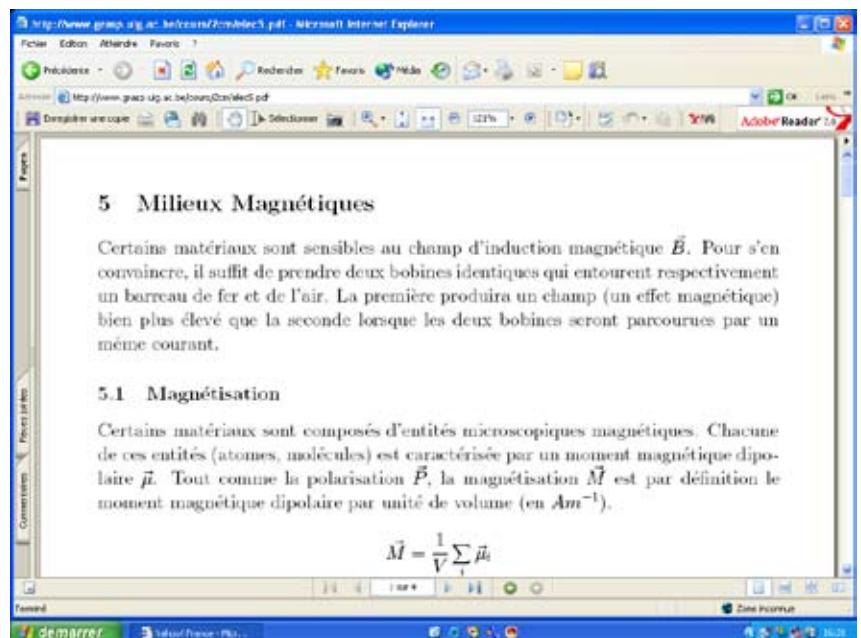
Description: It is a paper on magnetic media. It describes the magnetization, magnetic susceptibility and the relationship that binds the vectors B, H and M.

Rationale: A visit to this site introduces the learner to experimental approaches and can help to strengthen understanding on the issues in learning activity 3 of Unit 3 of this module.

Useful link # 19

Title : http://www.univ-lemans.fr/enseignements/physique/02/electri/oem1.html
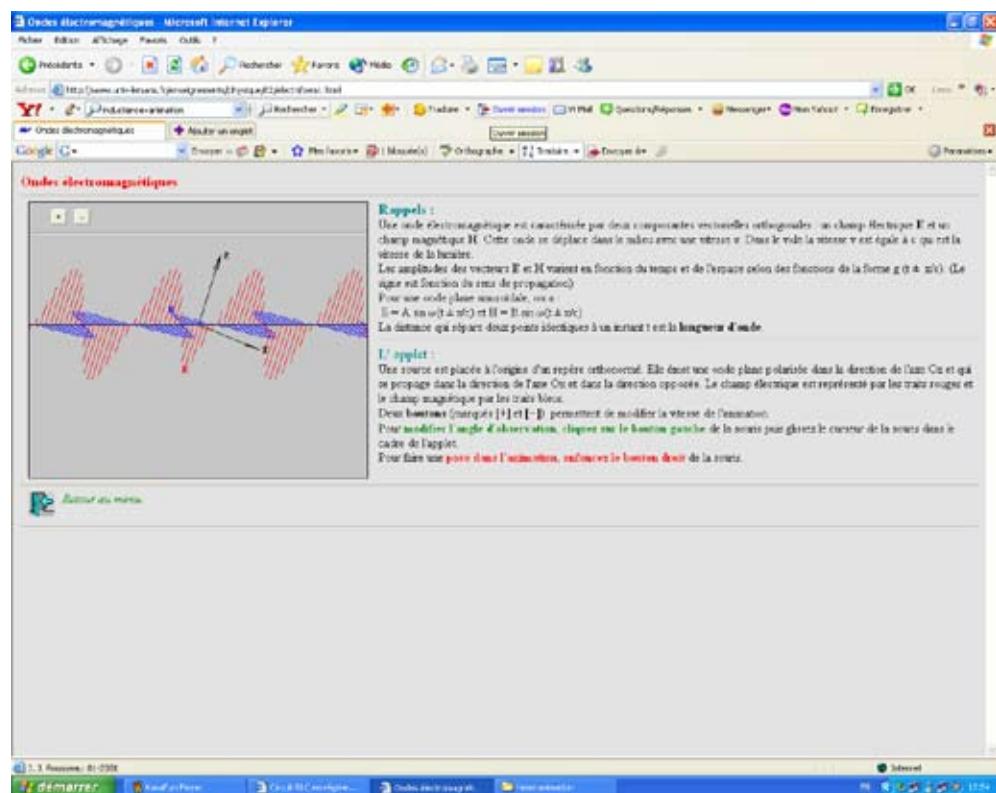
URL : http:www.

Screen capture:

Description: This site offers an animation of the propagation of an electromagnetic plane wave.

Rationale: The learner can see here that the fields that constitute the wave vibrate simultaneously and such that at any moment they are perpendicular to each other and perpendicular to the direction of propagation.

Useful link # 20

Title : http://www.walter-fendt.de/ph11f/emwave_f.htm

URL : http:www.

Screen capture:
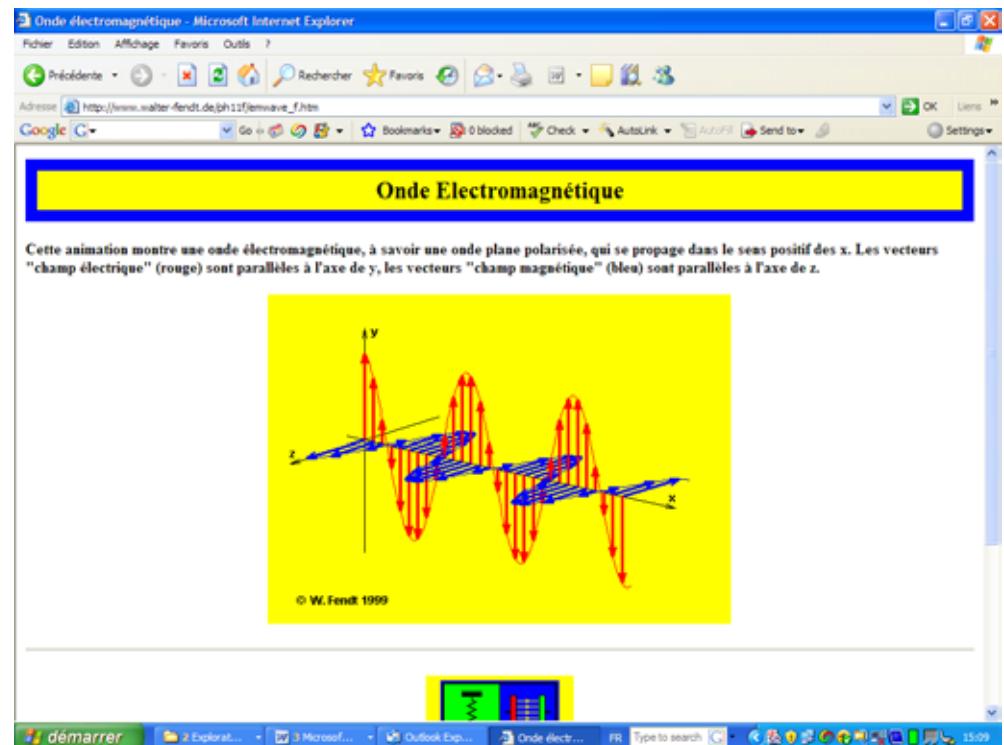
Description: This site offers an animation of the propagation of an electromagnetic plane wave.

Rationale: The learner can see here that the fields that constitute the wave vibrate simultaneously and that at any moment they are perpendicular to each other and perpendicular to the direction of propagation.

Useful link # 21

Title :

http://www.sciences.univ-nantes.fr/physique/perso/cortial/bibliohtml/2pcirc_
j.html

URL : http:www.

Screen capture:
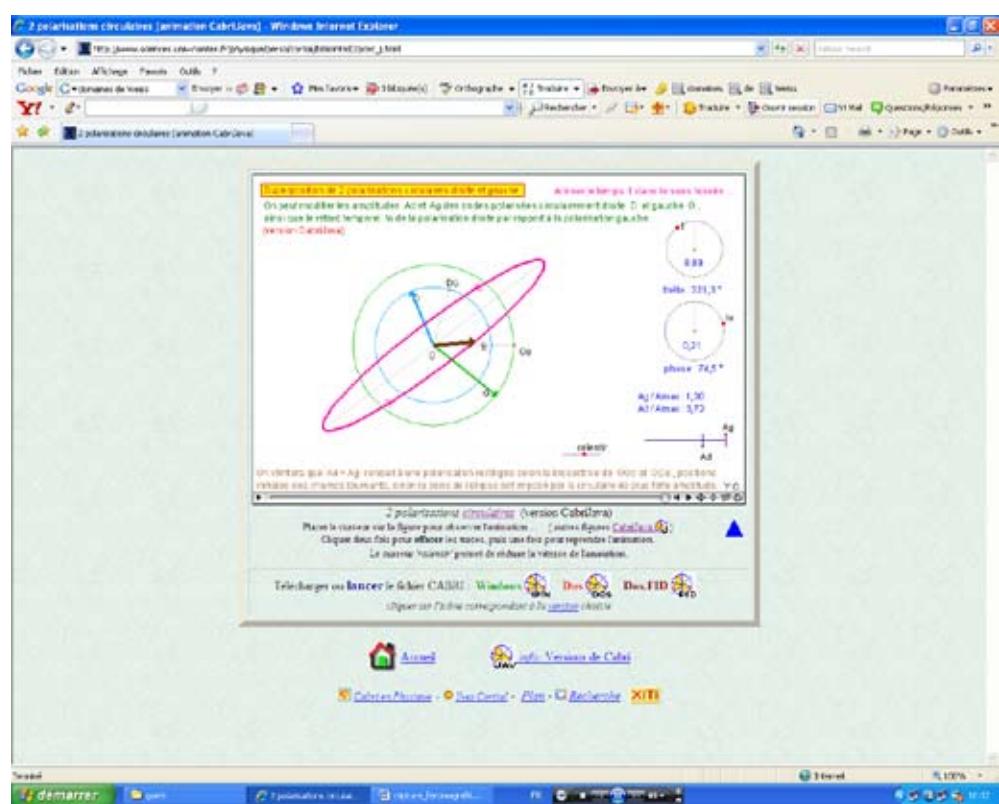
Description: This is an animation of the superposition of two circularly polarized waves.

Rationale: Visiting this site can enhance the learner's understanding on the polarization of electromagnetic waves.

Useful link # 22

Title :

http://www.sciences.univ-nantes.fr/physique/perso/cortial/bibliohtml/polond_
j.html

URL : http:www.

Screen capture:
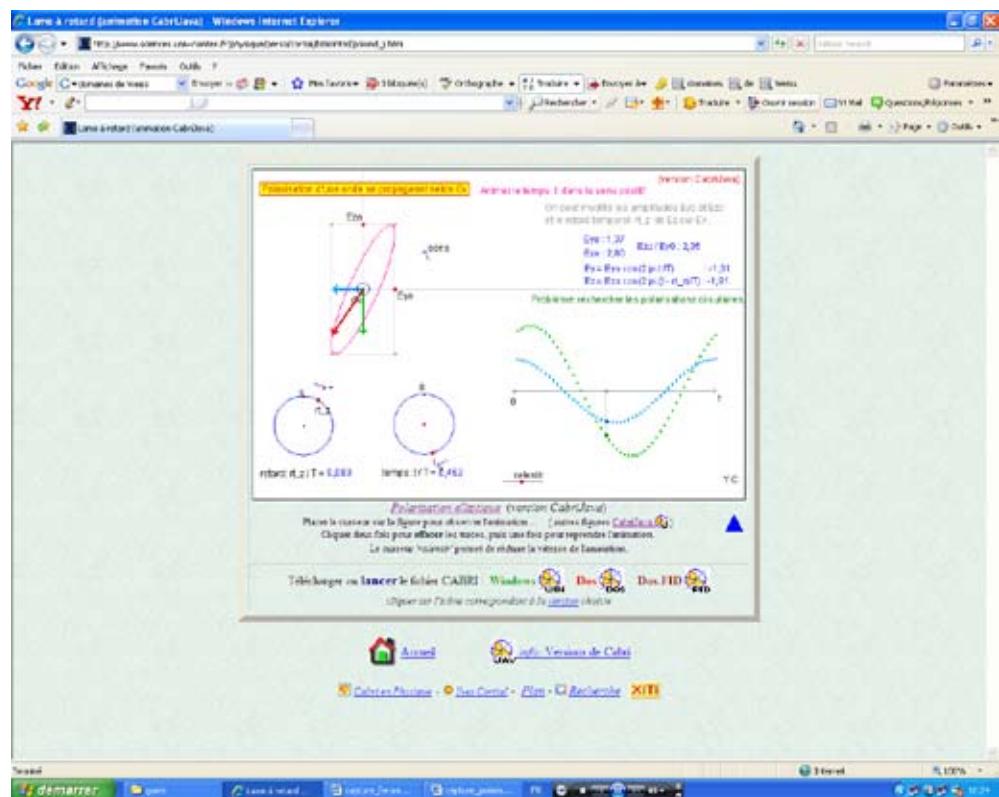
Description: This link shows an animation of an elliptically polarized wave.

Rationale: Visiting this site can enhance the learner's understanding on the polarization of electromagnetic waves.

XIV. Synthesis of the module

The module was divided into four parts.

1 - In the first part, the learner deals with the study of ac circuits and containing various combinations of resistance, inductance and capacity. Lessons include:

  - Analyze the fluctuations of voltage and current and calculate the phase shift between them;
  - Determine the impedance of a circuit
  - Describe, interpret and identify the characteristics of the condition of resonance of a series RLC circuit
  - Determined by calculation and graph the bandwidth of an RLC series circuit.

2 - In part two, the learner focuses on the movement of charged particles in an electric field and magnetic field and studies the experience of Thomson, the Hall effect, and the principle of operation of particle accelerators . It then examines the phenomenon of electromagnetic induction by the study of Lenz's law and phenomena of self induction and mutual induction. The learning will learn to:

  - detemine the ratio $\dfrac{e}{m}$
  - Describe and interpret the Hall effect
  - Calculate the velocity of a charge carrier to the output of a cyclotron
  - Apply the law of Lenz
  - Do calculations of inductance and mutual inductance.

3- In the third part, the learner is dedicated to the study of magnetic materials: the paramagnetic, the diamagnetic and ferromagnetic. They learn:

  - the origin of magnetization
  - The phenomenon of hysteresis.
  - To calculate the magnetic moment of a current loop, the magnetization of a substance.
  - To determine the coercive field.

4 - Maxwell's equations and electromagnetic waves are the subject of the last part. The learner studies the polarization of a plane electromagnetic wave, the energy it carries. They learn, among other things:
    - Describe, interpret and apply the Maxwell equations
    - Determine the polarization of a wave, the Poynting vector, its intensity and radiation pressure

XV. Summative Evaluation

Exercise 1

A circuit with a pure resistance R = 40 Ω is connected across a source of sinusoidal alternating voltage $U = U_m \sin\omega t$, frequency variable. It maintains the amplitude of voltage constant and equal to 21.21 V.

1 - Determine the intensity of effective current through the circuit when the source frequency is 100Hz and when it is 100kHz.

2 - What is this intensity if we replace the resistance by
   - A capacitor of C = 0.3 uF
   - Coil inductance L = 2mH

Exercise 2

A coil inductance L = 0.1 H and R = 10Ω resistor is connected across a source of sinusoidal alternating voltage $U = 10 \cos 100\pi t$.

1 - Determine the phase of current that traverses the circuit from the applied voltage.

2 - Give the expression of the instantaneous current

Exercise 3

A coil of resistance equal to 5Ω is supplied with sinusoidal current of frequency 50Hz. The delay Φ current on the applie voltage is such that pf = 0.8.
Calculate the inductance of the coil.

Exercise 4

A coil driven by a DC voltage of 120V, is traversed by a current of 2A intensity, fueled by a sinusoidal voltage of frequency 50Hz, an effective value 100V, it is traversed by a current of 0.5A RMS current.

   1 - Calculate the inductance of the coil
   2 - Give the value in degrees of phase shift of the voltage on the power for such a sinusoidal alternating voltage applied across the coil. Give expressions of instantaneous values of current through the coil and voltage.

Exercise 5

Resistance R = 20 W and a coil inductance L = 40 mH is connected in series across a voltage 150V sinusoidal amplitude and frequency f = 60Hz.

1 - At time t =T/6, calculating the instantaneous values:

   - The current I in the circuit
   - The source of sinusoidal voltage U
   - Voltage VR at the terminals of the resistance
   - The voltage VL at the terminals of the coil

2 - Compare and U (VR + VL) at time t =T/6 and conclude.

3 - Express the magnitude of U and amplitude voltage VRM and VLM at the terminals of the resistance and the coil.

Exercise 6

Consider the following circuit :
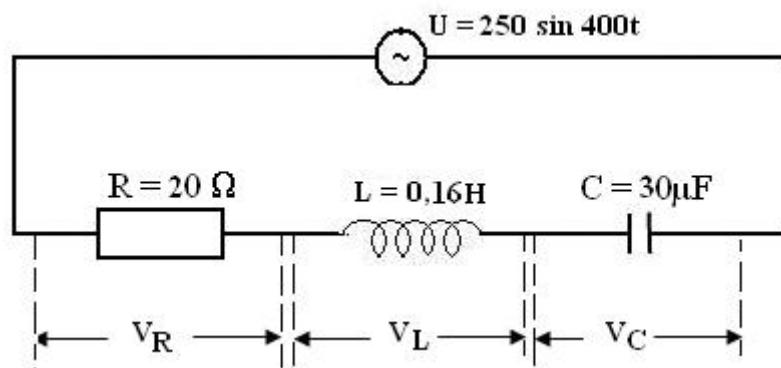
1-
Give the expression of the instantaneous current I in the circuit and the terms of instantaneous voltages $V_R,$ V_L and V_C.

2-
Calculate $V_R + V_L + V_C$ and compare with U.

We are given the trigonometric relation :

$A \sin a + B \cos a = (A^2 + B^2)^{1/2} \sin (\alpha + \beta)$ where $tg\ \beta = \dfrac{B}{A}$.

Exercise 7

Consider the following circuit:
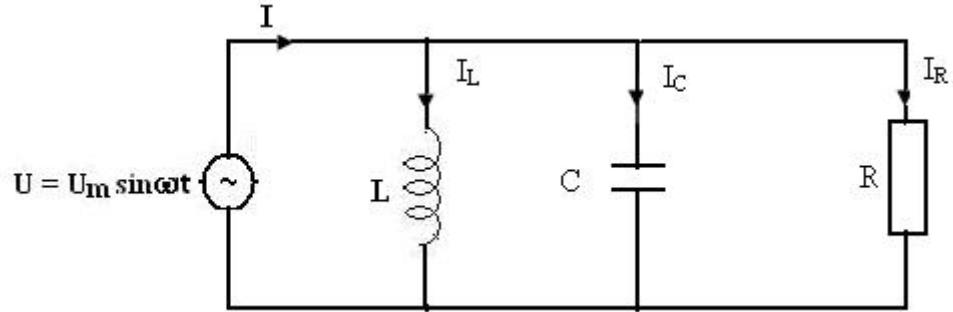
1- Find the instantaneous currents I_L , I_C , I_R and I

2- What is the impedance of the circuit ?

3- Find the phase angle between the applied voltage and the principal current I.

We are given the trigonometric relation :

$$A \sin a + B \cos a = (A^2 + B^2)^{1/2} \sin (\alpha + \beta) \text{ with } tg\ \beta = \frac{B}{A}$$

Exercise 8

Consider the following diagram
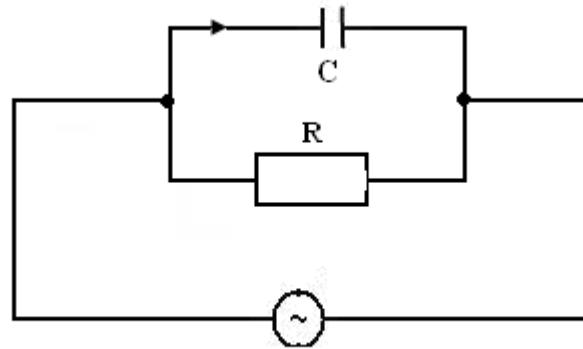
1 - Find the complex impedance of this circuit and derive its impedance.
2 – Is the applied voltage ahead or behind the current?

Exercise 9

Resistance R = 10Ω, a coil inductance L = 0.4 H, a capacitor of C = 0.4 mF and a lamp are connected in series across a source of applied voltage of U_{eff} = 10^{-2} V. Neglecting the resistance of the lamp. It increases the frequency f of the source while maintaining constant amplitude.

1 - Describe and explain how to vary the brightness of the lamp.

2 - Compute:
- The resonant frequency
- The maximum current
- The voltage across the capacitor at resonance

3 – We replace the resistor R by a resistor R1 = R / 2 then R2 = R / 4. Recalculate the maximum intensity.

Exercise 10

A coil of resistance R and inductance L and a capacitor of capacitance C are connected in series with an alternative sinusoidal source $U = U_m \sin (1000\pi t)$. The resonant frequency of this circuit is f_0 = 718Hz. Is the current ahead or behind the voltage?

Exercise 11

A capacitor of capacitance C and a coil resistance R, inductance L are connected in series across a source of sinusoidal voltage 10V and pulsation ω. It gives R = 10Ω, C = 5μF and L = 1H. Calculate:

- The resonant frequency of this circuit
- The effective intensity at resonance
- The quality factor and bandwidth of this circuit
- The effective voltage across the capacitor at resonance

Exercise 12

Consider a circuit (RLC) series powered by a sinusoidal voltage source of variable frequency and contant U_{eff} = 24V. The effective intensity is maximum and equal to 150mA for a frequency of 800Hz. The bandwidth of this circuit is 167.5 rad.s-1. Calculate the resistance R, inductance L and capacitance C.

Exercise 13

A coil inductance L = 4mH, a capacitor of capacitance C = 16μF and a resistance R = 20 Ω are connected in series with a sinusoidal voltage source whose frequency is equal to 400Hz.

a – Give a Fresnel representation, which corresponds to this circuit.

b - Find the phase angle between voltage and current.

c - Without explHeretly calculate the resonant frequency of the circuit is it higher or lower than 400Hz?

d - Calculate the resonance frequency.

Exercise 14

Consider the following circuit :
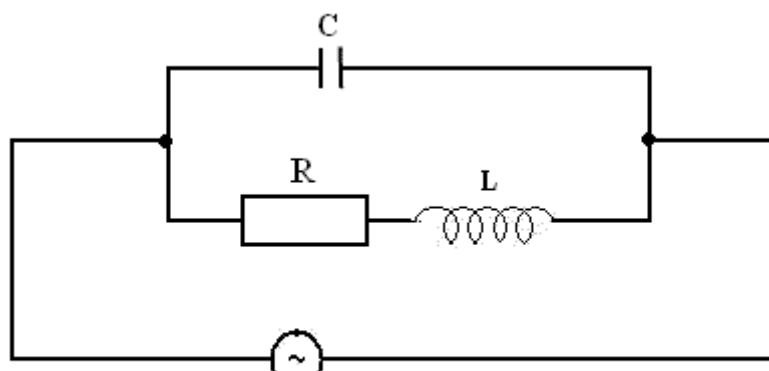
$$U = U_m \sin\omega t$$

1 - Find the complex impedance of the circuit.

2 - A parallel resonance is obtained when the imaginary part of complex impedance vanishes. Find the impedance of this circuit to parallel resonance in the case where $L^2 w^2 \gg R^2$.

Exercise 15

An electron is launched in a region where acting simultaneously are an electric field and magnetic field uniform and mutually perpendicular. These two fields are perpendicular to its initial velocity. It finds that:

- Under the simultaneous action of two fields, the electron is not deviated from its original path

- When it stops the action of the electric field while keeping the magnetic field, it describes a circle of radius R = 1.14 cm.

Calculate the ratio $\dfrac{e}{m}$ knowing that B = 2 x 10^{-3} T and E = 8kV/m

Exercise 16

Consider the device below. S is a source of charged particles. Between the two

plates P1 and P2 there is a uniform electric field $\vec{E}$ and a uniform magnetic field

$\vec{B}$. The electric field is parallel to the face of the figure and headed from P1 to P2.

$\vec{B}$ is normal to the plane of the face and headed backwards. The particles emerging

from O enter a region where there is a uniform magnetic field $\vec{B'}$ normal to the
plane of the face and headed backwards .
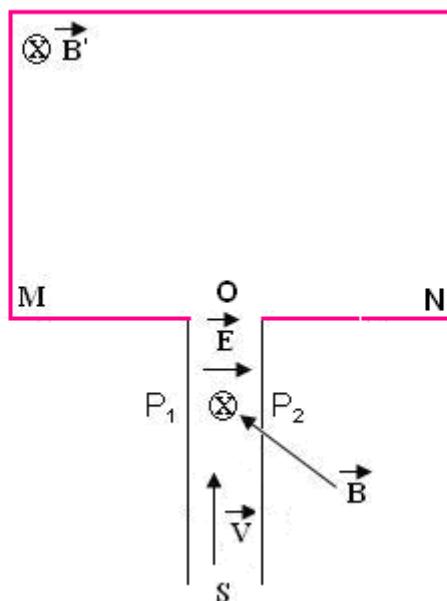
S sends a beam of positive ions of charge q, mass m and velocity $\vec{v}$ as shown in
the figure below.

1 - What should be the velocity of these ions so that they traverse the space
between S and O without being deflected?

2 - What part of the photographic plate will overlap when they enter the field

$\vec{B'}$? Calculate the distance d at which they intersect the photographic plate.

3 - Suppose the beam sent by S consists of isotopes of the element tin (Sn) whose isotope mass 116 u and 120 u. Determine the difference of distances corresponding to these two isotopes.

A.N. :

E = 15 kV/m, B = B' = 0,15 T

1 u = a unit of atomic mass = 1,66 x 10^{-27} kg

q = e

Exercise 17

In a cyclotron we accelerate a proton charge q = 1.6 x 10-19 C and mass m = 1,6610-27 kg. The dice of the cyclotron are placed in a uniform magnetic field normal to their plane and of magnitude B = 1.5 T

1. What should be the frequency of the oscillating electric field that exists in the space between the dice, which accelerates the protons?
2. Calculate the kinetic energy of the protons at the exit of the cyclotron knowing that just before their exit, the trajectory has a radius R = 50cm.
3. What is the number of accelerations effected by a proton if the potential difference between the dice is 40 kV?

Exercise 18

The figure below shows a metal strip of rectangular section with sides a and b. It has n conduction electrons per unit volume and has a current intensity whose density $\vec{j}$ is uniform and directed as shown in the figure below.
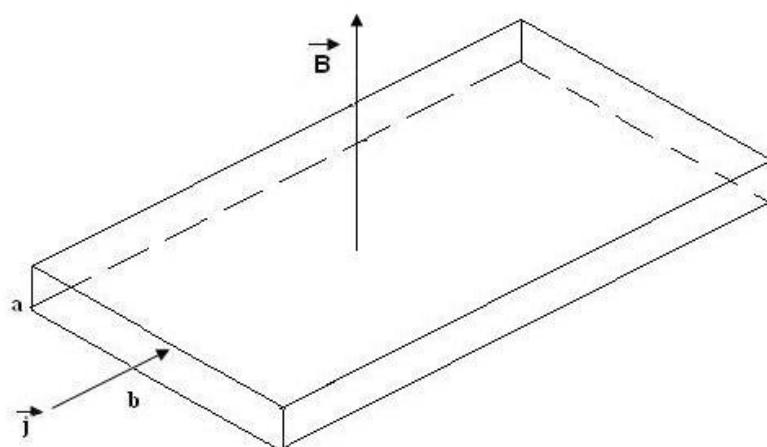
1 - Provide a sense of drift velocity of conduction electrons. Determine this speed.

2 - When the ribbon is placed in a magnetic field $\vec{B}$ perpendicular to the current density $\vec{j}$, electrons accumulate on one side. Explain why and indicate this on the front face.

3 – A steady state is quickly established where the electrons do not drift laterally. Explain why.

4 - What is the relationship that binds and when is the steady state reached.

5 - Calculate the potential difference between the front and rear edges of the ribbon

We are given :

$B = 2\ T\ ;\ I = 2A\ ;\ a = 2\ 10^{-3}\ m$

$ne = 2.10^{10}\ C\ m^{-3}$

Exercise 19
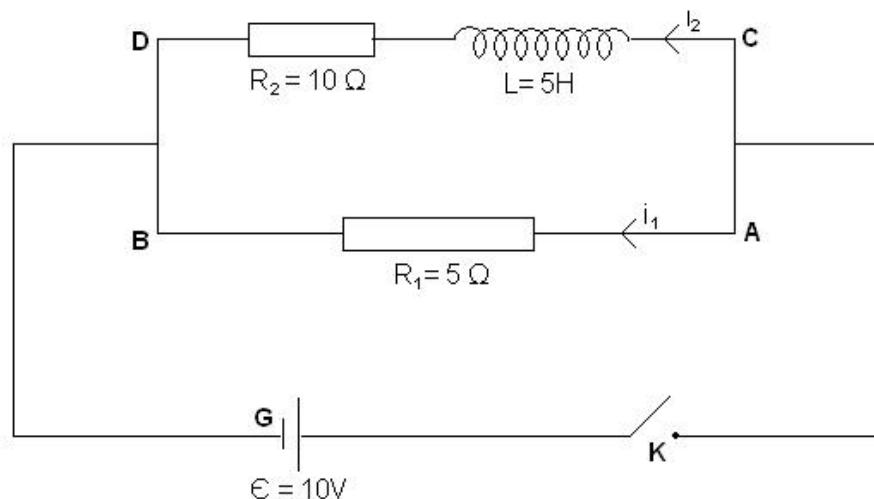
1 - Just after closing the switch K, calculate:

   - The intensities of the currents i1 and i2
   - The intensity of the current i passing through the switch K
   - The potential difference across the terminals of R2
   - The potential difference across the coil
   - The growth rate of the current i2

2 - Calculate again these quantities permanent regime of the system is reached.

Exercise 20

A coil inductance L = 0.1 mH and resistance r is connected in series with a resistor R1 = 0.5 W, a generator emf e = 20 V and a switch K.

1) Write the differential equation giving the evolution of the intensity of current i versus time after the closure of K.
2) Verify that the solution of this differential equation is:

$$i = \frac{\varepsilon}{R}(1 - e^{-\frac{R}{L}t}) \text{ with } R = R_1 + r$$

3) Calculate the resistance of the coil r given that the current intensity reaches $\frac{999}{1000}$ of its final value after $6{,}9 \ 10^{-4}$ s.

Exercise 21

A solenoid of infinite length and radius R has n turns per unit length. The coils are traversed by a current of intensity I.

1 - Provide the magnetic induction inside and outside the solenoid
2 - Find the coeffHereent of self induction of the solenoid
3 - It is now up around the solenoid coil plate consisting of N turns with radius greater than R. This coil has for an axis the axis D of the solenoid.
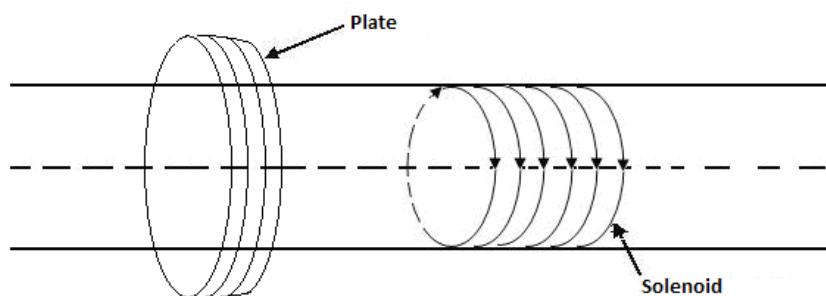
b) Find the coeffHereent of mutual induction of the two circuits
c) The current is now time-varying. Find e, the emf induced in the coil using Faraday's law.
d) In a very short time, the current in the solenoid is reduced to zero. Find the load through the flat coil of resistance r. Give the direction of the induced current.

Exercise 22

A toroidal coil of average circumference l = 40cm, with N = 400 turns is traversed by a current of intensity I = 2A. The extent of the field that exists inside of the coil gives 1T. Determine:

- Magnetic intensity

- Magnetization

- Magnetic susceptibility

- The equivalent surface current

- Relative permeability.

Exercise 23

Consider a toroid of mean radius R = 20cm, with 2000 turns and a current I = 15A. It contains liquid oxygen whose magnetic susceptibility is 4×10^{-3}.
Determine the magnetization of the substance and the magnetic field.

Exercise 24

A current I flows straight along the axis of a cylinder of radius a. Outside the cylinder, we have a vacuum. Determine:

1 - Intensity of magnetic excitation in the space

2 - The field in all of the space

3 - The magnetization vector in the space

4 - The Amperian current Ia whose density is defined by $\vec{j}_a = \vec{\nabla} \wedge \vec{M}$

Exercise 25

1.

Show that the Curie law $M = \dfrac{1}{3} m \dfrac{B_{app}}{kT} M_s$ implies that the susceptibility of a

paramagnetic substance can be written: $X_m = m \dfrac{\mu_0 M_s}{3kT}$

m is the magnetic moment of an atom of the paramagnetic substance, B_{app} the applied field, μ0 permeability of vacuum, Ms the maximum value of magnetization.

2-

Suppose that the magnetic material is aluminum here and admit that the magnetic moment of an atom of aluminum is one Bohr magneton: $\mu_B = 9.27 \times 10^{-24}\,Am^2$. Determine susceptibility Xm to the absolute temperature T = 300K

We are given : density of aluminum :
$\rho = 2,7g/cm^3$

Its molar mass :

M = 27g/mol

Boltzmann constant :   k = 1,38 10^{-23} J/K

Exercise 26

1 - Define the coercive field.

2 - We place a bar magnet inside a coil of 15cm in length and containing 600 turns. The current that must travel to demagnetize the solenoid rod is 11A. Calculate the corresponding coercive field.

Exercise 27

We put some magnetic material inside a long solenoid. The solenoid has 50 turns per cm and is traversed by a current of intensity I = 0.2 A. The magnetic field that exists in this system is 1.58 T. Calculate:

1 - the applied magnetic field
2 - magnetization M
3 - relative permeability

Exercise 28

The table below shows values of H and B for a given ferromagnetic substance.
Plot as a function of H the field B, the permeability μ, $\mu_0 H$, $\mu_0 M$ and the relative permeability Km

H (Am^{-1})	B (T)
0	0
10	0,050
20	0,15
40	0,43
50	0,54
60	0,62
80	0,74
100	0,83
150	0,98
200	1,07
500	1,27
1000	1,34
10 000	1,65
100 000	2,02
800 000	2,92

Exercise 29

Consider an electromagnetic wave whose electric field is described by:

$$\vec{E}(x,t) = E_0 \sin(kx-\omega t)\, \vec{e}_y + E_0 \cos(kx-\omega t)\, \vec{e}_z$$

$\vec{e}_y$ and $\vec{e}_z$ being the unit vectors following the x and y axes.

1- Determine the field $\vec{B}$ associated with this wave.

2- Study the evolutions of $\vec{E}(0,t)$ and $\vec{B}(0,t)$ over time and give the state of polarization of the wave.

3- Determine the Poynting vector $\vec{S}$ associated with this wave.

We are given $\vec{a} \wedge (\vec{b} \wedge \vec{c}) = (\vec{a} \cdot \vec{c})\,\vec{b} - (\vec{a} \cdot \vec{b})\,\vec{c}$

Exercise 30

An electromagnetic wave through empty space is described by:

$$\vec{E} = \vec{E}_0 \cos(\vec{k} \cdot \vec{r} - \omega t)$$

$$\vec{B} = \vec{B}_0 \cos(\vec{k} \cdot \vec{r} - \omega t)$$

FInd the intensity of the wave.

Exercise 31

A source of monochromatic light, with 30W of power, radiates uniformly in all directions.

1 - Determine the intensity and the radiation pressure of the light wave at a point M located 2m from the source.

2 - Determine the amplitudes of the electric field and the magnetic field associated with this wave at the point M.

Correct answers

Solution of exercise 1

1) For a pure resistance, no matter what the frequency, we have :

$$U_{eff} = RI_{eff}$$
$$U_{eff} = \frac{U_m}{\sqrt{2}}$$

$$I_{eff} = \frac{U_{eff}}{R} = \frac{U_m}{R\sqrt{2}}$$

A.N. $U_m = 21,21V$; $R = 40\Omega$

$$U_{eff} = \frac{21,21}{\sqrt{2}} = 15V \Rightarrow I_{eff} = \frac{15}{40} = \underline{\mathbf{0,375A}}$$

2) We replace the pure resistance by by a capacitor of capacitance $C = 0,3\mu F$

We have :

$$U_{eff} = X_C I_{eff}, \text{ where } I_{eff} = \frac{U_{eff}}{X_C}$$

with : $X_C = \dfrac{1}{C\omega}$ et $\omega = 2\pi f$, f being the frequency of the source

a) for f = 100Hz

$$X_C = \frac{1}{(0,3.10^{-6}\,F)(200\pi\,rad.s^{-1})} = 5308\Omega$$

where $I_{eff} = \dfrac{U_{eff}}{X_C} = \dfrac{15V}{5308\Omega} = 2,83\times10^{-3}A \Rightarrow \underline{\mathbf{I_{eff} = 2,83mA}}$

b) for f = 100kHz

$$I_{eff} = \frac{U_{eff}}{X_C} = U_{eff}(C\omega) = U_{eff}(2\pi\, fC)$$

When the frequency is 1000 times larger, the effective intensity is multiplied by 1000. We have :

$$\underline{\mathbf{I_{eff} = 2,83A}}$$

We replace the pure resistance by a coil inductor of $\underline{L = 2mH}$

In this case $I_{eff} = \dfrac{U_{eff}}{X_L}$ with $X_L = L\omega$

a) for f = 100Hz

$$L\omega = (2 \times 10^{-3}H)(200\pi\ rad.s^{-1}) = 1,256\Omega$$

$$I_{eff} = \frac{15V}{1,256\Omega} = 11,9A \implies \underline{\mathbf{I_{eff} = 11,9A}}$$

b) for f = 100kHz

$$I_{eff} = \frac{U_{eff}}{L\omega} = \frac{U_{eff}}{L(2\pi\ f)}$$

When the frequency is 1000 times larger, the effective intensity is divided by 1000, where :

$$\underline{\mathbf{I_{eff} = 11,9mA}}$$
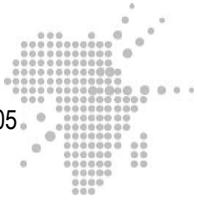
Solution of exercise 2

1- The phase angle Φ between the applied voltage and the current is such that :

$$\tan\Phi = \frac{X_L}{R}$$

$X_L = L\omega$ with $\omega = 100\pi$ rad.s^{-1} and $L = 0,1$ H $\Rightarrow L\omega = 31,4\Omega$

We obtain :

$$\tan\Phi = \frac{31,4\Omega}{10\Omega} = 3,14 \Rightarrow \Phi = 72,3° \text{ ou } \Phi = 1,26 \text{ rad}$$

The current is lagging by 72,3° with respect to the voltage.

1- Expression of the instantaneous current

$I = I_m \cos(100\pi t - \Phi)$ with Φ expressed in radians and I_m the amplitude of the current.

We have :

$$I_m = I_{eff}\sqrt{2} \quad \text{and} \quad I_{eff} = \frac{U_{eff}}{Z}$$

$$U_{eff} = \frac{U_m}{\sqrt{2}} \text{ with } U_m = 10\sqrt{2} \text{ where } U_{eff} = 10V$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + L^2\omega^2} = \sqrt{100 + (31,4)^2} = 32,95\Omega$$

The effective intensity is thus:

$$I_{eff} = \frac{U_{eff}}{Z} = \frac{10V}{32,95\Omega} = 0,303A$$

We obtain finally

$I = I_m \cos(100\pi t - \Phi) = I_{eff}\sqrt{2} \cos(100\pi t - \Phi) = 0,303\sqrt{2} \cos(100\pi t - 1,26)$

Solution of exercise 3

The impedance of the circuit is :

$$Z = \sqrt{R^2 + L^2\omega^2} \implies L^2 = \frac{Z^2 - R^2}{\omega^2} \implies L = \sqrt{\frac{Z^2 - R^2}{\omega^2}} \quad (1)$$
$$R = 5\Omega \; ; \; \omega = 2\pi f = 2\pi \times 50Hz = 100\pi \; rad.s^{-1}$$

We have:

$$\cos \Phi = \frac{R}{Z} = 0{,}8 \implies Z = \frac{R}{\cos\phi} \implies \frac{5\Omega}{0,8} = 6{,}25 \; \Omega$$

Take the values of Z , R and ω in (1). We obtain :

$$L = \sqrt{\frac{(6,25)^2 - (5)^2}{(100\pi)^2}} \implies L = \underline{\mathbf{1{,}19.10^{-2}H}}$$

Solution of exercise 4

1) Inductance of the coil

In alternating current, the impedance is :

$$Z = \sqrt{R^2 + L^2\omega^2} \implies L^2 = \frac{Z^2 - R^2}{\omega^2} \implies L = \sqrt{\frac{Z^2 - R^2}{\omega^2}}$$

Here $\omega = 2\pi f = 2\pi \times 50Hz = 100\pi \; rad.s^{-1}$

$$Z = \frac{U_{eff}}{I_{eff}} = \frac{100V}{0,5A} = 200 \; \Omega$$

Determine the resistance of the coil. When it is powered by the voltage of 120V, it is traversed by a current I = 2A. Its resistance is:

$$R = \frac{120V}{2A} = 60 \; \Omega$$

The inductance is thus :

$$L = \sqrt{\frac{(200)^2 - (60)^2}{(100\pi)^2}} \quad \Rightarrow \quad L = \underline{\mathbf{0,61H}}$$

2) Phase shift Φ of the voltage on the current

$$\tan\Phi = \frac{X_L}{R} = \frac{L\omega}{R} = \frac{(0,61)(100\pi)}{60} = 3,192 \Rightarrow \underline{\mathbf{\Phi = 72,6° = 1,26\ rad}}$$

The voltage is ahead by 72,6° on the current

Instantaneous voltage

Consider $U = U_m \sin\omega t$ the instantaneous voltage across the terminals of the coil:

$$U_m = 100\sqrt{2} \quad \text{et} \quad \omega = 2\pi f = 2\pi \times 50Hz = 100\pi\ \text{rad.s}^{-1}, \text{where:}$$

$$\mathbf{U = 100\sqrt{2}\ \sin(100\pi\ t\ )}$$

Instantaneous current

$$I = I_m \sin(100\pi\ t - \Phi) \quad \text{with } \Phi \text{ expressed in radians}$$

$$\left. \begin{array}{l} I_m = I_{eff}\sqrt{2} \\ I_{eff} = 0,5A \\ \Phi = 1,26\ \text{rad} \end{array} \right\} \Rightarrow \mathbf{I = 0,5\sqrt{2}\ \sin(100\pi\ t - 1,26)}$$

Solution of exercise 5
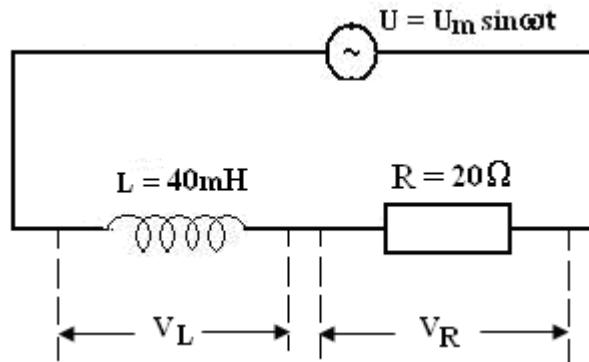
1) First find the phase F between voltage and current. For this, calculate the inductive reactance XL and trace the Fresnel representation, which corresponds to the circuit. We have:

$\omega = 2\pi f = 120\pi \text{rad/s}$

$X_L = L\omega = L \times (2\pi f) = (40 \times 10^{-3}) 2\pi \times 60 = 15,07\Omega$.

$$\tan \Phi = \frac{V_{Lm}}{V_{Rm}} = \frac{X_L}{R} = \frac{15,07}{20} = 0,754$$

$\Rightarrow \Phi = 37°$

The phase angle is positive $\Rightarrow$ the voltage source is ahead by 37° on the current.
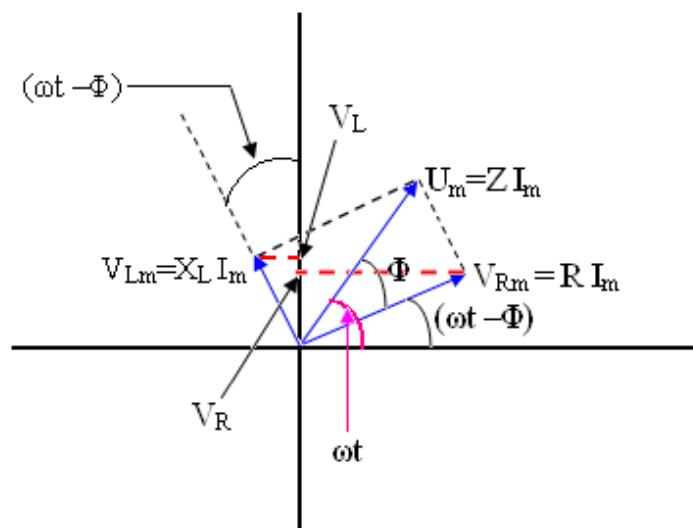
Instantaneous current à $t = \dfrac{T}{6}$

The instantaneous current is written :

$I = I_m \sin(\omega t - \Phi) = I_m \sin(2\pi ft - \Phi)$ (1)   with F in radians .

Here

$ZI_m = U_m \Rightarrow I_m = \dfrac{U_m}{Z}$

$Z = \sqrt{R^2 + X_L^2} \; \Omega$

$U_m = 150 \text{ V}$

$\Rightarrow I_m = \dfrac{150 \text{ V}}{\sqrt{(20)^2 + (15{,}07)^2}} = 6 \text{ A}$

Equation (1) becomes :

$I = 6\,(\sin 2\pi ft - \Phi)$

At the instant $t = \dfrac{T}{6}$

$I = 6 \sin \left(\dfrac{2\pi}{T} \dfrac{T}{6} - n \right)$

$I = 6 \sin \left(\dfrac{\pi}{3} - n \right) \Rightarrow 6 \sin(60° - 37°) = 6 \sin 23°$

I = 2,34 A

Value of the sinusoidal voltage source at the instant $t = \dfrac{T}{6}$.

$U = U_m \sin wt \Rightarrow U = 150 \sin \dfrac{2\pi}{T} \dfrac{T}{6} = 150 \sin \dfrac{\pi}{3} = 150 \sin 60°$

U = 130V

The instantaneous voltage across the terminals of the resistor

The projection of V_{Rm} on the vertical axis gives the instantaneous voltage V_R at the terminals of the resistor

$V_R = V_{Rm} \sin(\omega t - \Phi)$

and $V_{Rm} = RI_m = 20 \times 6 = 120$ V.

Note : We can also use the fact that V_R and I are in phase

We have $V_R = 120 \sin(2\pi ft - \Phi)$

And at the instant $t = \dfrac{T}{6} \Rightarrow V_R = 120 \sin 23° = \underline{\mathbf{46,88V}}$

Instantaneous voltage across the terminals of the coil

V_L is the projection of V_{Lm} on the vertical axis

$V_L = V_{Lm} \cos(wt - F)$

Note : We can also directly use the fact that V_L is ahead by 90° on the current.

$\Rightarrow V_L = X_L I_m \cos(2\pi ft - \Phi)$

à $t = \dfrac{T}{6}$ we have :

$V_L = 15,07 \times 6 \cos 23° = (15,07)(6)(0,920)$

$\underline{\mathbf{V_L = 83,23\ V}}$

2) Compare $V_R + V_L$ and U at the instant $= \dfrac{T}{6}$

$V_R + V_L = 46,88$ V $+ 83,23$ V $= 130, 11$ V
U = 130 V $\Biggr\}\Rightarrow \underline{\mathbf{U = V_R + V_L}}$

The instantaneous voltages add algebraically.

3) Calculate the amplitude of U

The Fresnel diagram shows that by applying Pythagoras theorem, we have :

$$U_m^2 = V_{Rm}^2 + V_{Lm}^2 \Rightarrow U_m = \sqrt{(RI_m)^2 + (X_L I_m)^2}$$

$$U_m = I_m Z = 6\sqrt{(20)^2 + (15,07)^2} \quad \underline{\textbf{150 V}}$$

Solution of exercise 6

1)
First we determine the phase shift between the applied voltage and current in the circuit. We have :

$$U = 250 \sin 400t$$

$$R = 20 \ \Omega \qquad L = 0,16H \qquad C = 30\mu F$$

$$v_R \qquad v_L \qquad v_C$$

$$\tan\Phi = \frac{X_L - X_C}{R}$$

$$X_L = L n$$

$$X_C = \frac{1}{C\omega}$$

$$\omega = 400 \text{rads}^{-1}$$

$\Rightarrow$
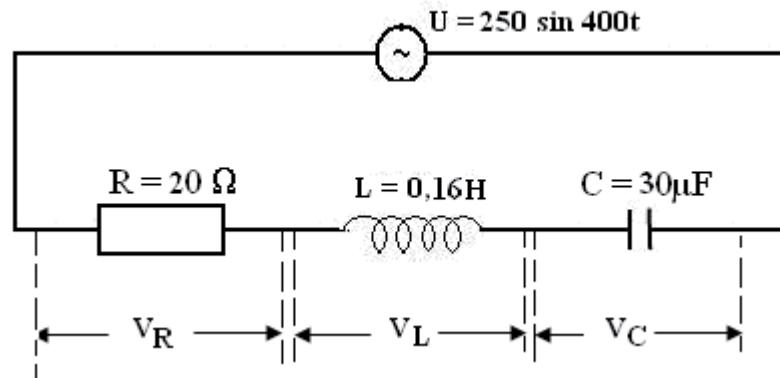
$$X_L = 0,16 \times 400 = 64 \ \Omega \ ;$$

$$X_C = \frac{1}{(30 \times 10^{-6}) \times 400} = 83,33\Omega \ ;$$

$$\tan\Phi = \frac{64 - 83,333}{20} = -0,96665$$

The pahse angle is negative, and thus the voltage lags by F behind the current.

Instantaneous current : The current is thus written :

$I = I_m \sin (400t + \theta)$ with $\theta = - \Phi$. θ est Expressed in radians :

$\tan \Phi = - 0,96665 \Rightarrow \Phi. = - 0,7685$ rad where $I = I_m \sin (400t + 0,7685)$

Calculate I_m :

$$U_m = ZI_m \Rightarrow I_m = \frac{U_m}{Z}$$
$$U_m = 250 \text{ V}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(20)^2 + (64 - 83,333)^2} = 27,82 \Omega$$
$$I_m = 8,99 \text{ A}$$

We obtain :

$\Rightarrow$ **I = 8,99 sin (400t + 0,7685)**

Instantaneous voltage V_R

V_R and I are in phase and we have :

$V_R = V_{Rm} \sin (400t + 0,7685)$ with $V_{Rm} = RI_m = 20 \times 8,987 = 179,8W$

$\Rightarrow$ **V_R = 179,8 sin (400t + 0,7685)**

Instantaneous voltage V_L

V_L est en avance de 90° sur I

Where $V_L = V_{Lm} \sin (400t + 0,7685 + \frac{\pi}{2}) = V_{Lm} \cos (400t + 0,7685)$

With $V_{Lm} = X_L I_m = 64 \times 8,978 = 575,17W$

$\Rightarrow$ **V_L = 575,17 cos(400t + 0,7685)**

Instantaneous voltage V_C

V_C is lagging 90° on I

Where $V_C = V_{Cm} \sin (400t + 0,7685 - \frac{\pi}{2}) = - V_{Cm} \cos (400t + 0,7685)$

With $V_{Cm} = X_C I_m = 83,333 \times 8,987 = 748,91W$

$\Rightarrow$ $V_C = - 748,91 \cos(400t + 0,7685)$

2) Calculate $V_R + V_L + V_C$

Let $V = V_R + V_L + V_C$

$V = 179,8 \sin (400t + 0,7685) +(575,17 – 748,91) \cos (400t + 0,7685)$

Apply the trigonometric relation

$A \sin a + b \cos a = (A^2 + B^2)^{1/2} \sin(a + b)$ or $\tan \beta = \dfrac{B}{A}$

We obtain :

$V = [(179,8)^2 + (-173,74)^2]^{1/2} \sin (400t + 0,7685 + \beta)$

$\tan\beta = \dfrac{(-173,74)}{179,8} = - 0,9665 \Rightarrow \beta = - 0,7685$

$V = 250 \sin (400t + 0,7685 + \beta) = 250 \sin (400t)$

We find that $V = V_R + V_L + V_C = U$

Solution of Exercise 7
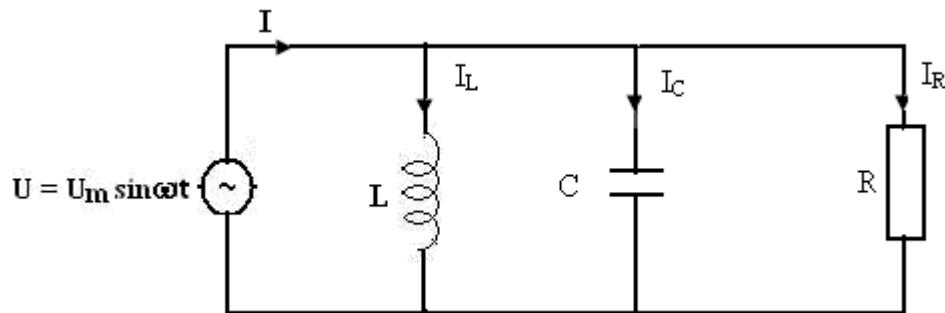
1) Instantaneous current I_L, I_C, I_R et I

The instantaneous voltages V_L, V_C et V_R at the terminals of the coil, of the capacitor and of the resistor are equal.

$V_L = \dfrac{LdI_L}{dt} = U ; \quad V_C = \dfrac{q}{C} = U ; \quad V_R = RI_R = U$

Expression of I_L

$$\frac{L dI_L}{dt} = U \Rightarrow dI_L = \frac{1}{L} U dt \Rightarrow I_L = \frac{1}{L} \int U_m \sin\omega t \, dt$$

Where $I_L = -\dfrac{U_m}{Ln} \cos\omega t = -\dfrac{U_m}{X_L} \cos\omega t = \dfrac{U_m}{X_L} \sin(\omega t - \frac{\pi}{2}) = I_{Lm} \sin(\omega t - \frac{\pi}{2})$

Expression of I_C

$$\frac{q}{C} = U \Rightarrow \frac{1}{C} \frac{dq}{dt} = \frac{dU}{dt} \Rightarrow \frac{I_C}{C} = \frac{dU}{dt} \Rightarrow C\omega U_m \cos\omega t$$

We have : $I_C = \dfrac{U_m}{X_C} \cos\omega t = \dfrac{U_m}{X_C} \sin(\omega t + \frac{\pi}{2}) = I_{Cm} \sin(\omega t + \frac{\pi}{2})$

Expression of I_R

$$R I_R = U \Rightarrow I_R = \frac{U_m}{R} \sin\omega t = I_{Rm} \sin\omega t$$

Expression of I

The principal current I is :

$$I = I_R + I_L + I_C$$

$$\Rightarrow I = \frac{U_m}{R} \sin\omega t - \frac{U_m}{X_L} \cos\omega t + \frac{U_m}{X_C} \cos\omega t$$

$$\Rightarrow I = U_m \left[\frac{1}{R} \sin\omega t + \left(\frac{1}{X_C} - \frac{1}{X_L} \right) \cos\omega t \right]$$

By using the trigonometric relation :

$$A \sin\alpha + B \cos\alpha = (A^2 + B^2)^{1/2} \sin(\alpha + \beta) \quad \text{with } \tan\beta = \frac{B}{A}$$

We can put I under the form $I = I_m \sin(\omega t + \Phi)$

Here
$\beta = \Phi$

$$A = \frac{1}{R}$$

$$B = \left(\frac{1}{X_C} - \frac{1}{X_L} \right)$$

$$\tan\beta = \tan\Phi = \left(\frac{1}{X_C} - \frac{1}{X_L} \right) / \frac{1}{R}$$

Φ is the phase angle between the applied voltage and the principal current

$$I_m = U_m \left[\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_C} - \frac{1}{X_L} \right)^2 \right]^{1/2}$$

2) Impedance of the circuit

$$Z = \frac{U_m}{I_m}$$

$$\Rightarrow Z = \left[\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_C} - \frac{1}{X_L} \right)^2 \right]^{-1/2}$$

3) Phase angle

We obtain this angle from :

$$\tan F = \left(\frac{1}{X_C} - \frac{1}{X_L} \right) / \frac{1}{R} = R \left(\frac{1}{X_C} - \frac{1}{X_L} \right)$$

Solution of exercise 8
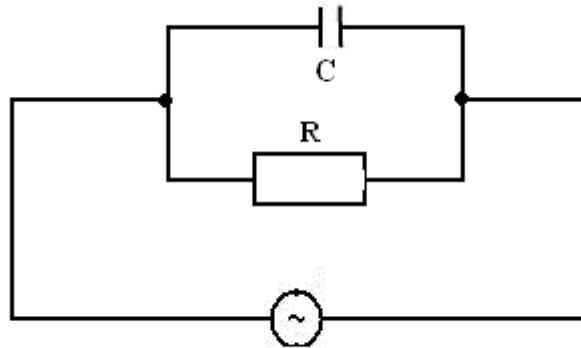
$$U = U_m \sin \omega t$$

The complex impedance of the circuit is $\overline{Z}$ such that :

$$\frac{1}{\overline{Z}} = \frac{1}{R} + \frac{1}{\left(\dfrac{1}{j\omega\,C}\right)} = \frac{1}{R} + j\omega\,C$$

Where $\overline{Z} = \dfrac{1}{\dfrac{1}{R} + j\omega\,C} = \dfrac{\dfrac{1}{R} - j\omega\,C}{\left(\dfrac{1}{R} + j\omega\,C\right)\left(\dfrac{1}{R} - j\omega\,C\right)}$

$$\overline{Z} = \frac{\dfrac{1}{R} - j\omega\,C}{\dfrac{1}{R^2} + \omega^2 C^2}$$

$$\overline{Z} = \frac{R - j\omega\,C\,R^2}{1 + \omega^2 C^2 R^2} = X + jY$$

The impedance of the circuit is $Z = \sqrt{X^2 + Y^2}$

$$Z = \frac{R}{\sqrt{1 + \omega^2 C^2 R^2}}$$

j of $\overline{Z}$ is such that $\tan\varphi = \dfrac{Y}{X} = -\omega c R$

The angle j is negative and thus the voltage is lagging by j behind the current.

Solution of exercise 9

1-Luminosity of the lamp

When we increased the frequency of the source, the impedance Z of the circuit decreases, reaches a minimum and then increases again. The minimum of Z corresponds to the resonance condition
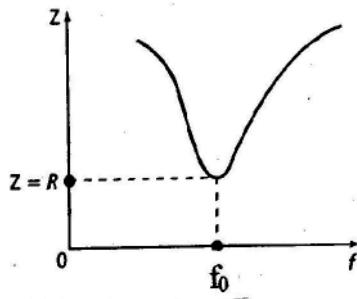
$$Z = \sqrt{R^2 + (X_L - X_c)^2}$$

At the resonance $X_L = X_c$ and $f = f_0 \Rightarrow Z = R$

The effective intensity (I_{eff}) that flows through the circuit is $I_{eff} = \dfrac{U_{eff}}{Z}$

I_{eff} is inversely proportional to Z for constant U_{eff}. When Z diminishes, I_{eff} increases and vice-versa. Thus when f increases, I_{eff} increases, passes by a maximum when $Z = R$ i.e when $f = f_0$ and diminishes
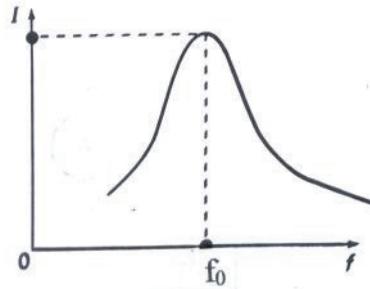
The lamp brightness varies with the variation I_{eff}: the lamp shines more bright as we increase the frequency f. The most intense light comes when f = f0, then the lamp shines less and less when f continues to grow.

2- Frequency of the resonance

At the resonance $X_L = X_C \Rightarrow L\omega_0 = \dfrac{1}{C\omega_0} \Rightarrow L(2\pi\, f_0) = \dfrac{1}{C(2\pi\, f_0)}$

Where $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{0,4(0,4.10^{-6})}}$

$f_0 = \dfrac{10^3}{2\pi(0,4)} \approx 400$ Hz

Maximum intensity of the current

$I_{eff} = \dfrac{U_{eff}}{Z(f)}$. I_{eff} is maximized when $Z(f) = Z(f_0) = R$

$(I_{eff})_m = \dfrac{U_{eff}}{R} = \dfrac{10^{-2}V}{10\Omega}$

$(I_{eff})_m = 10^{-3}A = 1mA$

Voltage at the terminals of the capacitor at resonance

Consider U_C as this voltage : $U_C = X_C \, I_{eff}$

At the resonance $X_C = \dfrac{1}{C\omega_0} = \dfrac{1}{C(2\pi \, f_0)} = \dfrac{1}{(0,4)10^{-6}2\pi(400)} = 1000\Omega$

$\Rightarrow X_C = 1000\Omega$

$(I_{eff})_m = 10^{-3}A$

$\Rightarrow U_C = X_C \, (I_{eff})_m = 1000\Omega \times 10^{-3}A = 1V$

3- Effective intensity for different values of resistance

For $R_1 = \dfrac{R}{2}$, the maximum current becomes:

$(I_{eff})_m = \dfrac{U_{eff}}{R_1} = \dfrac{U_{eff}}{\dfrac{R}{2}} = \dfrac{2U_{eff}}{R} = 2mA$

For $R_2 = \dfrac{R}{4}$, the amximum current becomes:

$(I_{eff})_m = \dfrac{U_{eff}}{R_2} = \dfrac{U_{eff}}{\dfrac{R}{4}} = \dfrac{4U_{eff}}{R} = 4mA$

When the resistance diminishes, the resonance is sharper.

Solution of exercise 10

The phase angle Φ between voltage and current is such that :

$\tan \Phi = \dfrac{X_L - X_C}{R}$

$$\cos \Phi = \frac{R}{Z}$$

If $\tan \Phi > 0$ i.e $X_L > X_C$ the voltage is ahead by Φ with respect to current.

If $\tan \Phi < 0$ i.e $X_L < X_C$ the voltage is lagging by Φ with respect to current.

The voltage and current are in phase if $\tan \Phi = 0$ and $\cos \Phi = 1$. This situation happens when:

$$X_L = X_C$$

$$Z = R$$

This corresponds to a resonance condition

Find the sign of ($X_L - X_C$). For this, draw the types of X_L and X_C as a function of f.

$$X_L = L\omega = L\,2\pi\,f$$

$$X_C = \frac{1}{C\omega} = \frac{1}{C(2\pi)f}$$
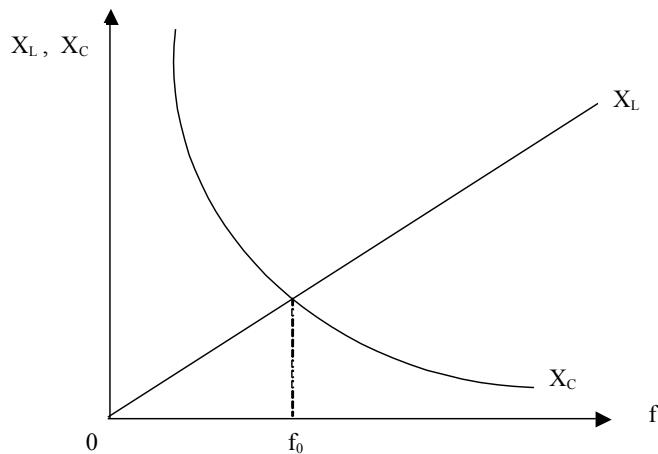
X_L is larger than X_C if the frequency f of the source alternating voltage exceeds the resonant frequency f_0. X_L is inferior to X_C in the opposite case. Compare f with $f_0 = 718Hz$:

$U = U_m \sin (1000\pi t) = U_m \sin\omega t = U_m \sin 2\pi f t \implies 2\pi f = 1000\pi \implies f = 500Hz$

On voit que $f < f_0 \implies X_L < X_C$:

$\tan \Phi = \dfrac{X_L - X_C}{R} < 0 \implies$ the voltage is behind Φ with respect to current
the circuit has the behaviour of a capacitor

Solution of exercise 11

Frequency of resonance

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(1)(5.10^{-6})}}$$
$$f_0 = 71,2Hz$$

Effective intensity of the resonance

At resonance, the impedance Z of the circuit is equal to the resonance R : $Z(f_0) = R$

Consider $I_{eff}(f_0)$ the effective intensity at resonance :

$$I_{eff}(f_0) = \frac{U_{eff}}{Z(f_0)} \implies I_{eff}(f_0) = \frac{U_{eff}}{R} = \frac{10V}{10\Omega}$$
$$I_{eff}(f_0) = 1A$$

Quality factor

$Q = \dfrac{\omega_0}{\Delta\omega} = \dfrac{L\omega_0}{R}$ with $\omega_0 = (2\pi)f_0 = 2\pi (71,2) = 447,14$ rad.s^{-1}

We obtain :

$$Q = \frac{L\omega_0}{R} = \frac{(1H)(447,14\,\text{rad.s}^{-1})}{10\Omega}$$
$$Q = 44,7$$

Bandpass

$$\Delta\omega = \frac{\omega_0}{Q} = \frac{447{,}14\,\text{rad.s}^{-1}}{44{,}7}$$

$$\Delta\omega = 10\,\text{rad.s}^{-1}$$

Effective voltage across the terminals of the capacitor at the resonance

Consider $(U_C)_{eff}$ as this voltage :

$(U_C)_{eff} = X_C(\omega_0)I_{eff}(f_0)$, $X_C(\omega_0)$ being the capacitive reactance at resonance.

$$X_C(\omega_0) = \frac{1}{C\omega_0}$$

Where $(U_C)_{eff} = \dfrac{1}{C\omega_0} I_{eff}(f_0) = \dfrac{1A}{(5.10^{-6})(2\pi)(71,2)}$

$$(U_C)_{eff} = 447{,}3V$$

This tension is much greater than the effective voltage of the source, which is 10V

Solution of exercise 12

Resistance R

The effective intensity is maximized at resonance and is equal to $(I_{eff})_m = 150\text{mA}$.

The frequency of resonance is thus $f_0 = 800\text{Hz}$.

We have : $(I_{eff})_m = \dfrac{U_{eff}}{Z(f_0)}$ with $Z(f_0) = R$ the impedance of the circuit at resonance.

We obtain : $(I_{eff})_m = \dfrac{U_{eff}}{Z(f_0)} = \dfrac{U_{eff}}{R} \Rightarrow R = \dfrac{U_{eff}}{(I_{eff})_m} = \dfrac{24V}{150.10^{-3}A}$

$$R = 160\Omega$$

Inductance L

The bandpass is : $\Delta\omega = \dfrac{R}{L} \Rightarrow L = \dfrac{R}{\Delta\omega} = \dfrac{160\Omega}{167,5\,\mathrm{rad.s^{-1}}}$

$$L = 0,995\mathrm{H}$$

Capacitance C

The quality factor of the circit is :

$$Q = \dfrac{\omega_0}{\Delta\omega} = \dfrac{1}{CR\omega_0} \Rightarrow C = \dfrac{1}{QR\omega_0}$$

But $\omega_0 = (2\pi)\, f_0 = 2\pi\,(800\,\mathrm{Hz}) = 1600\pi\ \mathrm{rad.s^{-1}}$

$$\Delta\omega = 167,5\ \mathrm{rad.s^{-1}}$$

We have $\quad Q = \dfrac{1600\pi.\mathrm{rad.s^{-1}}}{167,5.\mathrm{rad.s^{-1}}} = 30$

We obtain :

$$C = \dfrac{1}{QR\omega_0} = \dfrac{1}{(30)(160\Omega)(1600\pi.\mathrm{rad.s^{-1}})}$$

$$C = (4,14)\ 10^{-8}\,\mathrm{F}$$

Solution of exercise 13

1- Fresnel representation.

Calculate and compare the inductive and capacitive reactance X_L and X_C :

$$X_L = Lw = L\,(2pf)$$

$$X_C = \dfrac{1}{Cn} = \dfrac{1}{C(2\pi)f}$$

with $w = 2pf$, $f = 400\mathrm{Hz}$, $L = 4.10^{-3}\mathrm{H}$ et $C = 1,6.10^{-5}\mathrm{F}$

After numerical application, we obtain :

$X_L = 10\ W$

$X_C = 25\ W$

$\Rightarrow X_C > X_L$

The Fresnel representation is thus :

$V_{Lm} = X_L\ I_m$

$V_{Rm} = R\ I_m$

Φ

$U_m = Z\ I_m$

$V_{Cm} - V_{Lm} = (X_C - X_L)\ I_m$

$V_{Cm} = X_C\ I_m$
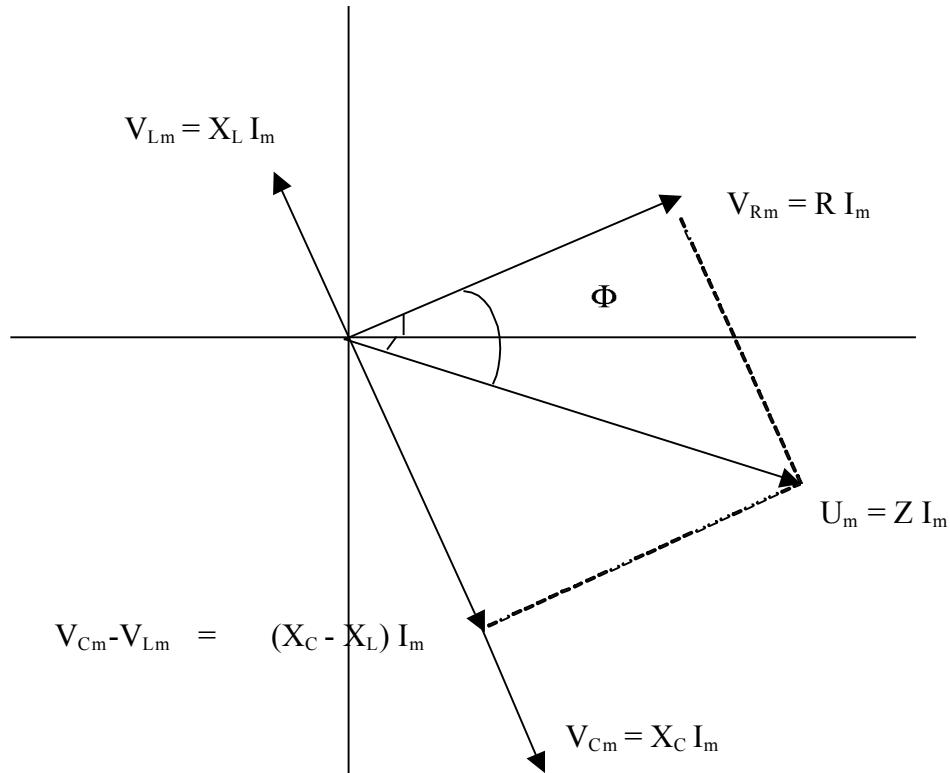
The voltage is lagging by F on the current :

2- Phase angle F

$$\tan F = \frac{V_{Lm} - V_{Cm}}{R} = \frac{X_L - X_C}{R} = \frac{10 - 25}{20} = -0,75 \Rightarrow F = -37°$$

The phase angle is negative $\Rightarrow$ the voltage lags behind the current.

3- Frequency of resonance

Draw X_L and X_C as a function of the frequency f

$X_L = L\,(2pf)$

$X_C = \dfrac{1}{C(2\pi)f}$
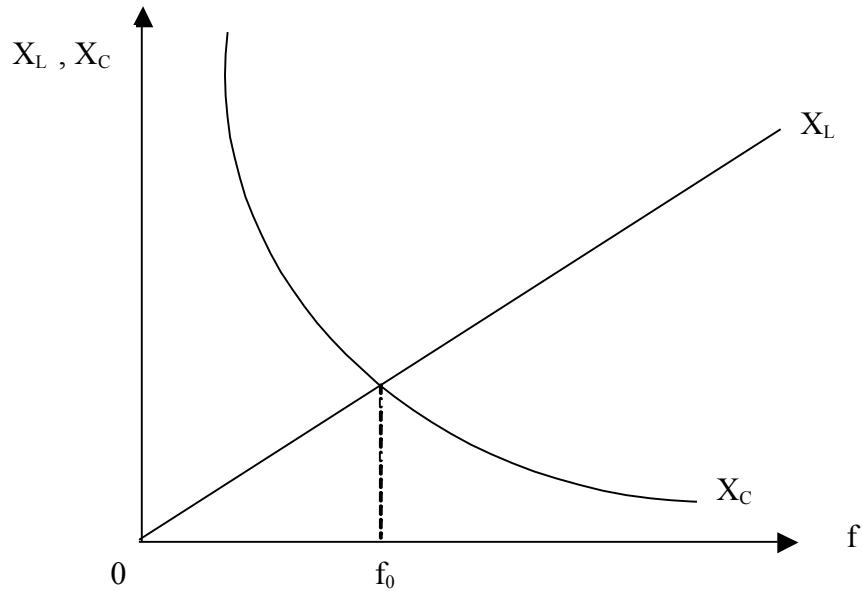
At the resonance f_0 we must have $X_L = X_C$; at the frequency $f = 400Hz$ we see that $X_L < X_C$: ($X_L = 10\Omega$ et $X_C = 25\Omega$)

We deduce the above figure where $f < f_0$: the frequency of resonance f_0 is greater than 400Hz.

4. Value of the frequency of resonance.

$X_L = X_C \Rightarrow L\,(2\pi f_0) = \dfrac{1}{C(2\pi)\,f_0}$

Where $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

$$f_0 = \frac{1}{6,28\sqrt{4.10^{-3}(16.10^{-6})}} = 629Hz.$$

Solution of exercise 14

$$U = U_m \sin\omega t$$
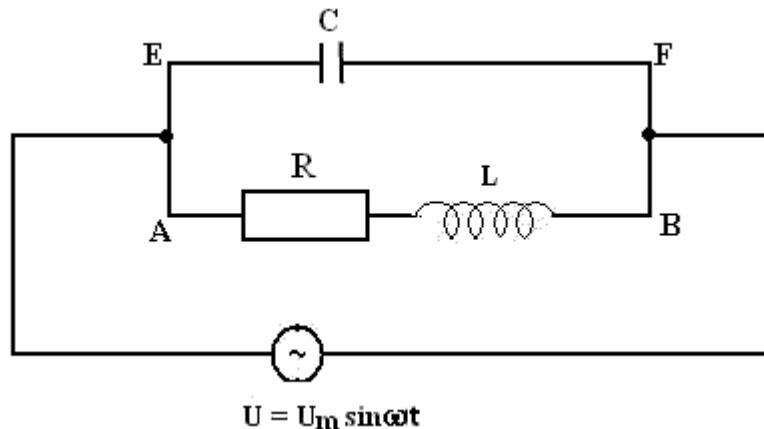
1) Complex impedance of the circuit

Consider $\overline{Z_1}$ et $\overline{Z_2}$ respectively the complex impedances of the branches AB and EF.

We have : $\overline{Z_1} = R + jLw$ et $\overline{Z_2} = \dfrac{1}{jC\omega}$

We have here a parallel connection, where :

$$\frac{1}{\overline{Z}} = \frac{1}{\overline{Z_1}} + \frac{1}{\overline{Z_2}} , \ \overline{Z} \text{ being the complex impedance of the circuit.}$$

$$\Rightarrow \frac{1}{\overline{Z}} = \frac{1}{R + jL\omega} + jC\omega = \frac{1 + (jC\omega)(R + jL\omega)}{R + jL\omega}$$

$$\Rightarrow \overline{Z} = \frac{R + jL\omega}{1 + (jC\omega)(R + jL\omega)} = \frac{R + jL\omega}{(1 + LC\omega^2) + jRC\omega}$$

$$\Rightarrow \overline{Z} = \frac{(R + jL\omega)\left\{(1\text{-}LC\omega^2)\text{-}jRC\omega\right\}}{(1 - LC\omega^2)^2 + R^2C^2\omega^2}$$

$$\Rightarrow \bar{Z} = \frac{R + j\omega\left\{L(1-LC\omega^2)-R^2C\right\}}{(1 - LC\omega^2)^2 + R^2C^2\omega^2}$$

2) The parallel resonance is obtained when the imaginary part of $\bar{Z}$ is zero, when $L(1 - LC\omega^2) - R^2C = 0$

In the case where $\omega^2L^2 >> R^2$, this condition is true for a value of ω_0 of ω such

that $L(1 - LC\omega_0^2) = 0 \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$ (value of ω at resonance). The impedance becomes :

$$\bar{Z} = \frac{R}{\omega_0^2R^2C^2} = \frac{1}{\omega_0^2RC^2} = \frac{LC}{RC^2} = \frac{L}{RC}$$

At resonance the impedance of the circuit is the modulus $\bar{Z}$ which is $\frac{L}{RC}$

$$Z_{resonance} = \frac{L}{RC}$$

Solution of exercise 15

In the presence of $\vec{E}$ and $\vec{B}$, the electron undergoes a force :

$$\vec{F}_e + \vec{F}_m = -e\vec{E} - e\vec{v}\wedge\vec{B} = 0 \Rightarrow \vec{E} = \vec{v}\wedge\vec{B} \Rightarrow E = vB$$

The force exerted on the electron when the field $\vec{E}$ is removed is :

$$m\frac{v^2}{R} = e\,v\,B$$

v being the velocity of the electron R the radius of its trajectory. We obtain :

$$R = \frac{mv}{eB}$$

By replacing v with $\dfrac{E}{B}$, the radius becomes :

$$R = \dfrac{m}{eB}\dfrac{E}{B} = \dfrac{mE}{eB^2}$$

It follows that :

$$\dfrac{e}{m} = \dfrac{E}{RB^2}$$

A.N. : E = 8 kV/m ; R = 1,14cm ; B = 2 x 10^{-3} T

$$\dfrac{e}{m} = \dfrac{8.10^3}{(1,14 \times 10^{-2})(2 \times 10^{-3})^2}$$

$$\dfrac{e}{m} = \mathbf{1,75 \times 10^{11}\ C/kg}$$

Solution of exercise 16

1- Velocity of ions

In the space between S and O, and ion is subjected to the force :

$$\vec{F} = \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v} \wedge \vec{B}$$

For it to travel through space without being deflected from its original path, it must.

$\vec{F} = \vec{o}$ i. e. $\vec{F}_E = -\vec{F}_B \Rightarrow qE = qvB$ (remarque $\vec{v} \perp \vec{B}$)

We obtain :

$$v = \dfrac{E}{B}$$

Only the ions with this velocity will leave O.

2- <u>Distance d on the photographic plate</u>.

When an ion penetrates the region reigned by $\vec{B'}$, it is subjected to the force :

$$\vec{F'} = q\vec{v} \wedge \vec{B'} \quad \text{with} \quad \vec{v} \perp \vec{B'}$$

The charge q is positive, the ion undergoes a magnetic deflection to the left. The ions that enter in the field therefore overlap the OM part of the photographic plate.

The ions move along a semicircle of diameter d.
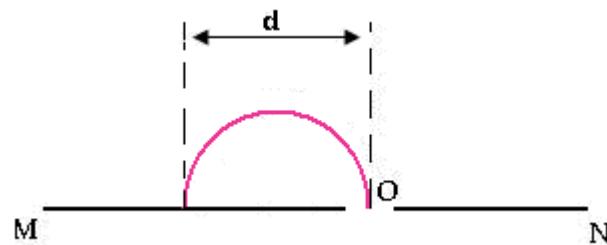
$\vec{F'}$ is a centripetal force.

$$m\frac{v^2}{R} = q\,V\,B'$$

The radius of trajectory of an ion is thus :

$$R = \frac{mv}{qB'}$$

Il follows that

$$d = 2R = \frac{2mv}{qB'}$$

3- Isotopes of tin

These isotopes have the same charge Q But not the same mass. Consider d1 the distance at which the isotope of mass m1 = 116 u cut the photographic plate and d2 is the distance corresponding to the isotope mass m2 = 120 u

We have :

$$d_2 - d_1 = \frac{2v}{qB'}(m_2 - m_1)$$

where $\Delta d = \frac{2v}{qB'}\Delta m$ with $\Delta d = d_2 - d_1$ et $\Delta m = m_2 - m_1$

Replace $v = \frac{E}{B}$, We obtain :

$$\Delta d = \frac{2E}{qBB'}\Delta m$$

A.N. : E = 15 kV/m; B = B' = 0,15 T

1 u = 1,66 x 10⁻²⁷ kg

q = e = 1,6 x 10⁻¹⁹ C ; Δm = 4 u

$$\Delta d = \frac{2(15 \times 10^3)(4)(1,66 \times 10^{-27})}{(1,6 \times 10^{-19})(0,15)^2}$$
$$\Delta d = 55,3 \text{ mm}$$

Solution of exercise 17

1- Frequency of oscillating field

Within a dee, a proton forms a half-circle of radius $R = \frac{mv}{qB}$, v being its input velocity in the dee. It is accelerated whenever it passes from one dee to the other if the half period of the oscillating electric field is equal to the time t to go for a half-circle. Calculate this time t.

For do a complete revolution, there must be a time T such that:

$$vT = 2pR \implies T = \frac{(2\pi)R}{v} = \frac{2\pi}{v}(\frac{mv}{qB}) = \frac{(2\pi)m}{qB}$$

Thus : $t = \frac{T}{2} = \frac{(\pi)m}{qB}$

The period of the electric field is :

$$2\,t = \frac{(2\pi)m}{qB} = T$$

This is just the period of revolution of the proton.

The frequency of the oscillating electric field is thus:

$$f = \frac{1}{T} = \frac{qB}{(2\pi)m}$$

A.N. $q = 1{,}6\ 10^{-19}\,C$; $B = 1{,}5\,T$; $m = 1{,}66\ x10^{-27}\,kg$

$$f = \frac{(1{,}6x10^{-19})(1{,}5)}{2\pi(1{,}6610^{-27}\,k)}$$

f = 23 MHz

2- Kinetic energy

Calculate the velocity of the proton leaving the cyclotron :

$$R_s = \frac{m}{qB}v_s \Rightarrow v_s = \frac{qBR_s}{m}$$

A.N. $q = 1{,}6\ 10^{-19}\,C$; $B = 1{,}5\,T$; $m = 1{,}66\ x10^{-27}\,kg$; $R = 0{,}5m$

$$v_s = \frac{(1{,}610^{-19})(1{,}5)(0{,}5)}{(1{,}6610^{-27})}$$

$$v_s = 72{,}3\ 10^6\ ms^{-1}$$

The kinetic energy is thus :

$$E_C = \frac{1}{2}mv_s^2$$

A.N. $m = 1{,}66\ x10^{-27}\,kg$; $v_s = 72{,}3\ 10^6\ ms^{-1}$

$E_C = 43,38 \times 10^{-13}$ J $\Rightarrow E_C = 27,11$ MeV

3- Number of accelerations

The potential difference between the dice is 40 kV. Whenever the proton moves from one die to another, it gains an energy of 40 keV = 4 x 10⁴ eV.

The final energy being E_C, the number of accelerations is :

$$N = \frac{Ec}{40keV} = \frac{27,11 \times 10^3 keV}{40 keV}$$

$$N = \frac{27110}{40} \approx 678$$

The proton travels 678 times in the space between the dice.

Solution of exercise 18

1) Training velocity

Current density :

$$\vec{j} = - n e \vec{v_d}, \vec{v_d}$$

The velocity and current density vector are opposite to each other. We have a module:

$$j = n e v_d \Rightarrow v_d = \frac{j}{ne} \qquad (1)$$

Intensity of the current is : $I = j$ ab. Thus :

$$v_d = \frac{I}{neab} \qquad (2)$$

2) Accumulation of electrons on one of the faces

Each electron is subjected to a force $\vec{F_m} = -e\vec{v_d} \wedge \vec{B}$ and drifted towards the edge N as shown in the figure below. Therefore, there is accumulation of electrons on the N side and a lack of electrons on the trailing edge M.
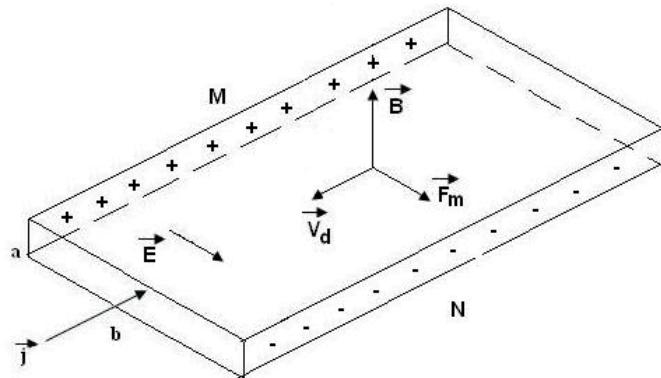
3) Permanent regime

The accumulation of charges leads to the appearance of a potential difference between M and N edges then an electric field oriented towards the decreasing potential from M to N.

An electric force $\vec{F}_e = -e\vec{E}$ is implied on electrons in addition to the magnetic force. This force opposes the accumulation of electrons on the edge N.

A permanent regime is established when $\vec{F}_e$ et $\vec{F}_m$ compensate each other :

$$\vec{F}_e + \vec{F}_m = \vec{o}$$

4) Relation between $\vec{E}$ and $\vec{B}$

When the permanent regime is established :

$$-e\vec{E} + (-e \; \vec{v}_d \wedge \vec{B}) = \vec{0} \qquad \Rightarrow \vec{E} = -\vec{v}_d \wedge \vec{B}$$

$\vec{B}$ and $\vec{v}_d$ being perpendicular to each other, we obtain

$$E = v_d B \qquad (3)$$

5- Potential difference $V_M - V_N$

The potential difference between the front and rear edges of the ribbon is connected to the movement of $\vec{E}$ along the trajectory M ® N

$$V_M - V_N = \int_M^N \vec{E}.dl$$

$\vec{E}$ is uniform within these coundaries. It follows that :

$$V_M - V_N = E \int_M^N dl = Eb$$

$$\Rightarrow E\, b = V_M - V_N$$

Remplace E by using relation (3). We obtain :

$$v_d\, B\, b = V_M - V_N \qquad (4)$$

But $v_d = \dfrac{I}{neab}$

Relation (4) thus gives :

$$\frac{I\, B\, b}{neab} = V_M - V_N$$

$$\frac{I\, B}{(ne)a} = V_M - V_N$$

A.N. $I = 2\ A;\ B = 2\ T;\ ne = 2\ 10^{10}\ Cm^{-3};\ a = 2\ 10^{-3}\ m$

$$V_M - V_N = 10^{-7}\ \textbf{Volt}$$

Solution of exercise 19

The mesh law of Kirchhoff gives :

Mesh GABG : $e - R_1\, i_1 = 0 \qquad (1)$

Mesh GCDG : $e - L\dfrac{di_2}{dt} - R_2 i_2 = 0 \qquad (2)$

1) Just after closing switch K.

I. intensity i_1

The connection traversed by i_1 is a pure resistance :

$$e - R_1 i_1 = 0 \implies i_1 = \frac{\varepsilon}{R_1} = \frac{10V}{5\Omega} = 2A$$

Intensity i_2

At this initial instant $i_2 = 0$

Intensity i traversing the connection K

The law of nodes gives :

$i = i_1 + i_2 \implies i = 2A + 0A$

$i = 2\ A$

Potential difference across the terminals of R_2

$$V_{R_2} = R_2 i_2 \implies V_{R_2} = 0$$

Potential difference across the terminals of the plate

$$V_L = L \frac{di_2}{dt}$$

Take relation (2) : $e - L \dfrac{di_2}{dt} - R_2 i_2 = 0$

$i_2 = 0 \implies e = L \dfrac{di_2}{dt} = V_L$

$V_L = 10V$

Growth rate of current i_2

Relation (2) gives :

$$e = L \frac{di_2}{dt} \implies \frac{di_2}{dt} = \frac{\varepsilon}{L} = \frac{10V}{5H}$$

$$\frac{di_2}{dt} = 2\ A \cdot s^{-1}.$$

2) when the permanent regime is attained :

Intensity i_1

i_1 n'a pas changé $\Rightarrow i_1 = 2$ A

Intensity i_2

i_2 attains its maximum value

$e = R_2 i_2 \Rightarrow i_2 = \dfrac{\varepsilon}{R_2} = \dfrac{10V}{10\Omega}$

$i_2 = 1A$

Intensity i

La loi des nœuds donne : $i = i_1 + i_2 = 2A + 1A$

$i = 3$ A

Potential difference between the terminals of R_2

$V_{R_2} = R_2 i_2 = 10$ W x 1 A

$V_{R_2} = 10$ V

Potential difference between the terminals of the coil.

From relation (2) we obtain :

$e - L \dfrac{di_2}{dt} - R_2 i_2 = 0 \Rightarrow L \dfrac{di_2}{dt} = 0$

(Car $e = R_2 i_2$)

$V_L = 0$ V

Growth rate of current i_2,

The intensity does not increase

$\dfrac{di_2}{dt} = 0$ As^{-1}

Solution of exercise 20
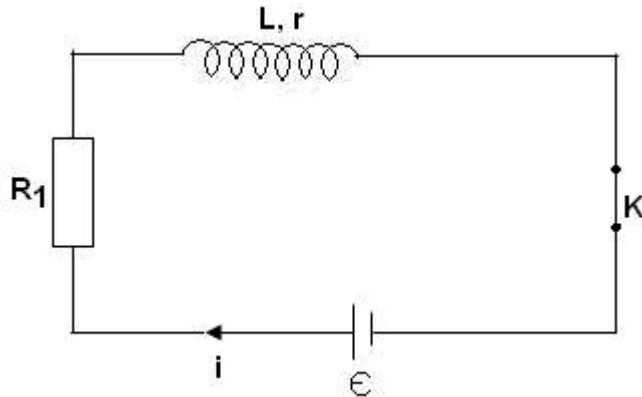
1) The law of mesh of Kirchhoff applied to this circuit gives

$$e = L\frac{di}{dt} - (R_1 + r)\, i = 0 \qquad\qquad (1)$$

2) Verify that $i = \dfrac{\varepsilon}{R}(1 - e^{-\frac{R}{L}t})$ is the solution to the differential equation.

$$i = \frac{\varepsilon}{R}(1 - e^{-\frac{R}{L}t}) \;\Rightarrow\; \frac{di}{dt} = \frac{\varepsilon}{L}e^{-\frac{R}{L}t}$$

Take i and $\dfrac{di}{dt}$ in equation (1) :

$$e - L\left(\frac{\varepsilon}{L}e^{-\frac{R}{L}t}\right) - (R_1 + r)\left[\frac{\varepsilon}{R}(1 - e^{-\frac{R}{L}t})\right]$$

$$= e - e\,e^{-\frac{R}{L}t} - R\frac{\varepsilon}{R}(1 - e^{-\frac{R}{L}t}) \qquad \text{car } R_1 + r = R$$

$$= e - e\,e^{-\frac{R}{L}t} - \varepsilon + (\varepsilon)e^{-\frac{R}{L}t} = 0$$

$i = \dfrac{\varepsilon}{R}(1\text{-}e^{-\frac{R}{L}t})$ is indeed the solution

3) calculate the resistance r of the coil

The final value of i is observed to be sufficiently long, which implies t ® . Thus :

$$i_f = \frac{\varepsilon}{R}$$

We have :

$$\frac{999}{1000} i_f = \frac{\varepsilon}{R}(1 - e^{-\frac{R}{L}t_1}) \text{ with } t_1 = 6,9.10^{-4} \text{ s}$$

$$\frac{999}{1000} \frac{\varepsilon}{R} = \frac{\varepsilon}{R}(1 - e^{-\frac{R}{L}t_1})$$

$$0,999 = 1 - e^{-\frac{R}{L}t_1} \Rightarrow \frac{R}{L}t_1 = \ln 1000$$

$$\frac{R}{L}t_1 = 3 \ln 10 \Rightarrow R = \frac{3 L \ln 10}{t_1} \Rightarrow R_1 + r = \frac{3 L \ln 10}{t_1} \Rightarrow r = \frac{3 L \ln 10}{t_1} - R_1$$

A.N. L = 0,1 mH ; ln 10 = 2,3, t_1 = 6,910^{-4}s ; R_1 = 0,5 W

$$r = \frac{3 \times 10^{-4} \times 2,3}{6,9.10^{-4}} - 0,5$$

r = (1 - 0,5 W)

r = 0,5 W

Solution of exercise 21

1) Magnetic field inside and outside of a solenoid

The solenoid is infinitely long :

$\Rightarrow \vec{B}$ is uniform inside : $B = \mu_0 n I$

$\Rightarrow \vec{B}$ is zero outside

2) Self-induction coefficient

$$\Phi_1 = B\,S = \mu_0\,n\,I\,S$$

The total flux through all of the turns is :

$$\Phi = \mu_0\,n\,N'I\,S$$

N' number of turns

$$\Phi = L\,I \implies \qquad \mathbf{L = \mu_0\,n\,N'\,S} \quad \text{with } S = \pi\,R^2$$

3) Mutual induction coefficient

a) the coil is traversed by the flux Φ of field $\vec{B}$ created by the solenoid

$\vec{B}$ is uniform in the solenoid : $B = \mu_0\,n\,I$ is zero outside

$$\Phi = \underbrace{\mu_0\ n\ I\ \pi\,R^2}\ N$$

Flux through one turn of the coil

Total number of turns in the coil

$$\Phi = MI$$
$$\implies M = \mu_0 n\,Np\ R^2$$

b) emf

when I varies, the flux varies and an emf appears in the coil :

$$e_{ind} = -\frac{d\Phi}{dt} = -\mu_0 nN\pi R^2 \frac{dI}{dt}$$

c) induced current

$$i = \frac{dq}{dt} = \frac{\varepsilon_{in}}{r} = -\frac{\mu_0 nN\pi R^2}{r}\frac{dI}{dt}$$

$$q = \int_{I}^{0} -\frac{\mu_0 nN\pi R^2}{r} dI$$

$$q = \frac{\mu_0 nN\pi R^2 I}{r}$$

According to Lenz's law the direction of induced current is such that it opposes the reduction of Φ, that is to say, the decrease of $\vec{B}$.

$\Rightarrow$ the direction of the current induced, thus the same as the direction of the current running through the turns of the solenoid.

Solution of exercise 22

Magnetic intensity
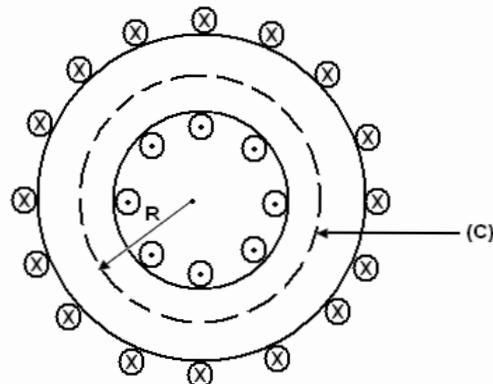
Ampere's law gives $\oint_{(C)} \vec{H}.\vec{dl} = NI$ (1)

Consider R as the mean radius of the coil.

Relation (1) gives $H(2pR) = NI$

or $Hl = NI$, $l = 2pR$ being the mean circumference of the coil.

Thus: $H = \frac{NI}{l}$

<u>A.N</u> : N = 400 ; I = 2A ; l = 0,40m

$$H = \frac{400 \times 2}{0,4}$$

Magnetization

Use the relation $\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M} \Rightarrow M = \frac{B - \mu_0 H}{\mu_0}$

<u>A.N.</u> : B = 1 T ; H = 2000 Am^{-1} ; μ_0 = 4p x 10^{-7} $\frac{Tm}{A}$

$$M = \frac{1 - (4\pi)10^{-7} \times 2000}{4\pi 10^{-7}}$$

M = 7,94 x 10^5 Am^{-1}

Magnetic susceptibility

$\vec{M}$ et $\vec{H}$ are related by $\vec{M} = X_m \vec{H}$ where

$M = X_m H \Rightarrow X_m = \frac{M}{H}$
<u>A.N.</u> : M = 7,94 x 10^5 Am^{-1} ; H = 2000Am^{-1}

$$X_m = \frac{7,94 \times 10^5}{2000}$$

X$_m$ = 397

Current of equivalent surface

The magnetization M here is just the surface current per unit length. Consider I$_s$ as this surface current.

$M = \frac{I_s}{l} \Rightarrow I_s = M l$
<u>A.N.</u> : M = 7,94 x 10^5 Am^{-1} ; l = 0,40m

I$_s$ = 7,94 x 10^5 x 0,4

I$_s$ = 317 x 10^{3}A.

Relative permeability

$\mu_r = 1 + X_m$

Here $X_m = 397$ where : $\mu_r = 398$

Solution of exercise 23

Aimantation

The magnetization $\vec{M}$ and magnetic intensity $\vec{H}$ are linked by :

$\vec{M} = X_m \vec{H} \Rightarrow M = X_m H$ (X_m is positive)

But $H = n I \Rightarrow M = X_m n I$

n is the number of turns per unit length .

Here $n = \dfrac{N}{l}$ with N being the number of turns and l the mean circumference
We have:

$$M = X_m \dfrac{N}{l} I$$

<u>A.N.</u>: $X_m = 4 \times 10^{-3}$; $N = 2000$; $l = 2pR = 2p(0,2)$ m; $I = 15A$

$$M = (4 \times 10^{-3}) \times \dfrac{2000}{2\pi(0,2)} 15$$

$M = 95, 54$ Am^{-1}

Field B

$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M} \Rightarrow B = \mu_0 (H + M)$

$H = n I = \dfrac{N}{l} I$

Where $B = \mu_0 (\dfrac{N}{l} I + X_m \dfrac{N}{l} I)$

$$B = \mu_0 \, (1 + X_m) \, \frac{NI}{l}$$

<u>A.N.</u>: $\mu_0 = 4p \times 10^{-7} \dfrac{Tm}{A}$; $X_M = 4 \times 10^{-3}$; N = 2000; l = 2p (0, 2) m; I = 15A

$$B = 4px \; 10^{-7} \, (1 + 0{,}004) \, \frac{2000}{2\pi(0{,}2)} \; x \; 15$$

B = 0,0301 T

Solution of exercise 24
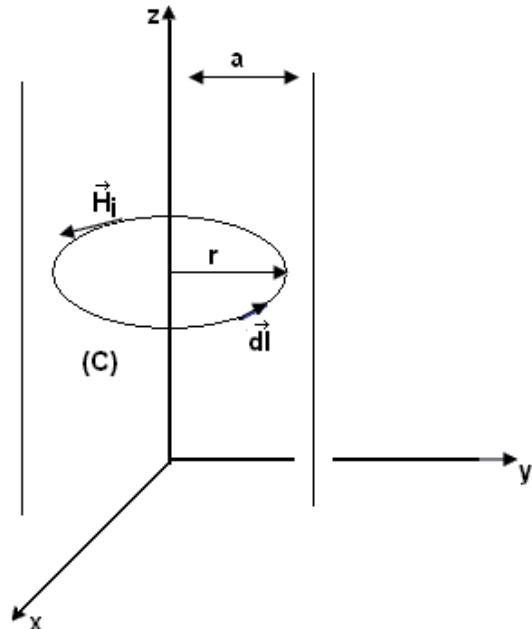
1- Magnetic intensity $\vec{H}$ in all of the space

$r < a$:

Take $\vec{H}_i$ as the magnetic intensity in this region

We know that $\vec{\nabla} \wedge \vec{H}_i = \vec{j}_{ex}$

Thus, $\int_{(S)} (\vec{\nabla} \wedge \vec{H}_i) \vec{dS} = \int_{(S)} \vec{j}_{ex} . \vec{dS} = I$

By Stoke's theorem, we have :

$$\int_{(S)} (\vec{\nabla} \wedge \vec{H}_i) \vec{dS} = \int_{(C)} \vec{H}_i . \vec{dl}$$

Where $\int_{(C)} \vec{H}_i . \vec{dl} = I$

$\vec{dl}$ is tangent to circle (C)

$\vec{H}_i$ is tangent to circle (C)

$\Rightarrow H_i (2pr) = I$

$$H_i = \frac{I}{2\pi \, r} \quad \text{or} \quad \vec{H}_i = \frac{I}{2 \, \pi \, r} \vec{\varphi}_0$$

$\vec{\varphi}_0$ is a unit vector on circle (C)

$r > a$

Consider $\vec{H}_e$ the field in this region. We do the same and we obtain :

$$H_e (2pr) = I \quad \Rightarrow \vec{H}_e = \frac{I\vec{\varphi}_0}{2 \, \pi \, r}$$

2- Field $\vec{B}$ on all points in the space

 r < a

$$\vec{B}_i = \mu \vec{H}_i \qquad\qquad \Rightarrow \vec{B}_i = \mu \frac{I}{2\pi r}\vec{\varphi}_0$$

μ being the permeability of the material of the cylinder.

 r > a : $\vec{B}_e = \mu_0 \vec{H}_e = \dfrac{I}{2\pi r}\vec{\varphi}_0$ (since we have a void outside the cylinder).

3- Magnetization vector $\vec{M}$

We have the relation : $\vec{B} = \mu_0\vec{H} + \mu_0\vec{M} \Rightarrow \vec{M} = \dfrac{\vec{B}}{\mu_0} - \vec{H}$

 r < a

$$\vec{M}_i = \frac{\vec{B}_i}{\mu_0} - \vec{H}_i$$

Replace $\vec{B}_i$ and $\vec{H}_i$; we obtain :

$$\vec{M}_i = \frac{\mu}{\mu_0}\frac{I}{2\pi r}\vec{\varphi}_0 - \frac{I}{2\pi r}\vec{\varphi}_0$$

$$\vec{M}_i = \frac{I}{2\pi r}(\frac{\mu}{\mu_0} - 1)\vec{\varphi}_0$$

 r > a

$\vec{M}_e = \vec{0}$ since we are in a void

4- Amperian current I_a

It is the current due to the inner magnetization.

We have $\vec{j}_a = \vec{\nabla} \wedge \vec{M}_i$

$I_a = \int_{(S)} \vec{j}_a \cdot \vec{dS} = \int_{(S)} (\vec{\nabla} \wedge \vec{M}_i) \cdot \vec{dS} = \oint_{(C)} \vec{M}_i \cdot \vec{dl}$ (Stoke's theorem)

$\vec{M}_i$ et $\vec{dl}$ are paralle. After integrating, we obtain

$M_i (2p \ r) = I_a \Rightarrow I_a = \dfrac{I}{2\pi \ r}(\dfrac{\mu}{\mu_0} - 1)(2\pi \ r)$

$$I_a = I \ (\dfrac{\mu}{\mu_0} - 1)$$

Solution of exercise 25

1- Expression ofsusceptibility

Curie's law : $M = \dfrac{1}{3}m\dfrac{B_{app}}{kT}M_s$

But $B_{app} = \mu_0 H$ and $M = X_m H$, where :

$X_m H = \dfrac{1}{3}m\dfrac{\mu_0 H}{kT}M_s$

$\Rightarrow X_m = \dfrac{1}{3}m\dfrac{\mu_0 H}{kT}M_s$

2- Calculate susceptibility X_m à T = 300K

Calculate Ms :

$M_s = N_A \ r/ \ M \ m$

<u>A.N.</u>: $N_A = 6,02 \times 10^{23}$ atomes / mol; $r = 2700$kg /m³,

$M = 27 \times 10^{-3}$ kg/mol ; $m = \mu_B = 9,27 \times 10^{-24}$ Am²

$$M_s = \frac{(6,02 \text{ x } 10^{23})(2700)(9,27 \text{ x } 10^{-24})}{27 \text{x } 10^{-3}}$$

$$M_s = 5,58 \text{ x} 10^5 \; \frac{A}{m}$$

Calculate X_m

$$X_m = \frac{1}{3} m \frac{\mu_0 M_s}{kT}$$

<u>A.N.</u> : $m = \mu_B = 9,27 \text{ x } 10^{-24} \text{ Am}^2$; $\mu_0 = 4 \text{ p x } 10^{-7} \frac{Tm}{A}$

$$M_s = 5,58 \; 10^5 \frac{A}{m} \; ; k = 1,38 \text{ x} 10^{-23} \frac{J}{K} \; ; T = 300K$$

$$X_m = \frac{1}{3} \frac{(9,27 \text{ x} 10^{-24})(4\pi 10^{-7})(5,58 \text{x } 10^5)}{(1,38 \text{ x } 10^{-23})(300)}$$
$$X_m = 5,23 \text{ x } 10^{-4}$$

Solution of exercise 26

1- Coercive field

The coercive field and coercive force is the applied magnetic field necessary for reducing the B field to zero along the hysteresis curve.

2- Calculation

$$B_{coercive} = \mu_0 \, n \, I$$

<u>A.N.</u> : $\mu_0 = 4 \text{ p x } 10^{-7} \frac{Tm}{A}$; $n = \frac{600}{0,15 \text{ m}}$; $I = 11A$

$$B_{coercive} = (4 \text{ p x } 10^{-7}) \left(\frac{600}{0,15}\right) (11)$$

$$B_{coercitif} = 5,53 \text{ x } 10^{-2} \text{ T}$$

Solution of exercise 27

The overall field is :

$B = B_{app} + \mu_0 M$ (1) , B_{app} being the applied field

1- Applied field

$B_{app} = \mu_0 n I$

A.N.: $\mu_0 = 4 p \times 10^{-7} \dfrac{Tm}{A}$; n = 5000 turns by m; I = 0,2A

$B_{app} = (4 p \times 10^{-7}) (5000) (0,2)$

$B_{app} = 1,26 \times 10^{-3} T$

2- Magnetization M

Relation (1) gives :

$M = \dfrac{B - B_{app}}{\mu_0}$

A.N.: B = 1,58 T; $B_{app} = 1,26 \times 10^{-3}$ T ; $\mu_0 = 4 p \times 10^{-7} \dfrac{Tm}{A}$

$M = \dfrac{1,58 - (1,26 \times 10^{-3})}{4 \pi \times 10^{-7}}$

$M = 1,26 \times 10^6 \dfrac{A}{m}$

3- Relative permeability

$\mu_r = \dfrac{B}{B_{app}}$

$$\mu_r = \frac{1{,}58\ T}{1{,}26 \times 10^{-3}T}\quad \text{in le tableau c}$$

$$\mu_r = 1254$$

Solution of exercise 28

Calculate $\mu = \dfrac{B}{H}$, $\mu_0 H$, $\mu_0 M = B - \mu_0 H$ et $K_m = \dfrac{\mu}{\mu_0}$ and place their values in the following table.

H (Am⁻¹)	B (T)	$\mu = \dfrac{B}{H}$ (TA⁻¹m)	$\mu_0 H$(T)	$\mu_0 M = B - \mu_0 H$ (T)	$K_m = \dfrac{\mu}{\mu_0}$
0	0				
10	0,050	0,005	$1{,}256 \times 10^{-5}$	0,04998	$3{,}98 \times 10^3$
20	0,15	0,0075	$2{,}512 \times 10^{-5}$	0,14997	$5{,}971 \times 10^3$
40	0,43	0,01075	$5{,}024 \times 10^{-5}$	0,42995	$8{,}558 \times 10^3$
50	0,54	0,0108	$6{,}28 \times 10^{-5}$	0,53993	$8{,}598 \times 10^3$
60	0,62	0,01033	$7{,}536 \times 10^{-5}$	0,61992	$8{,}224 \times 10^3$
80	0,74	0,00925	$10{,}048 \times 10^{-5}$	0,73989	$7{,}364 \times 10^3$
100	0,83	0,0083	$12{,}56 \times 10^{-5}$	0,82987	$6{,}608 \times 10^3$
150	0,98	0,00653	$18{,}84 \times 10^{-5}$	0,97981	$5{,}199 \times 10^3$
200	1,07	0,00535	$25{,}12 \times 10^{-5}$	1,06975	$4{,}259 \times 10^3$
500	1,27	0,00254	$62{,}80 \times 10^{-5}$	1,26937	$2{,}022 \times 10^3$
1000	1,34	0,00134	$125{,}60 \times 10^{-5}$	1,3387	$1{,}066 \times 10^3$
10 000	1,65	0,000165	1256×10^{-5}	1,63744	$1{,}31 \times 10^2$
100 000	2,02	$2{,}02 \times 10^{-5}$	12560×10^{-5}	1,8944	$1{,}6 \times 10^1$
800 000	2,92	$3{,}6 \times 10^{-6}$	100480×10^{-5}	1,9152	$2{,}87 \times 10^1$

Comments

• B = f (H)

The curve B = f (H) shows a nonlinear variation of B according to the magnetic Intensity (or magnetic stimulation) H. There has been a rapid growth of B and early growth becomes increasingly slow as the ferromagnetic substance magnetizes.
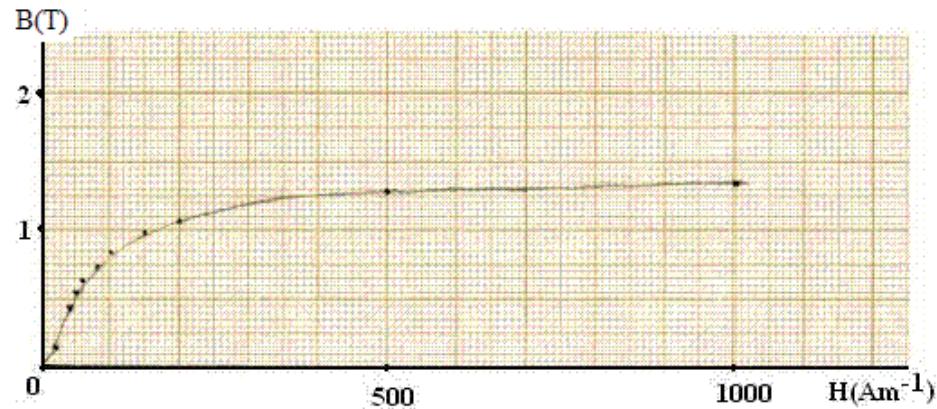
- $\mu_0 M = f(H)$

$\mu_0 M$ also increases rapidly at first, then more slowly and tends towards a maximum value which corresponds to magnetic saturation.
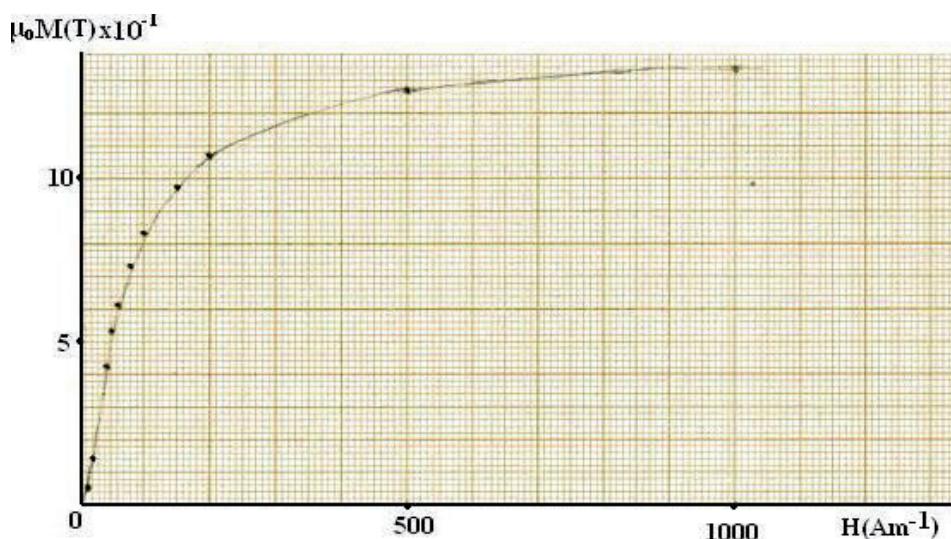
- $\mu = f(H)$

The permeability μ is not constant. When H increases, μ increases from an initial value of about $3 \times 10^{-3} TA^{-1}m$, then reaches a maximum equal to $10.8 \times 10^{-3} TA^{-1}m$ then decreases and tends towards a value close to $10^{-3} TA^{-1}m$.

$\mu(T A^{-1} m) x 10^{-3}$
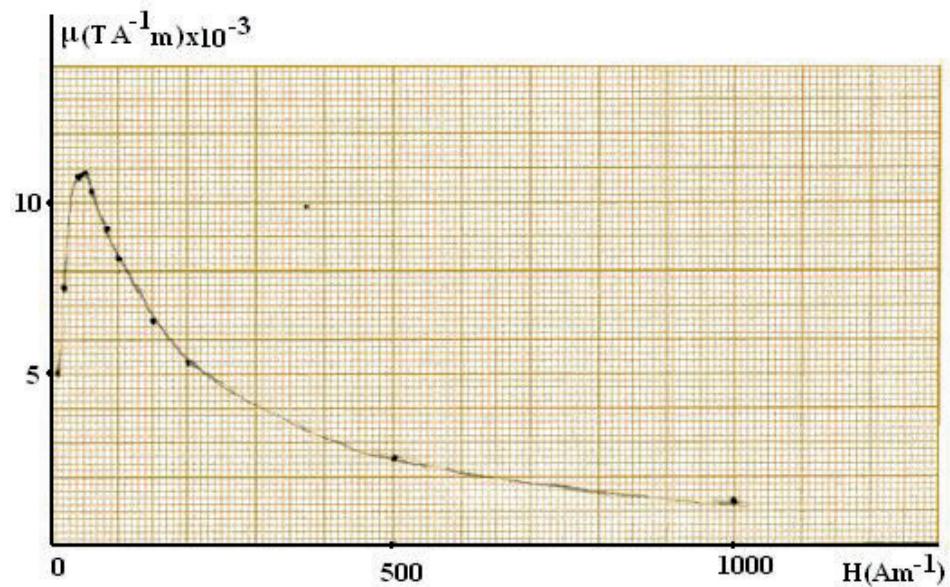
• $K_m = f(H)$

$K_m = \dfrac{\mu}{\mu_0}$ and μ_0 stay equal at $4 p \times 10^{-7} \dfrac{Tm}{A}$ and $K_m = f(H)$ et $\mu = f(H)$ have the same shape.

$(K_m = \dfrac{\mu}{\mu_0}) x 10^{3}$
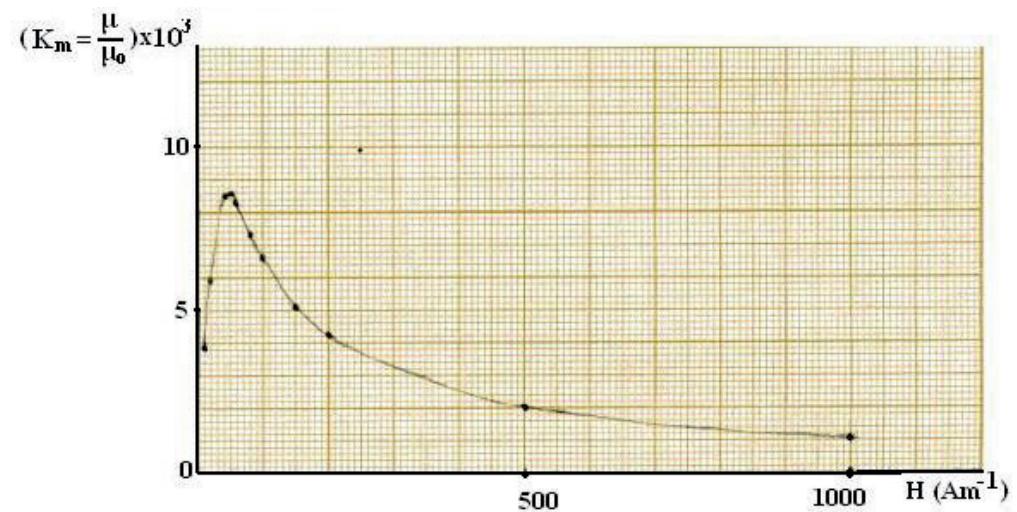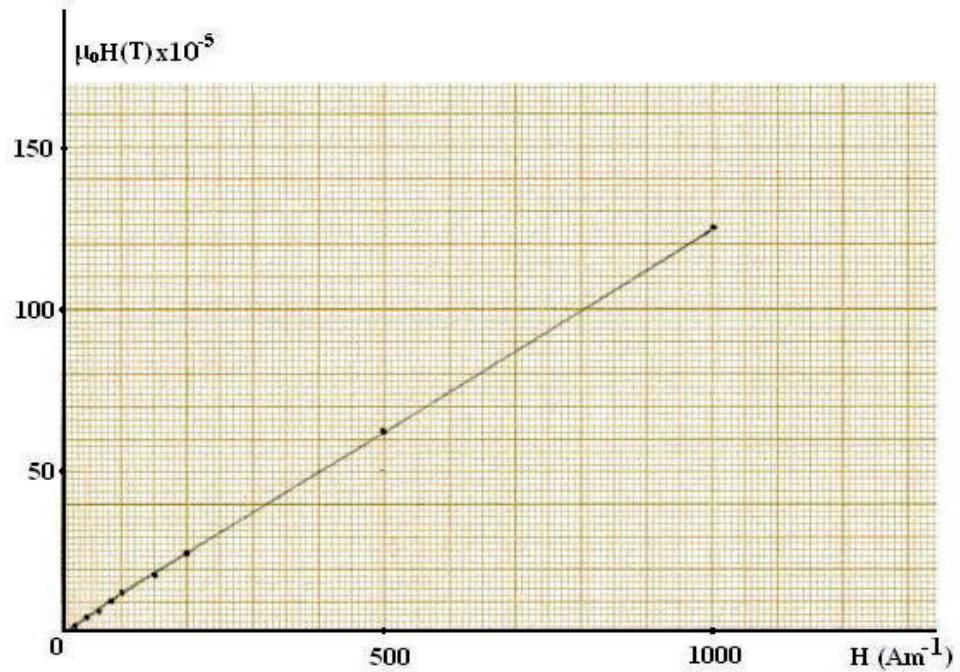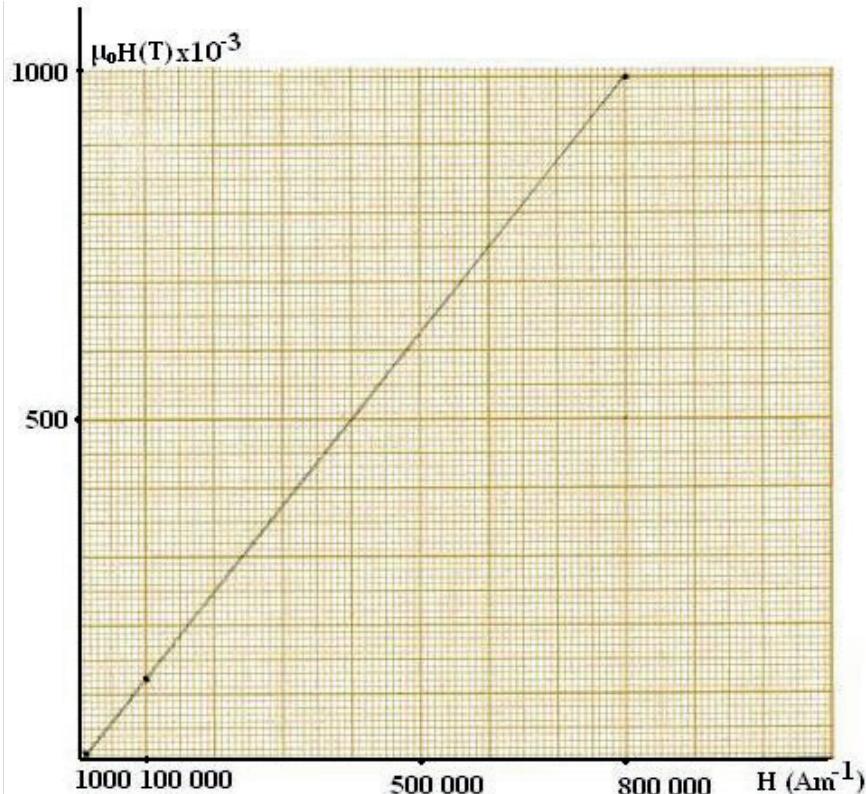
Solution of exercise 29

Field $\vec{B}$

$$\vec{B} = \frac{\vec{k} \wedge \vec{E}}{\omega}$$

$\vec{k}$ is the wave vector and defines the direction of wave propagation. Here we have a wave that propagates along the axis x, then we have $\vec{k} = k \, \vec{e}_x$, $\vec{e}_x$ being the unit vector following the x-axis. We have :

$$\vec{B} = \frac{\vec{k} \wedge \vec{E}}{\omega} = \frac{k}{\omega} \, \vec{e}_x \wedge \vec{E}$$

But $\dfrac{k}{\omega} = \dfrac{1}{c}$ where $\vec{B} = \dfrac{\vec{e}_x \wedge \vec{E}}{c}$. Determine $\vec{e}_x \wedge \vec{E}$:

$$\vec{e_x} \wedge \vec{E} = \vec{e_x} \wedge \left[E_0 \sin(kx - \omega t) \vec{e_y} + E_0 \cos(kx - \omega t) \vec{e_z} \right]$$

$$= \vec{e_z} E_0 \sin(kx - \omega t) - \vec{e_y} E_0 \cos(kx - \omega t)$$

$$= (\vec{e_x} \wedge \vec{e_y}) E_0 \sin(kx - \omega t) + (\vec{e_x} \wedge \vec{e_z}) E_0 \cos(kx - \omega t)$$

We obtain :

$$\vec{B}(x,t) = - \frac{E_0}{c} \cos(kx - \omega t) \vec{e_y} + \frac{E_0}{c} \sin(kx - \omega t) \vec{e_z}$$

Evolution of $\vec{E}(0,t)$ over time

We have :

$$\vec{E}(0,t) = - E_0 \sin(\omega t) \vec{e_y} + E_0 \cos(\omega t) \vec{e_z}$$

$$E_y(0,t) = - E_0 \sin(\omega t) = - E_0 \sin 2\pi \, ft$$

$$E_z(0,t) = E_0 \cos(\omega t) = E_0 \cos 2\pi \, ft$$

f is the frequency of the wave. The evolutions of $E_y(0,t)$, $E_z(0,t)$ and $\vec{E}(0,t)$ are described below
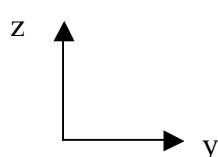
t	0	$\dfrac{1}{8f}$	$\dfrac{2}{8f}$	$\dfrac{3}{8f}$	$\dfrac{4}{8f}$	$\dfrac{5}{8f}$
$E_y(0,t)$	0	$-E_0\,\sin\dfrac{\pi}{4}$	$-E_0$	$-E_0\,\sin\dfrac{3\pi}{4}$	0	$-E_0\,\sin\dfrac{5\pi}{4}$
$E_z(0,t)$	E_0	$E_0\,\cos\dfrac{\pi}{4}$	0	$E_0\,\cos\dfrac{3\pi}{4}$	$-E_0$	$E_0\,\cos\dfrac{5\pi}{4}$
$\vec{E}\,(0,t)$						

We see that $\vec{E}\,(0,t)$ rotates counterclockwise and its amplitude is constant. We can verify that $E_y^2 + E_z^2 = E_0^2$. It is the equation of a circle with radius E_0.

Evolution of $\vec{B}\,(0,t)$ with time

$\vec{B}\,(x,t) = -\dfrac{E_0}{c}\cos(kx - \omega t)\,\vec{e}_y + \dfrac{E_0}{c}\sin(kx - \omega t)\,\vec{e}_z$. Let $B_0 = \dfrac{E_0}{c}$. We have:

$$\vec{B}\,(x,t) = -B_0\cos(kx - \omega t)\,\vec{e}_y + B_0\sin(kx - \omega t)\,\vec{e}_z$$

$$\vec{B}\,(0,t) = -B_0\cos\omega t\,\vec{e}_y + B_0\sin\omega t\,\vec{e}_z$$

$$B_y\,(0,t) = -B_0\cos\omega t = -B_0\cos 2\pi\,ft$$

$$B_z\,(0,t) = B_0\sin\omega t = B_0\sin 2\pi\,ft$$

$$z$$

$$y$$
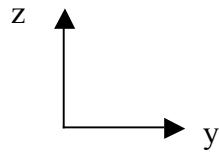
t	0	$\dfrac{1}{8f}$	$\dfrac{2}{8f}$	$\dfrac{3}{8f}$	$\dfrac{4}{8f}$	$\dfrac{5}{8f}$
$B_y(0,t)$	$-B_0$	$-B_0 \cos\dfrac{\pi}{4}$	0	$-B_0 \cos\dfrac{3\pi}{4}$	B_0	$-B_0 \cos\dfrac{5\pi}{4}$
$B_z(0,t)$	0	$B_0 \sin\dfrac{\pi}{4}$	B_0	$B_0 \sin\dfrac{3\pi}{4}$	0	$B_0 \sin\dfrac{5\pi}{4}$
$\vec{B}(0,t)$	←	↖	↑	↗	→	↘

$\vec{B}(0,t)$ turns counterclockwise with constant amplitude. We can verify that

$B_y^2 + B_z^2 = B_0^2$. It is the equation of a circle with radius B_0

The electromagnetic wave has circular polarization. For a given value of x vectors $\vec{E}$

and $\vec{B}$ turn a circle in a plane perpendicular to the direction of propagation which

is the x axis. $\vec{E}$ is circularly polarized to the left, and $\vec{B}$ circularly to the right.

Poynting vector related to this wave

$$\vec{S} = \dfrac{\vec{E} \wedge \vec{B}}{\mu_0}$$

Determine $\vec{E} \wedge \vec{B}$:

$$\vec{E} \wedge \vec{B} = \left[\vec{e}_y\, E_0 \sin(kx - \omega t) + \vec{e}_z\, E_0 \cos(kx - \omega t)\right] \wedge \left[-\vec{e}_y\, B_0 \cos(kx - \omega t) + \vec{e}_z\, B_0 \sin(kx - \omega t)\right]$$

Recall that :

$$\vec{e}_y \wedge \vec{e}_y = \vec{e}_z \wedge \vec{e}_z = \vec{0}$$

$$\vec{e}_y \wedge \vec{e}_z = \vec{e}_x$$

$$\vec{e}_z \wedge \vec{e}_y = -\vec{e}_x$$

We obtain :

$$\vec{E} \wedge \vec{B} = \vec{e}_x E_0 B_0 \sin^2(kx - \omega t) + \vec{e}_x E_0 B_0 \cos^2(kx - \omega t) = \vec{e}_x E_0 B_0$$

The Poynting vector is :

$$\vec{S} = \frac{\vec{E} \wedge \vec{B}}{\mu_0} = \frac{E_0 B_0}{\mu_0} \vec{e}_x = \frac{E_0^2}{\mu_0 c} \vec{e}_x$$

The Poynting vector has the direction of the propagation of the wave

Other method

We can also start with the following relations :

$$\vec{S} = \frac{\vec{E} \wedge \vec{B}}{\mu_0} \text{ with } \vec{B} = \frac{\vec{k} \wedge \vec{E}}{\omega} = \frac{k}{\omega} \vec{e}_x \wedge \vec{E} = \frac{1}{c} \vec{e}_x \wedge \vec{E}$$

We have :

$$\vec{S} = \frac{1}{c \mu_0} \vec{E} \wedge (\vec{e}_x \wedge \vec{E})$$

By using $\vec{a} \wedge (\vec{b} \wedge \vec{c}) = (\vec{a} \cdot \vec{c}) \vec{b} - (\vec{a} \cdot \vec{b}) \vec{c}$, We obtain:

$$\vec{S} = \frac{E_0^2}{\mu_0\, c}\ \vec{e}_x$$

Solution of exercise 30

Poynting vector :

$$\vec{S} = \frac{\vec{E}\ \wedge\ \vec{B}}{\mu_0} = \frac{1}{\mu_0}\ \vec{E}_0\ \wedge\ \vec{B}_0\ \cos^2(\vec{k}\cdot\vec{r}\ -\ \omega t)$$

But $\vec{E}_0\ \perp\ \vec{B}_0$ et $E_0 = B_0\, c$. We have:

$$\left|\vec{S}\right| = S = \frac{1}{\mu_0}\ \left|\vec{E}_0\ \wedge\ \vec{B}_0\right|\ \cos^2(\vec{k}\cdot\vec{r}\ -\ \omega t)$$

$$S = \frac{1}{\mu_0}\ E_0 B_0\ \cos^2(\vec{k}\cdot\vec{r}\ -\ \omega t) = \frac{1}{\mu_0\, c}\ E_0^2\ \cos^2(\vec{k}\cdot\vec{r}\ -\ \omega t)$$

The intensity I of the wave is the average value of $\left|\vec{S}\right|$. Consider $< S >$ this mean value. We have:

$$I = <S> = \frac{1}{\mu_0\, c}\ E_0^2\ <\cos^2(\vec{k}\cdot\vec{r}\ -\ \omega t)>$$

The mean value $<\cos^2(\vec{k}\cdot\vec{r}\ -\ \omega t)>$ est $<\cos^2(\vec{k}\cdot\vec{r}\ -\ \omega t)> = \frac{1}{2}$. We obtain:

$$I = <S> = \frac{1}{2\,\mu_0\, c}\ E_0^2$$

$c = \dfrac{1}{\sqrt{\varepsilon_0\,\mu_0}}$, this intensity can also be written as :

$$I = <S> = \frac{1}{2\,\mu_0\,c}\,E_0^2 = \frac{c}{2\,\mu_0\,c^2}\,E_0^2 = \frac{c\,E_0^2}{2\,\mu_0}\,(\mu_0\,\varepsilon_0) = \frac{1}{2}\,c\,\varepsilon_0\,E_0^2$$

Solution of exercise 31

Intensity of the wave

Consider $\vec{S}$ as the Poynting vector. Let $\left|\vec{S}\right| = S$. The intensity I is the mean value $<S>$ of S. But S is the power P of the wave per unit area. We have $I = \dfrac{P}{A}$; A is the area of a sphere with the center as a light source and radius r as the distance between the point M of this source. We obtain:

$$I = \frac{P}{A} = \frac{P}{4\,\pi\,r^2}$$

Numerical application : $P = 30W$; $r = 2m$

$$I = \frac{P}{A} = \frac{30W}{4\,\pi\,(2m)^2}$$

$$I = 0{,}6\,\frac{W}{m^2}$$

Radiation pressure

Consider P_r as this pressure :

$$P_r = \frac{I}{c}$$

Numerical application : $I = 0,6 \dfrac{W}{m^2}$; $c = 3 \times 10^8 \dfrac{m}{s}$

$$P_r = \dfrac{0,6}{3 \times 10^8}$$

$$P_r = 0,2 \times 10^{-8} \text{ Pa}$$

Amplitude of the electric field

Designate E_0 as this amplitude. We have :

$$I = \dfrac{1}{2} c \varepsilon_0 E_0^2 \Rightarrow P_r = \dfrac{I}{c} = \dfrac{1}{2} \varepsilon_0 E_0^2$$

It follows that :

$$E_0 = \sqrt{\dfrac{2 P_r}{\varepsilon_0}}$$

Numerical application : $P_r = 0,2 \times 10^{-8}$ Pa ; $\varepsilon_0 = 8,85 \times 10^{-12}$ C^2 N^{-1} m^{-2}

$$E_0 = \sqrt{\dfrac{2 \,(0,2 \times 10^{-8})}{8,85 \times 10^{-12}}}$$

$$E_0 = 21,26 \dfrac{V}{m}$$

Amplitude of the magnetic field

Consider B_0 as this amplitude. We deduce B_0 from the relation $E_0 = B_0 c$:

$$B_0 = \dfrac{E_0}{c}$$

Numerical application : $E_0 = 21{,}26 \dfrac{V}{m}$; $c = 3 \times 10^8 \dfrac{m}{s}$

$B_0 = \dfrac{21{,}26}{3 \times 10^8}$

$B_0 = 7{,}09 \times 10^{-8}$ T

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ELECTRICITY AND MAGNETISM

Required Readings

Source: Wikipedia.org

Table of Contents

Lenz's law

Lenz's law (pronounced /ˈlɛntsɪz ˌlɔː/) is an extension of the law of conservation of energy to the non-conservative forces in electromagnetic induction. It can be used to give the direction of the induced electromotive force (emf) and current resulting from electromagnetic induction. Heinrich Lenz postulated the following law;

"An induced current is always in such a direction as to oppose the motion or change causing it"

The law provides a physical interpretation of the choice of sign in Faraday's law of induction, indicating that the induced emf and the change in flux have opposite signs.

[] Connection with law of conservation of energy

The law of conservation of energy relates exclusively to irrotational (conservative) forces. Lenz's Law extends the principles of energy conservation to situations that involve non-conservative forces in electromagnetism. To see an example, move a magnet towards the face of a closed loop of wire (e.g. a coil or solenoid). An electric current is induced in the wire, because the electrons within it are subjected to an increasing magnetic field as the magnet approaches. This produces an EMF (electro-motive force) that acts upon them. The direction of the induced current depends on whether the north or south pole of the magnet is approaching: an approaching north pole will produce a counter-clockwise current (from the perspective of the magnet), and south pole approaching the coil will produce a clockwise current.[need citation]

To understand the implications for conservation of energy, suppose that the induced currents' directions were opposite to those just described. Then the north pole of an approaching magnet would induce a south pole in the near face of the loop. The attractive force between these poles would accelerate the magnet's approach. This would make the magnetic field increase more quickly, which in turn would increase the loop's current, strengthening the magnetic field, increasing the attraction and acceleration, and so on. Both the kinetic energy of the magnet and the rate of energy dissipation in the loop (due to Joule heating) would increase. A small energy input would produce a large energy output, violating the law of conservation of energy.[need citation]

This scenario is only one example of electromagnetic induction. Lenz's Law states that the magnetic field of any induced current opposes the change that induces it.

For a rigorous mathematical treatment, see electromagnetic induction and Maxwell's equations.

Hall effect
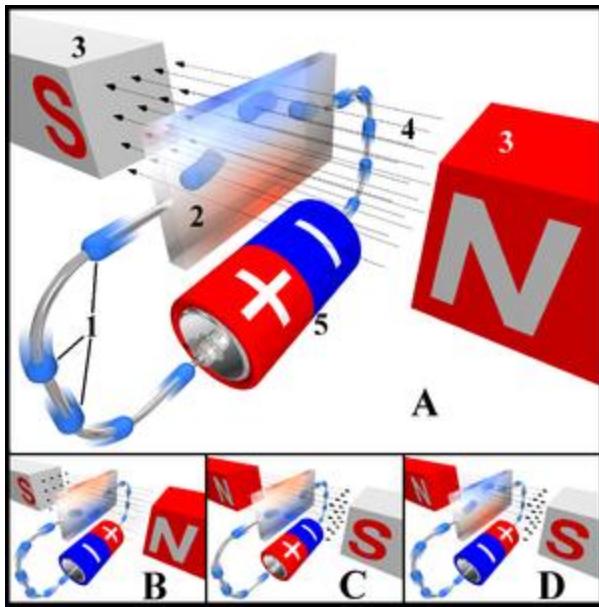
Hall effect diagram, showing electron flow (rather than conventional current).
Legend:
1. Electrons (not conventional current!)
2. Hall element, or Hall sensor
3. Magnets
4. Magnetic field
5. Power source
Description
In drawing "A", the Hall element takes on a negative charge at the top edge (symbolised by the blue color) and positive at the lower edge (red color). In "B" and "C", either the electric current or the magnetic field is reversed, causing the polarization to reverse. Reversing both current and magnetic field (drawing "D") causes the Hall element to again assume a negative charge at the upper edge.

The **Hall effect** is the production of a voltage difference (the **Hall voltage**) across an electrical conductor, transverse to an electric current in the conductor and a magnetic field perpendicular to the current. Edwin Hall discovered this effect in 1879.[1]

The Hall coefficient is defined as the ratio of the induced electric field to the product of the current density and the applied magnetic field. It is a characteristic of the material from which the conductor is made, as its value depends on the type, number and properties of the charge carriers that constitute the current.

[] Discovery

The Hall effect was discovered in 1879 by Edwin Herbert Hall while working on his doctoral degree at the Johns Hopkins University in Baltimore, Maryland, USA. His measurements of the tiny effect produced in the apparatus he used was an experimental tour de force, accomplished 18 years before the electron was discovered.

[] Theory

The Hall effect comes about due to the nature of the current in a conductor. Current consists of the movement of many small charge carriers, typically electrons, holes, or both. Moving charges experience a force, called the Lorentz Force, when a magnetic field is present that is not parallel to their motion.[2] When such a magnetic field is absent, the charges follow an approximately straight, 'line of sight' path. However, when a perpendicular magnetic field is applied, their path is curved so that moving charges accumulate on one face of the material. This leaves equal and opposite charges exposed on the other face, where there is a scarcity of mobile charges. The result is an asymmetric distribution of charge density across the Hall element that is perpendicular to both the 'line of sight' path and the applied magnetic field. The separation of charge establishes an electric field that opposes the migration of further charge, so a steady electrical potential builds up for as long as the charge is flowing.
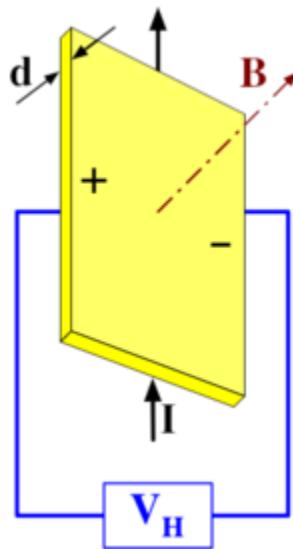
For a simple metal where there is only one type of charge carrier (electrons) the Hall voltage V_H is given by

$$V_H = \frac{-IB}{dne}$$

where I is the current across the plate length, B is the magnetic flux density, d is the depth of the plate, e is the electron charge, and n is the charge carrier density of the carrier electrons.

The Hall coefficient is defined as

$$R_H = \frac{E_y}{j_x B}$$

where *j* is the current density of the carrier electrons. In SI units, this becomes

$$R_H = \frac{E_y}{j_x B} = \frac{dV_H}{IB} = -\frac{1}{ne}$$

As a result, the Hall effect is very useful as a means to measure either the carrier density or the magnetic field.

One very important feature of the Hall effect is that it differentiates between positive charges moving in one direction and negative charges moving in the opposite. The Hall effect offered the first real proof that electric currents in metals are carried by moving electrons, not by protons. The Hall effect also showed that in some substances (especially p-type semiconductors), it is more appropriate to think of the current as positive "holes" moving rather than negative electrons. A common source of confusion with the Hall Effect is that holes moving to the left are really electrons moving to the right, so one expects the same sign of the Hall coefficient for both electrons and holes. However, the actual electrons that correspond to the fictional "holes" have negative effective mass, and thus are deflected in the opposite direction [3]

It must be noted though that the sample inhomogeneity might result in spurious sign of the Hall effect, even in ideal van der Pauw configuration of electrodes. For example, positive Hall effect was observed in evidently n-type semiconductors.[4]

[] Hall effect in semiconductors

When a current-carrying semiconductor is kept in a magnetic field, the charge carriers of the semiconductor experience a force in a direction perpendicular to both the magnetic field and the current. At equilibrium, a voltage appears at the semiconductor edges.

The simple formula for the Hall coefficient given above becomes more complex in semiconductors where the carriers are generally both electrons and holes which may be present in different concentrations and have different mobilities. For moderate magnetic fields the Hall coefficient is[5]

$$R_H = \frac{-n\mu_e^2 + p\mu_h^2}{e(n\mu_e + p\mu_h)^2}$$

where n is the electron concentration, p the hole concentration, μ_e the electron mobility , μ_h the hole mobility and e the absolute value of the electronic charge.

For large applied fields the simpler expression analogous to that for a single carrier type holds.

$$R_H = \frac{(p - nb^2)}{e(p + nb)^2}$$

with $\quad b = \dfrac{\mu_e}{\mu_h}$

[] Technological applications

Hall effect sensors are readily available from a number of different manufacturers, and may be used in various sensors such as rotating speed sensors, fluid flow sensors, current sensors, and pressure sensors. Other applications may be found in some electric airsoft guns and on the triggers of electropneumatic paintball guns, as well as current smart phones, and some global positioning systems.

[] Quantum Hall effect

Main article: Quantum Hall effect

For a two dimensional electron system which can be produced in a MOSFET. In the presence of large magnetic field strength and low temperature, one can observe the quantum Hall effect, which is the quantization of the Hall voltage.

[] Spin Hall effect

Main article: Spin Hall effect

The spin Hall effect consists in the spin accumulation on the lateral boundaries of a current-carrying sample. No magnetic field is needed. It was predicted by M.I. Dyakonov and V.I. Perel in 1971 and observed experimentally more than 30 years later, both in semiconductors and in metals, at cryogenic as well as at room temperatures.

[] Quantum spin Hall effect

Main article: Quantum Spin Hall Effect

For HgTe two dimensional quantum wells with strong spin-orbit coupling, in zero magnetic field, at low temperature, the Quantum Spin Hall effect has been recently observed.

[] Anomalous Hall effect

In ferromagnetic materials (and paramagnetic materials in a magnetic field), the Hall resistivity includes an additional contribution, known as the **anomalous Hall effect** (or the **extraordinary Hall effect**), which depends directly on the magnetization of the material, and is often much larger than the ordinary Hall effect. (Note that this effect is *not* due to the contribution of the

magnetization to the total magnetic field.) Although a well-recognized phenomenon, there is still debate about its origins in the various materials. The anomalous Hall effect can be either an *extrinsic* (disorder-related) effect due to spin-dependent scattering of the charge carriers, or an *intrinsic* effect which can be described in terms of the Berry phase effect in the crystal momentum space (*k*-space).[6]

[] Hall effect in ionized gases

(See electrothermal instability)

The Hall effect in an ionized gas (plasma) is significantly different from the Hall effect in solids (where the **Hall parameter** is always very inferior to unity). In a plasma, the Hall parameter can take any value. The Hall parameter, *β*, in a plasma is the ratio between the electron gyrofrequency, Ω_e, and the electron-heavy particle collision frequency, *v*:

$$\beta = \frac{\Omega_e}{\nu} = \frac{eB}{m_e\nu}$$

where

> *e* is the elementary charge (approx. 1.6×10^{-19} C)
>
> *B* is the magnetic field (in teslas)
>
> m_e is the electron mass (approx. 9.1×10^{-31} kg).

The Hall parameter value increases with the magnetic field strength.

Physically, the trajectories of electrons are curved by the Lorentz force. Nevertheless when the Hall parameter is low, their motion between two encounters with heavy particles (neutral or ion) is almost linear. But if the Hall parameter is high, the electron movements are highly curved. The current density vector, *J*, is no more colinear with the electric field vector, *E*. The two vectors *J* and *E* make the **Hall angle**, *θ*, which also gives the Hall parameter:

> β = tan(θ)

[] Applications

Hall effect devices produce a very low signal level and thus require amplification. While suitable for laboratory instruments, the vacuum tube amplifiers available in the first half of the 20th century were too expensive, power consuming, and unreliable for everyday applications. It was only with the development of the low cost integrated circuit that the Hall effect sensor became suitable for mass application. Many devices now sold as Hall effect sensors in fact contain both the sensor as described above plus a high gain integrated circuit (IC) amplifier in a single package. Recent advances have further added into one package an ADC (Analog to Digital)

converter and I²C (Inter-integrated circuit communication protocol) IC for direct connection to a microcontroller's I/O port.

Hall probes are often used to measure magnetic fields, or inspect materials (such as tubing or pipelines) using the principles of magnetic flux leakage.

[] Advantages over other methods

Hall effect devices when appropriately packaged are immune to dust, dirt, mud, and water. These characteristics make Hall effect devices better for position sensing than alternative means such as optical and electromechanical sensing.

Hall effect current sensor with internal integrated circuit amplifier. 8 mm opening. Zero current output voltage is midway between the supply voltages that maintain a 4 to 8 volt differential. Non-zero current response is proportional to the voltage supplied and is linear to 60 amperes for this particular (25 A) device.

When electrons flow through a conductor, a magnetic field is produced. Thus, it is possible to create a non-contacting current sensor. The device has three terminals. A sensor voltage is applied across two terminals and the third provides a voltage proportional to the current being sensed. This has several advantages; no additional resistance (a *shunt*, required for the most common current sensing method) need be inserted in the primary circuit. Also, the voltage present on the line to be sensed is not transmitted to the sensor, which enhances the safety of measuring equipment.

[] Ferrite toroid Hall effect current transducer

Hall sensors can detect stray magnetic fields easily, including that of Earth, so they work well as electronic compasses: but this also means that such stray fields can hinder accurate

measurements of small magnetic fields. To solve this problem, Hall sensors are often integrated with magnetic shielding of some kind. For example, a Hall sensor integrated into a ferrite ring (as shown) can reduce the detection of stray fields by a factor of 100 or better (as the external magnetic fields cancel across the ring, giving no residual magnetic flux). This configuration also provides an improvement in signal-to-noise ratio and drift effects of over 20 times that of a bare Hall device. The range of a given feedthrough sensor may be extended upward and downward by appropriate wiring. To extend the range to lower currents, multiple turns of the current-carrying wire may be made through the opening. To extend the range to higher currents, a current divider may be used. The divider splits the current across two wires of differing widths and the thinner wire, carrying a smaller proportion of the total current, passes through the sensor.

The principle of increasing the number of windings a conductor takes around the ferrite core is well understood, each turn having the effect of multiplying the current under measurement. Often these additional turns are carried out by a staple on the PCB. CZV

[] *Split ring clamp-on sensor*

A variation on the ring sensor uses a split sensor which is clamped onto the line enabling the device to be used in temporary test equipment. If used in a permanent installation, a split sensor allows the electrical current to be tested without dismantling the existing circuit.

[] Analog multiplication

The output is proportional to both the applied magnetic field and the applied sensor voltage. If the magnetic field is applied by a solenoid, the sensor output is proportional to product of the current through the solenoid and the sensor voltage. As most applications requiring computation are now performed by small (even tiny) digital computers, the remaining useful application is in power sensing, which combines current sensing with voltage sensing in a single Hall effect device.

[] Current sensing

By sensing the current provided to a load and using the device's applied voltage as a sensor voltage it is possible to determine the power dissipated by a device.

[] Position and motion sensing

Hall effect devices used in motion sensing and motion limit switches can offer enhanced reliability in extreme environments. As there are no moving parts involved within the sensor or magnet, typical life expectancy is improved compared to traditional electromechanical switches. Additionally, the sensor and magnet may be encapsulated in an appropriate protective material.

[] *Automotive ignition and fuel injection*

Commonly used in distributors for ignition timing (and in some types of crank and camshaft position sensors for injection pulse timing, speed sensing, etc.) the Hall effect sensor is used as a

direct replacement for the mechanical breaker points used in earlier automotive applications. Its use as an ignition timing device in various distributor types is as follows. A stationary permanent magnet and semiconductor Hall effect chip are mounted next to each other separated by an air gap, forming the Hall effect sensor. A metal rotor consisting of windows and tabs is mounted to a shaft and arranged so that during shaft rotation, the windows and tabs pass through the air gap between the permanent magnet and semiconductor Hall chip. This effectively shields and exposes the Hall chip to the permanent magnet's field respective to whether a tab or window is passing though the Hall sensor. For ignition timing purposes, the metal rotor will have a number of equal-sized tabs and windows matching the number of engine cylinders. This produces a uniform square wave output since the on/off (shielding and exposure) time is equal. This signal is used by the engine computer or ECU to control ignition timing. It is worth noting that many automotive Hall effect sensors have a built-in internal NPN transistor with an open collector and grounded emitter, meaning that rather than a voltage being produced at the Hall sensor signal output wire, the transistor is turned on providing a circuit to ground though the signal output wire.

[] Wheel rotation sensing

The sensing of wheel rotation is especially useful in anti-lock brake systems. The principles of such systems have been extended and refined to offer more than anti-skid functions, now providing extended vehicle handling enhancements.

[] Electric motor control

Some types of brushless DC electric motors use Hall effect sensors to detect the position of the rotor and feed that information to the motor controller. This allows for more precise motor control.

[] Industrial applications

Applications for Hall Effect sensing have also expanded to industrial applications, which now use Hall Effect Joysticks to control hydraulic valves, replacing the traditional mechanical levers. Such applications include; Mining Trucks, Backhoe Loaders, Cranes, Diggers, Scissor Lifts, etc.

[] Spacecraft propulsion

A Hall effect thruster (HET) is a relatively low power device that is used to propel some spacecraft, once they get into orbit or farther out into space. In the HET, atoms are ionized and accelerated by an electric field. A magnetic field is used to trap the ionized electrons. After the positive ions are accelerated by the electric field, trapped electrons are recaptured by the positive ions and result in fast neutral atoms exiting the HET and providing thrust. Neutralizing the ions eliminates the electrostatic attraction that positively charged ions would have had for the electrons that they left behind. Thus the neutral atoms propelled out the rear won't exert an attractive electrostatic force on the spacecraft, which would have partly canceled the forward propulsion.

[] The Corbino effect

The Corbino effect is a phenomenon similar to the Hall effect, but a disc-shaped metal sample is used in place of a rectangular one. A radial current through a circular disc subjected to a magnetic field perpendicular to the plane of the disc, produces a "circular" current through the disc.[7]

The absence of the free transverse boundaries render the interpretation of the Corbino effect simpler than that of the Hall effect.

Paramagnetism
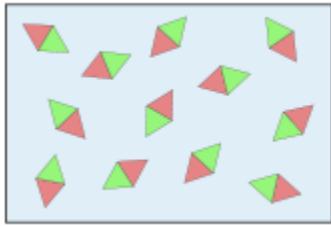
Simple Illustration of a paramagnetic probe made up from miniature magnets.

Paramagnetism is a form of magnetism which occurs only in the presence of an externally applied magnetic field. Paramagnetic materials are attracted to magnetic fields, hence have a relative magnetic permeability of ≥1 (1 or more, a positive magnetic susceptibility). The magnetic moment induced by the applied field is *linear* in the field strength and rather *weak*. It typically requires a sensitive analytical balance to detect the effect and modern measurements on paramagnetic materials are often conducted with a SQUID magnetometer. Unlike ferromagnets, paramagnets do not retain any magnetization in the absence of an externally applied magnetic field, because thermal motion causes the spins to become *randomly oriented* without it. Thus the total magnetization will drop to zero when the applied field is removed. Even in the presence of the field there is only a small *induced* magnetization because only a small fraction of the spins will be oriented by the field. This fraction is proportional to the field strength and this explains the linear dependency. The attraction experienced by ferromagnets is non-linear and much stronger, so that it is easily observed, for instance, in magnets on one's refrigerator.

[] Relation to electron spins

Constituent atoms or molecules of paramagnetic materials have permanent magnetic moments (dipoles), even in the absence of an applied field. This generally occurs due to the spin of unpaired electrons in the atomic/molecular electron orbitals (see Magnetic moment). In pure paramagnetism, the dipoles do not interact with one another and are randomly oriented in the

absence of an external field due to thermal agitation, resulting in zero net magnetic moment. When a magnetic field is applied, the dipoles will tend to align with the applied field, resulting in a net magnetic moment in the direction of the applied field. In the classical description, this alignment can be understood to occur due to a torque being provided on the magnetic moments by an applied field, which tries to align the dipoles parallel to the applied field. However, the truer origins of the alignment can only be understood via the quantum-mechanical properties of spin and angular momentum.

If there is sufficient energy exchange between neighbouring dipoles they will interact, and may spontaneously align or anti-align and form magnetic domains, resulting in ferromagnetism (permanent magnets) or antiferromagnetism, respectively. Paramagnetic behavior can also be observed in ferromagnetic materials that are above their Curie temperature, and in antiferromagnets above their Néel temperature. At these temperatures the available thermal energy simply overcomes the interaction energy between the spins.

In general paramagnetic effects are quite small: the magnetic susceptibility is of the order of 10^{-3} to 10^{-5} for most paramagnets, but may be as high as 10^{-1} for synthetic paramagnets such as ferrofluids.

[] Delocalization

Selected Pauli-paramagnetic metals[1]

Material ⊠	Magnetic susceptibility ⊠
Tungsten	6.8
Cesium	5.1
Aluminium	2.2
Lithium	1.4
Magnesium	1.2

Sodium   0.72

In many metallic materials the electrons are itinerant, i.e. they travel through the solid more or less as an electron gas. This is the result of very strong interactions (overlap) between the wave functions of neighboring atoms in the extended lattice structure. The wave functions of the valence electrons thus form a band with equal numbers of spins up and down. When exposed to an external field only those electrons close to the Fermi level will respond and a small surplus of one type of spins will result. This effect is a weak form of paramagnetism known as Pauli-paramagnetism. The effect always competes with a diamagnetic response of opposite sign due to all the core electrons of the atoms. Stronger forms of magnetism usually require localized rather than itinerant electrons. However in some cases a bandstructure can result in which there are two delocalized subbands with states of opposite spins that have different energies. If one subband is preferentially filled over the other, one can have itinerant ferromagnetic order. This usually only happens in relatively narrow (d-)bands, which are poorly delocalized.

[] s and p electrons

In general one can say that strong delocalization in a solid due to large overlap with neighboring wave functions tends to lead to pairing of spins (*quenching*) and thus weak magnetism. This is why s- and p-type metals are typically either Pauli-paramagnetic or as in the case of gold even diamagnetic. In the latter case the diamagnetic contribution from the closed shell inner electrons simply wins from the weak paramagnetic term of the almost free electrons.

[] d and f electrons

Stronger magnetic effects are typically only observed when d- or f-electrons are involved. Particularly the latter are usually strongly localized. Moreover the size of the magnetic moment on a lanthanide atom can be quite large as it can carry up to 7 unpaired electrons. This is one reason why superstrong magnets are typically based on lanthanide elements like neodymium or samarium.

[] Molecular localization

Of course the above picture is a *generalization* as it pertains to materials with an extended lattice rather than a molecular structure. Molecular structure can also lead to localization of electrons. Although there are usually energetic reasons why a molecular structure results such that it does not exhibit partly filled orbitals (i.e. unpaired spins), some non-closed shell moieties do occur in nature. Molecular oxygen is a good example. Even in the frozen solid it contains di-radical molecules resulting in paramagnetic behavior. The unpaired spins reside in orbitals derived from oxygen p wave functions, but the overlap is limited to the one neighbor in the O_2 molecules. The distances to other oxygen atoms in the lattice remain too large to lead to delocalization and the magnetic moments remain unpaired.

[] Curie's law

For low levels of magnetization, the magnetization of paramagnets follows Curie's law to good approximation:

$$M = \chi \cdot H = C \cdot \frac{H}{T}$$

where

> **M** is the resulting magnetization
>
> *χ* is the magnetic susceptibility
>
> **H** is the auxiliary magnetic field, measured in amperes/meter
>
> *T* is absolute temperature, measured in kelvins
>
> *C* is a material-specific Curie constant

This law indicates that the susceptibility χ of paramagnetic materials is inversely proportional to their temperature. Curie's law is valid under the commonly encountered conditions of low magnetization ($\mu_B H \lesssim k_B T$), but does not apply in the high-field/low-temperature regime where saturation of magnetization occurs ($\mu_B H \gtrsim k_B T$) and magnetic dipoles are all aligned with the applied field. When the dipoles are aligned, increasing the external field will not increase the total magnetization since there can be no further alignment.

For a paramagnetic ion with noninteracting magnetic moments with angular momentum J, the Curie constant is related the individual ions' magnetic moments,

$$C = \frac{N_A}{3k_B}\mu_{\text{eff}}^2 \text{ where } \mu_{\text{eff}} = g_J\sqrt{J(J+1)}\mu_B$$

.

The parameter μ_{eff} is interpreted as the effective magnetic moment per paramagnetic ion. If one uses a classical treatment with molecular magnetic moments represented as discrete magnetic dipoles, μ, a Curie Law expression of the same form will emerge with μ appearing in place of μ_{eff}.

[show]Click "show" to see a derivation of this law:

When orbital angular momentum contributions to the magnetic moment are small, as occurs for most organic radicals or for octahedral transition metal complexes with d^3 or high-spin d^5 configurations, the effective magnetic moment takes the form ($g_e = 2.0023... \approx 2$),

$$\mu_{\text{eff}} \simeq 2\sqrt{S(S+1)}\mu_B = \sqrt{n(n+2)}\mu_B$$, where n is the number of unpaired electrons. In other transition metal complexes this yields a useful, if somewhat cruder, estimate.

[] Examples of paramagnets

Materials that are called 'paramagnets', are most often those which exhibit, at least over an appreciable temperature range, magnetic susceptibilities that adhere to the Curie or Curie–Weiss laws. In principle any system that contains atoms, ions or molecules with unpaired spins can be called a paramagnet, but the interactions between them need to be carefully considered.

[] Systems with minimal interactions

The narrowest definition would be: a system with unpaired spins that *do not interact* with each other. In this narrowest sense, the only pure paramagnet is a dilute gas of monatomic hydrogen atoms. Each atom has one non-interacting unpaired electron. Of course, the latter could be said about a gas of lithium atoms but these already possess two paired core electrons that produce a diamagnetic response of opposite sign. Strictly speaking Li is a mixed system therefore, although admittedly the diamagnetic component is weak and often neglected. In the case of heavier elements the diamagnetic contribution becomes more important and in the case of metallic gold it dominates the properties. Of course, the element hydrogen is virtually never called 'paramagnetic' because the monatomic gas is stable only at extremely high temperature; H atoms combine to form molecular H_2 and in so doing, the magnetic moments are lost (*quenched*), because the spins pair. Hydrogen is therefore *diamagnetic* and the same holds true for most elements. Although the electronic configuration of the individual atoms (and ions) of most elements contain unpaired spins, it is not correct to call these elements 'paramagnets' because at ambient temperature quenching is very much the rule rather than the exception. However, the quenching tendency is weakest for f-electrons because *f* (especially 4*f*) orbitals are radially contracted and they overlap only weakly with orbitals on adjacent atoms. Consequently, the lanthanide elements with incompletely filled 4f-orbitals are paramagnetic or magnetically ordered.[2]

μ_{eff} values for typical d^3 and d^5 transition metal complexes.[3]

Material ✖	μ_{eff}/μ_B ✖
$[Cr(NH_3)_6]Br_3$	3.77
$K_3[Cr(CN)_6]$	3.87

$K_3[MoCl_6]$	3.79
$K_4[V(CN)_6]$	3.78
$[Mn(NH_3)_6]Cl_2$	5.92
$(NH_4)_2[Mn(SO_4)_2] \bullet 6H_2O$	5.92
$NH_4[Fe(SO_4)_2] \bullet 12H_2O$	5.89

Thus, condensed phase paramagnets are only possible if the interactions of the spins that lead either to quenching or to ordering are kept at bay by structural isolation of the magnetic centers. There are two classes of materials for which this holds:

1. Molecular materials with a (isolated) paramagnetic center.
    1. Good examples are coordination complexes of d- or f-metals or proteins with such centers, e.g. myoglobin. In such materials the organic part of the molecule acts as an envelope shielding the spins from their neighbors.
    2. Small molecules can be stable in radical form, oxygen O_2 is a good example. Such systems are quite rare because they tend to be rather reactive.
2. Dilute systems.
    1. Dissolving a paramagnetic species in a diamagnetic lattice at small concentrations, e.g. Nd^{3+} in $CaCl_2$ will separate the neodymium ions at large enough distances that they do not interact. Such systems are of prime importance for what can be considered the most sensitive method to study paramagnetic systems: EPR.

[] Systems with interactions
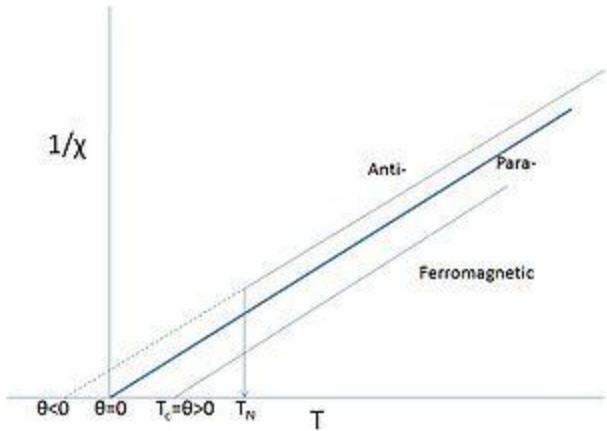
Idealized Curie–Weiss behavior; N.B. $T_C=\theta$, but T_N is not θ. Paramagnetic regimes are denoted by solid lines. Close to T_N or T_C the behavior usually deviates from ideal.

As stated above many materials that contain d- or f-elements do retain unquenched spins. Salts of such elements often show paramagnetic behavior but at low enough temperatures the magnetic moments may order. It is not uncommon to call such materials 'paramagnets', when referring to their paramagnetic behavior above their Curie or Néel-points, particularly if such temperatures are very low or have never been properly measured. Even for iron it is not uncommon to say that *iron becomes a paramagnet* above its relatively high Curie-point. In that case the Curie-point is seen as a phase transition between a ferromagnet and a 'paramagnet'. The word paramagnet now merely refers to the linear response of the system to an applied field, the temperature dependence of which requires an amended version of Curie's law, known as the Curie–Weiss law:

$$M = C\frac{H}{T-\theta}$$

This amended law includes a term θ that describes the exchange interaction that is present albeit overcome by thermal motion. The sign of θ depends on whether ferro- or antiferromagnetic interactions dominate and it is seldom exactly zero, except in the dilute, isolated cases mentioned above.

Obviously, the paramagnetic Curie–Weiss description above T_N or T_C is a rather different interpretation of the word 'paramagnet' as it does *not* imply the *absence* of interactions, but rather that the magnetic structure is random in the absence of an external field at these sufficiently high temperatures. Even if θ is close to zero this does not mean that there are no interactions, just that the aligning ferro- and the anti-aligning antiferromagnetic ones cancel. An additional complication is that the interactions are often different in different directions of the crystalline lattice (anisotropy), leading to complicated magnetic structures once ordered.

Randomness of the structure also applies to the many metals that show a net paramagnetic response over a broad temperature range. They do not follow a Curie type law as function of

temperature however, often they are more or less temperature independent. This type of behavior is of an itinerant nature and better called Pauli-paramagnetism, but it is not unusual to see e.g. the metal aluminium called a 'paramagnet', even though interactions are strong enough to give this element very good electrical conductivity.

[] Superparamagnets

Further information: Superparamagnetism

There are materials that show induced magnetic behavior that follows a Curie type law but with exceptionally large values for the Curie constants. These materials are known as superparamagnets. They are characterized by a strong ferro- or ferrimagnetic type of coupling into domains of a limited size that behave independently from one another. The bulk properties of such a system resembles that of a paramagnet, but on a microscopic level they are ordered. The materials do show an ordering temperature above which the behavior reverts to ordinary paramagnetism (with interaction). Ferrofluids are a good example, but the phenomenon can also occur inside solids, e.g. when dilute paramagnetic centers are introduced in a strong itinerant medium of ferromagnetic coupling such as when Fe is substituted in $TlCu_2Se_2$ or the alloy AuFe. Such systems contain ferromagnetically coupled clusters that freeze out at lower temperatures. They are also called mictomagnets.

Diamagnetism
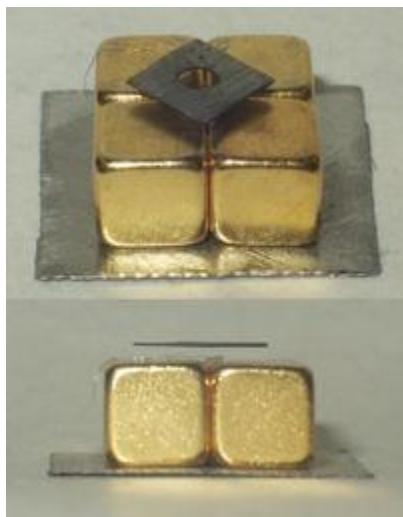
Levitating pyrolytic carbon

Diamagnetism is the property of an object which causes it to create a magnetic field in opposition of an externally applied magnetic field, thus causing a repulsive effect. Specifically, an external magnetic field alters the orbital velocity of electrons around their nuclei, thus changing the magnetic dipole moment in the direction opposing the external field. Diamagnets are materials with a magnetic permeability less than μ_0 (a relative permeability less than 1).

Consequently, diamagnetism is a form of magnetism that is only exhibited by a substance in the presence of an externally applied magnetic field. It is generally quite a weak effect in most materials, although superconductors exhibit a strong effect.

Diamagnetic materials cause lines of magnetic flux to curve away from the material, and superconductors can exclude them completely (except for a very thin layer at the surface).

[] Diamagnetic properties of materials

Notable diamagnetic materials[1]

Material	$\chi_m=K_m-1 \times 10^{-5}$
Bismuth	-16.6
Carbon (diamond)	-2.1
Carbon (graphite)	-1.6
Copper	-1.0
Lead	-1.8
Mercury	-2.9
Silver	-2.6
Water	-0.91
Superconductor	-10^5

Diamagnetism is a very general phenomenon, because all paired electrons, including the electrons of an atom, will always make a weak contribution to the material's response. However, for materials that show some other form of magnetism (such as ferromagnetism or paramagnetism), the diamagnetism is completely overpowered. Substances that mostly display diamagnetic behaviour are termed diamagnetic materials, or diamagnets. Materials that are said to be diamagnetic are those that are usually considered by non-physicists to be "non-magnetic", and include water, wood, most organic compounds such as petroleum and some plastics, and many metals including copper, particularly the heavy ones with many core electrons, such as mercury, gold and bismuth. The diamagnetism of various molecular fragments are called Pascal's constants.

Diamagnetic materials have a relative magnetic permeability that is less than 1, thus a magnetic susceptibility which is less than 0, and are therefore repelled by magnetic fields. However, since diamagnetism is such a weak property its effects are not observable in every-day life. For example, the magnetic susceptibility of diamagnets such as water is $\chi_v = -9.05 \times 10^{-6}$. The most strongly diamagnetic material is bismuth, $\chi_v = -1.66 \times 10^{-4}$, although pyrolytic graphite may have a susceptibility of $\chi_v = -4.00 \times 10^{-4}$ in one plane. Nevertheless, these values are orders of magnitudes smaller than the magnetism exhibited by paramagnets and ferromagnets.
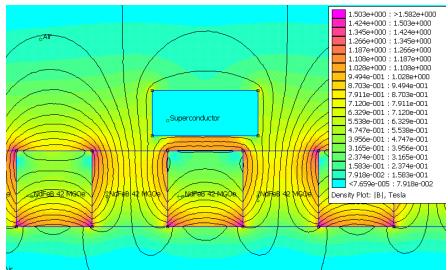
A superconductor acts as an essentially perfect diamagnetic material when placed in a magnetic field and it excludes the field, and the flux lines avoid the region

Superconductors may be considered to be perfect diamagnets ($\chi_v = -1$), since they expel all fields from their interior due to the Meissner effect. However this effect is not due to eddy currents, as in ordinary diamagnetic materials (see the article on superconductivity).

Additionally, all conductors exhibit an effective diamagnetism when they experience a changing magnetic field. The Lorentz force on electrons causes them to circulate around forming eddy currents. The eddy currents then produce an induced magnetic field which opposes the applied field, resisting the conductor's motion.

[] History

In 1778 S. J. Bergman was the first individual to observe that bismuth and antimony were repelled by magnetic fields. However, the term "*diamagnetism*" was coined by Michael Faraday in September 1845, when he realized that all materials in nature possessed some form of diamagnetic response to an applied magnetic field.

[] Demonstrations of diamagnetism

[] Curving water surfaces

If a powerful magnet (such as a supermagnet) is covered with a layer of water (that is thin compared to the diameter of the magnet) then the field of the magnet significantly repels the water. This causes a slight dimple in the water's surface that may be seen by its reflection.[2][3]

[] Diamagnetic levitation

A live frog levitates inside a 32 mm diameter vertical bore of a Bitter solenoid in a magnetic field of about 16 teslas at the Nijmegen High Field Magnet Laboratory. Various videos

Diamagnets may be levitated in stable equilibrium in a magnetic field, with no power consumption. Earnshaw's theorem seems to preclude the possibility of static magnetic levitation. However, Earnshaw's theorem only applies to objects with positive moments, such as ferromagnets (which have a permanent positive moment) and paramagnets (which induce a positive moment). These are attracted to field maxima, which do not exist in free space. Diamagnets (which induce a negative moment) are attracted to field minima, and there can be a field minimum in free space.

A thin slice of pyrolytic graphite, which is an unusually strong diamagnetic material, can be stably floated in a magnetic field, such as that from rare earth permanent magnets. This can be done with all components at room temperature, making a visually effective demonstration of diamagnetism.

The Radboud University Nijmegen, the Netherlands, has conducted experiments where water and other substances were successfully levitated. Most spectacularly, a live frog (see figure) was levitated.[4]

In September 2009, NASA's Jet Propulsion Laboratory in Pasadena, California announced they had successfully levitated mice using a superconducting magnet,[5] an important step forward since mice are closer biologically to humans than are frogs.[6] They hope to perform experiments regarding the effects of microgravity on bone and muscle mass.

Recent experiments studying the growth of protein crystals has led to a technique using powerful magnets to allow growth in ways that counteract Earth's gravity.[7]

A simple homemade device for demonstration can be constructed out of bismuth plates and a few permanent magnets that will levitate a permanent magnet. [8]

Ferromagnetism
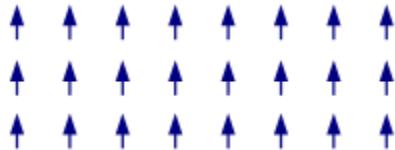
Ferromagnetic ordering of microscopic magnets (the magnetic moments of individual particles).

Ferromagnetism is the basic mechanism by which certain materials (such as iron) form permanent magnets, or are attracted to magnets. In physics, several different types of magnetism are distinguished. Ferromagnetism is the only type that can produce forces strong enough to be felt, and is responsible for the common phenomena of magnetism encountered in everyday life. One example is refrigerator magnets. The attraction between a magnet and ferromagnetic material is "the quality of magnetism first apparent to the ancient world, and to us today," according to a classic text on ferromagnetism.[1]

All permanent magnets (materials that can be magnetized by an external magnetic field and which *remain* magnetized after the external field is removed) are either ferromagnetic or ferrimagnetic, as are other materials that are noticeably attracted to them.

Historically, the term **ferromagnet** was used for any material that could exhibit spontaneous magnetization: a net magnetic moment in the absence of an external magnetic field. This general definition is still in common use. More recently, however, different classes of spontaneous magnetization have been identified when there is more than one magnetic ion per primitive cell of the material, leading to a stricter definition of "ferromagnetism" that is often used to distinguish it from ferrimagnetism. In particular, a material is "ferromagnetic" in this narrower sense only if *all* of its magnetic ions add a positive contribution to the net magnetization. If some of the magnetic ions *subtract* from the net magnetization (if they are partially *anti*-aligned), then the material is "ferrimagnetic". If the moments of the aligned and anti-aligned ions balance completely so as to have zero net magnetization, despite the magnetic ordering, then it is an antiferromagnet. All of these alignment effects only occur at temperatures below a certain critical temperature, called the Curie temperature (for ferromagnets and ferrimagnets) or the Néel temperature (for antiferromagnets).

Among the first investigations of ferromagnetism are the pioneering works of Aleksandr Stoletov on measurement of the magnetic permeability of ferromagnetics, known as the Stoletov curve.

[] Ferromagnetic materials

See also: Category:Ferromagnetic materials

Curie temperatures for some crystalline ferromagnetic (* = ferrimagnetic) materials (Kittel, p. 449.)

Material	Curie temp. (K)
Co	1388
Fe	1043
$FeOFe_2O_3$*	858
$NiOFe_2O_3$*	858
$CuOFe_2O_3$*	728
$MgOFe_2O_3$*	713
MnBi	630
Ni	627
MnSb	587
$MnOFe_2O_3$*	573
$Y_3Fe_5O_{12}$*	560
CrO_2	386
MnAs	318
Gd	292
Dy	88
EuO	69

There are a number of crystalline materials that exhibit ferromagnetism (or ferrimagnetism). The table on the right lists a representative selection of them, along with their Curie temperatures, the temperature above which they cease to exhibit spontaneous magnetization (see below).

Ferromagnetism is a property not just of the chemical makeup of a material, but of its crystalline structure and microscopic organization. There are ferromagnetic metal alloys whose constituents are not themselves ferromagnetic, called Heusler alloys, named after Fritz Heusler. Conversely there are nonmagnetic alloys, such as types of stainless steel, composed almost exclusively of ferromagnetic metals.

One can also make amorphous (non-crystalline) ferromagnetic metallic alloys by very rapid quenching (cooling) of a liquid alloy. These have the advantage that their properties are nearly isotropic (not aligned along a crystal axis); this results in low coercivity, low hysteresis loss, high permeability, and high electrical resistivity. A typical such material is a transition metal-metalloid alloy, made from about 80% transition metal (usually Fe, Co, or Ni) and a metalloid component (B, C, Si, P, or Al) that lowers the melting point.

A relatively new class of exceptionally strong ferromagnetic materials are the rare-earth magnets. They contain lanthanide elements that are known for their ability to carry large magnetic moments in well-localized f-orbitals.

[] Actinide ferromagnets

A number of actinide compounds are ferromagnets at room temperature or become ferromagnets below the Curie temperature (T_C). PuP is one actinide pnictide that is a paramagnet and has cubic symmetry at room temperature, but upon cooling undergoes a lattice distortion to tetragonal when cooled to below its T_c = 125 K. PuP has an easy axis of <100>,[2] so that

- $(c - a)/a = -(31 \pm 1) \times 10^{-4}$

at 5 K.[3] The lattice distortion is presumably a consequence of strain induced by the magnetoelastic interactions as the magnetic moments aligned parallel within magnetic domains.

In $NpFe_2$ the easy axis is <111>.[4] Above T_C ~500 K $NpFe_2$ is also paramagnetic and cubic. Cooling below the Curie temperature produces a rhombohedral distortion wherein the rhombohedral angle changes from 60° (cubic phase) to 60.53°. An alternate discription of this distortion is to consider the length c along the unique trigonal axis (after the distortion has begun) and a as the distance in the plane perpendicular to c. In the cubic phase this reduces to c/a = 1.00. Below the Curie temperature

- $(c - a)/a = -(120 \pm 5) \times 10^{-4}$

which is the largest strain in any actinide compound.[3] $NpNi_2$ undergoes a similar lattice distortion below T_C = 32 K, with a strain of $(43 \pm 5) \times 10^{-4}$.[3] $NpCo_2$ is a ferrimagnet below 15 K.

[] Lithium gas

In 2009, a team of MIT physicists demonstrated that a lithium gas cooled to less than one Kelvin can exhibit ferromagnetism. [5] The team cooled fermionic lithium-6 to less than 150 billionths of

one Kelvin above absolute zero using infrared laser cooling. This demonstration is the first time that ferromagnetism has been demonstrated in a gas.

[] Physical origin

The property of ferromagnetism is due to the direct influence of two effects from quantum mechanics: spin and the Pauli exclusion principle.[6]

[] Origin of magnetism

The spin of an electron, combined with its electric charge, results in a magnetic dipole moment and creates a magnetic field. Although an electron can be visualized classically as a spinning ball of charge, spin is actually a quantum mechanical property with differences from the classical picture, such as the fact that it is quantized into discrete up/down states. The spin of the electrons in atoms is the main source of ferromagnetism, although there is also some contribution from the orbital angular momentum of the electron about the nucleus, whose classical analogy is a current loop.

However in many materials (specifically, those with a filled electron shell), the total dipole moment of all the electrons is zero because the spins are in up/down pairs. Only atoms with partially filled shells (i.e., unpaired spins) can have a net magnetic moment, so ferromagnetism only occurs in materials with partially filled shells. When these tiny magnetic dipoles are aligned in the same direction, their individual magnetic fields add together to create a measurable macroscopic field.

These unpaired dipoles (often called simply "spins" even though they also generally include angular momentum) tend to align in parallel to an external magnetic field, an effect called paramagnetism. Ferromagnetism involves an additional phenomenon, however: the dipoles tend to *align spontaneously*, without any applied field. This is a purely quantum-mechanical effect.

[] Exchange interaction

According to classical electromagnetism, two nearby magnetic dipoles will tend to align in *opposite* directions. However in a few materials, the ferromagnetic ones, they tend to align in the *same* direction because of a quantum mechanical effect called the exchange interaction. The Pauli exclusion principle says that two electrons with the same spin cannot also have the same "position". Therefore, under certain conditions, when the orbitals of the unpaired outer valence electrons from adjacent atoms overlap, the distribution of their electric charge in space is further apart when the electrons have parallel spins than when they have opposite spins. This reduces the electrostatic energy of the electrons when their spins are parallel compared to their energy when the spins are anti-parallel, so the parallel-spin state is more stable, and the spins of these electrons tend to line up. This difference in energy is called the exchange energy.

The exchange interaction is also responsible for the other types of spontaneous ordering of atomic magnetic moments occurring in magnetic solids, antiferromagnetism and ferrimagnetism. In most ferromagnets the exchange interaction is much stronger than the competing dipole-dipole

interaction. For instance, in iron (Fe) the exchange interaction between two atoms is about 1000 times stronger than the dipole interaction. There are a small number of "exotic" ferromagnets in which the exchange interactions are exceptionally weak, and then the classical dipole-dipole interactions may become the dominant ones. However, such systems become ferromagnetic only at very low temperatures, usually below 1 K. But if the Curie temperature in a given material is higher than a few kelvins, then its ferromagnetism is surely produced by exchange interactions. In such systems the classical dipole-dipole interactions may only give rise to secondary effects, e.g., to weak magnetic anisotropy.

[] Magnetic domains

The above would seem to suggest that every piece of ferromagnetic material should have a strong magnetic field, yet iron and other ferromagnets are often found in an "unmagnetized" state. The reason for this is that a bulk piece of ferromagnetic material is divided into many tiny **magnetic domains** (also known as *Weiss domains*). Within each domain, the spins are aligned, but (if the bulk material is "unmagnetized"), the spins of separate domains point in different directions and their magnetic fields cancel out, so the object has no net large scale magnetic field.

Ferromagnetic materials spontaneously divide into magnetic domains because this is a lower energy configuration. At long distances (after many thousands of ions), the exchange energy advantage is overtaken by the classical tendency of dipoles to anti-align. This is why, in an equilibriated (non-magnetized) ferromagnetic material, the dipoles in the whole material are not aligned. Rather, they organize into magnetic domains that are aligned (magnetized) at short range, but at long range adjacent domains are anti-aligned.[7] The boundary between two domains, where the magnetization flips, is called a domain wall (i.e., a Bloch/Néel wall, depending upon whether the magnetization rotates parallel/perpendicular to the domain interface) and is a gradual transition on the atomic scale (covering a distance of about 300 ions for iron).

Thus, an ordinary piece of iron generally has little or no net magnetic moment. However, if it is placed in a strong enough external magnetic field, the domains will re-orient in parallel with that field, and will remain re-oriented when the field is turned off, thus creating a "permanent" magnet. The domains don't go back to their original minimum energy configuration when the field is turned off because the domain walls tend to become 'pinned' or 'snagged' on defects in the crystal lattice, preserving their parallel orientation. This is shown by the Barkhausen effect: as the magnetizing field is changed, the magnetization changes in thousands of tiny discontinuous jumps as the domain walls suddenly "snap" past defects. This magnetization as a function of the external field is described by a hysteresis curve. Although this state of aligned domains is not a minimal-energy configuration, it is extremely stable and has been observed to persist for millions of years in seafloor magnetite aligned by the Earth's magnetic field (whose poles can thereby be seen to flip at long intervals). Alloys used for the strongest permanent magnets are "hard" alloys made with many defects in their crystal structure where the domain walls "catch" and stabilize. The net magnetization can be destroyed by heating and then cooling (annealing) the material without an external field, however. The thermal motion allows the domain boundaries to move, releasing them from any defects, to return to their low-energy unaligned state.

[] Curie temperature

As the temperature increases, thermal motion, or entropy, competes with the ferromagnetic tendency for dipoles to align. When the temperature rises beyond a certain point, called the **Curie temperature**, there is a second-order phase transition and the system can no longer maintain a spontaneous magnetization, although it still responds paramagnetically to an external field. Below that temperature, there is a spontaneous symmetry breaking and random domains form (in the absence of an external field). The Curie temperature itself is a critical point, where the magnetic susceptibility is theoretically infinite and, although there is no net magnetization, domain-like spin correlations fluctuate at all length scales.

The study of ferromagnetic phase transitions, especially via the simplified Ising spin model, had an important impact on the development of statistical physics. There, it was first clearly shown that mean field theory approaches failed to predict the correct behavior at the critical point (which was found to fall under a *universality class* that includes many other systems, such as liquid-gas transitions), and had to be replaced by renormalization group theory.

Maxwell's equations

Maxwell's equations are a set of four partial differential equations that relate the electric and magnetic fields to their sources, charge density and current density. These equations can be combined to show that light is an electromagnetic wave. Individually, the equations are known as Gauss's law, Gauss's law for magnetism, Faraday's law of induction, and Ampère's law with Maxwell's correction. The set of equations is named after James Clerk Maxwell.

These four equations, together with the Lorentz force law are the complete set of laws of classical electromagnetism. The Lorentz force law itself was actually derived by Maxwell under the name of *Equation for Electromotive Force* and was one of an earlier set of eight equations by Maxwell.

[] Conceptual description

This section will conceptually describe each of the four Maxwell's equations, and also how they link together to explain the origin of electromagnetic radiation such as light. The exact equations are set out in later sections of this article.

- Gauss' law describes how an electric field is generated by electric charges: The electric field tends to point away from positive charges and towards negative charges. More technically, it relates the electric flux through any hypothetical closed "Gaussian surface" to the electric charge within the surface.

- Gauss' law for magnetism states that there are no "magnetic charges" (also called magnetic monopoles), analogous to electric charges.[1] Instead the magnetic field is generated by a configuration called a dipole, which has no magnetic charge but resembles a positive and negative charge inseparably bound together. Equivalent technical statements are that the total magnetic flux through any Gaussian surface is zero, or that the magnetic field is a solenoidal vector field.
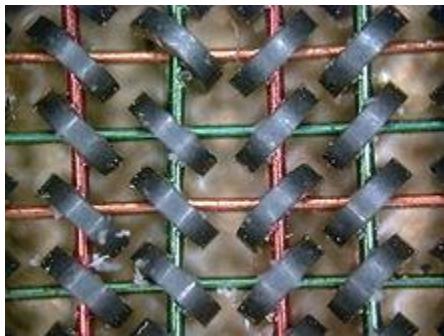
An Wang's magnetic core memory (1954) is an application of Ampere's law. Each core stores one bit of data.

- Faraday's law describes how a changing magnetic field can create ("induce") an electric field.[1] This aspect of electromagnetic induction is the operating principle behind many electric generators: A bar magnet is rotated to create a changing magnetic field, which in turn generates an electric field in a nearby wire. (Note: The "Faraday's law" that occurs in Maxwell's equations is a bit different than the version originally written by Michael Faraday. Both versions are equally true laws of physics, but they have different scope, for example whether "motional EMF" is included. See Faraday's law of induction for details.)

- Ampère's law with Maxwell's correction states that magnetic fields can be generated in two ways: by electrical current (this was the original "Ampère's law") and by changing electric fields (this was "Maxwell's correction").

Maxwell's correction to Ampère's law is particularly important: It means that a changing magnetic field creates an electric field, *and* a changing electric field creates a magnetic field.[1][2] Therefore, these equations allow self-sustaining "electromagnetic waves" to travel through empty space (see electromagnetic wave equation).

The speed calculated for electromagnetic waves, which could be predicted from experiments on charges and currents,[3] exactly matches the speed of light; indeed, light *is* one form of electromagnetic radiation (as are X-rays, radio waves, and others). Maxwell understood the connection between electromagnetic waves and light in 1864, thereby unifying the previously-separate fields of electromagnetism and optics.

[] General formulation

The equations in this section are given in SI units. Unlike the equations of mechanics (for example), Maxwell's equations are *not* unchanged in other unit systems. Though the general form remains the same, various definitions get changed and different constants appear at different places. Other than SI (used in engineering), the units commonly used are Gaussian units (based on the cgs system and considered to have some theoretical advantages over SI[4]), Lorentz-Heaviside units (used mainly in particle physics) and Planck units (used in theoretical physics). See below for CGS-Gaussian units.

Two equivalent, general formulations of Maxwell's equations follow. The first separates bound charge and bound current (which arise in the context of dielectric and/or magnetized materials) from free charge and free current (the more conventional type of charge and current). This separation is useful for calculations involving dielectric or magnetized materials. The second formulation treats all charge equally, combining free and bound charge into *total* charge (and likewise with current). This is the more fundamental or microscopic point of view, and is particularly useful when no dielectric or magnetic material is present. More details, and a proof that these two formulations are mathematically equivalent, are given in section 4.

Symbols in **bold** represent vector quantities, and symbols in *italics* represent scalar quantities. The definitions of terms used in the two tables of equations are given in another table immediately following.

Formulation in terms of *free* charge and current		
Name	**Differential form**	**Integral form**
Gauss's law	$\nabla \cdot \mathbf{D} = \rho_f$	$\oiint_{\partial V} \mathbf{D} \cdot \mathrm{d}\mathbf{A} = Q_f(V)$
Gauss's law for magnetism	$\nabla \cdot \mathbf{B} = 0$	$\oiint_{\partial V} \mathbf{B} \cdot \mathrm{d}\mathbf{A} = 0$
Maxwell–Faraday equation (Faraday's law of induction)	$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	$\oint_{\partial S} \mathbf{E} \cdot \mathrm{d}\mathbf{l} = -\dfrac{\partial \Phi_{B,S}}{\partial t}$
Ampère's circuital law (with Maxwell's correction)	$\nabla \times \mathbf{H} = \mathbf{J}_f + \dfrac{\partial \mathbf{D}}{\partial t}$	$\oint_{\partial S} \mathbf{H} \cdot \mathrm{d}\mathbf{l} = I_{f,S} + \dfrac{\partial \Phi_{D,S}}{\partial t}$

Formulation in terms of *total* charge and current[note 1]		
Name	**Differential form**	**Integral form**
Gauss's law	$\nabla \cdot \mathbf{E} = \dfrac{\rho}{\varepsilon_0}$	$\oiint_{\partial V} \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \dfrac{Q(V)}{\varepsilon_0}$

Gauss's law for magnetism	$\nabla \cdot \mathbf{B} = 0$	$\oiint_{\partial V} \mathbf{B} \cdot d\mathbf{A} = 0$
Maxwell–Faraday equation (Faraday's law of induction)	$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	$\oint_{\partial S} \mathbf{E} \cdot d\mathbf{l} = -\dfrac{\partial \Phi_{B,S}}{\partial t}$
Ampère's circuital law (with Maxwell's correction)	$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \dfrac{\partial \mathbf{E}}{\partial t}$	$\oint_{\partial S} \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_S + \mu_0 \varepsilon_0 \dfrac{\partial \Phi_{E,S}}{\partial t}$

The following table provides the meaning of each symbol and the SI unit of measure:

Definitions and units

Symbol	Meaning (first term is the most common)	SI Unit of Measure
$\mathbf{E}$	electric field	volt per meter or, equivalently, newton per coulomb
$\mathbf{B}$	magnetic field also called the magnetic induction also called the magnetic field density also called the magnetic flux density	tesla, or equivalently, weber per square meter, volt-second per square meter
$\mathbf{D}$	electric displacement field also called the electric induction also called the electric flux density	coulombs per square meter or equivalently, newton per volt-meter
$\mathbf{H}$	magnetizing field also called auxiliary magnetic field also called magnetic field intensity also called magnetic field	ampere per meter
$\nabla \cdot$	the divergence operator	per meter (factor contributed by applying either operator)
$\nabla \times$	the curl operator	
$\dfrac{\partial}{\partial t}$	partial derivative with respect to time	per second (factor contributed by applying the operator)
$d\mathbf{A}$	differential vector element of surface area A, with infinitesimally small magnitude and direction normal to surface S	square meters
$d\mathbf{l}$	differential vector element of *path length* tangential to the path/curve	meters
ε_0	permittivity of free space, also called the electric constant, a universal constant	farads per meter
μ_0	permeability of free space, also called the magnetic constant, a universal constant	henries per meter, or newtons per ampere

		squared
ρ_f	free charge density (not including bound charge)	coulombs per cubic meter
ρ	total charge density (including both free and bound charge)	coulombs per cubic meter
$\mathbf{J}_f$	free current density (not including bound current)	amperes per square meter
$\mathbf{J}$	total current density (including both free and bound current)	amperes per square meter
$Q_f(V)$	net free electric charge within the three-dimensional volume V (not including bound charge)	coulombs
$Q(V)$	net electric charge within the three-dimensional volume V (including both free and bound charge)	coulombs
$\oint_{\partial S} \mathbf{E} \cdot d\mathbf{l}$	line integral of the electric field along the boundary ∂S of a surface S (∂S is always a closed curve).	joules per coulomb
$\oint_{\partial S} \mathbf{B} \cdot d\mathbf{l}$	line integral of the magnetic field over the closed boundary ∂S of the surface S	tesla-meters
$\oiint_{\partial V} \mathbf{E} \cdot d\mathbf{A}$	the electric flux (surface integral of the electric field) through the (closed) surface ∂V (the boundary of the volume V)	joule-meter per coulomb
$\oiint_{\partial V} \mathbf{B} \cdot d\mathbf{A}$	the magnetic flux (surface integral of the magnetic B-field) through the (closed) surface ∂V (the boundary of the volume V)	tesla meters-squared or webers
$\iint_{S} \mathbf{B} \cdot d\mathbf{A} = \Phi_{B,S}$	magnetic flux through any surface S, not necessarily closed	webers or equivalently, volt-seconds
$\iint_{S} \mathbf{E} \cdot d\mathbf{A} = \Phi_{E,S}$	electric flux through any surface S, not necessarily closed	joule-meters per coulomb
$\iint_{S} \mathbf{D} \cdot d\mathbf{A} = \Phi_{D,S}$	flux of electric displacement field through any surface S, not necessarily closed	coulombs
$\iint_{S} \mathbf{J}_f \cdot d\mathbf{A} = I_{f,s}$	net free electrical current passing through the surface S (not including bound current)	amperes
$\iint_{S} \mathbf{J} \cdot d\mathbf{A} = I_S$	net electrical current passing through the surface S (including both free and bound current)	amperes

Maxwell's equations are generally applied to *macroscopic averages* of the fields, which vary wildly on a microscopic scale in the vicinity of individual atoms (where they undergo quantum mechanical effects as well). It is only in this averaged sense that one can define quantities such as the permittivity and permeability of a material. At microscopic level, Maxwell's equations, ignoring quantum effects, describe fields, charges and currents in free space—but at this level of detail one must include all charges, even those at an atomic level, generally an intractable problem.

[] History

Although James Clerk Maxwell is said by some not to be the originator of these equations, he nevertheless derived them independently in conjunction with his molecular vortex model of Faraday's "lines of force". In doing so, he made an important addition to Ampère's circuital law.

All four of what are now described as Maxwell's equations can be found in recognizable form (albeit without any trace of a vector notation, let alone ∇) in his 1861 paper *On Physical Lines of Force*, in his 1865 paper A Dynamical Theory of the Electromagnetic Field, and also in vol. 2 of Maxwell's "A Treatise on Electricity & Magnetism", published in 1873, in Chapter IX, entitled "General Equations of the Electromagnetic Field". This book by Maxwell pre-dates publications by Heaviside, Hertz and others.

[] The term *Maxwell's equations*

The term *Maxwell's equations* originally applied to a set of eight equations published by Maxwell in 1865, but nowadays applies to modified versions of four of these equations that were grouped together in 1884 by Oliver Heaviside,[6] concurrently with similar work by Willard Gibbs and Heinrich Hertz.[7] These equations were also known variously as the Hertz-Heaviside equations and the Maxwell-Hertz equations,[6] and are sometimes still known as the Maxwell–Heaviside equations.[8]

Maxwell's contribution to Science in producing these equations lies in the correction he made to Ampère's circuital law in his 1861 paper *On Physical Lines of Force*. He added the displacement current term to Ampère's circuital law and this enabled him to derive the electromagnetic wave equation in his later 1865 paper *A Dynamical Theory of the Electromagnetic Field* and demonstrate the fact that light is an electromagnetic wave. This fact was then later confirmed experimentally by Heinrich Hertz in 1887.

The concept of fields was introduced by, among others, Faraday. Albert Einstein wrote:

The precise formulation of the time-space laws was the work of Maxwell. Imagine his feelings when the differential equations he had formulated proved to him that electromagnetic fields spread in the form of polarised waves, and at the speed of light! To few men in the world has such an experience been vouchsafed . . it took physicists some decades to grasp the full significance of Maxwell's discovery, so bold was the leap that his genius forced upon the conceptions of his fellow-workers
—(*Science*, May 24, 1940)

The equations were called by some the Hertz-Heaviside equations, but later Einstein referred to them as the Maxwell-Hertz equations.[6] However, in 1940 Einstein referred to the equations as *Maxwell's equations* in "The Fundamentals of Theoretical Physics" published in the Washington periodical *Science*, May 24, 1940.

Heaviside worked to eliminate the potentials (electrostatic potential and vector potential) that Maxwell had used as the central concepts in his equations;[6] this effort was somewhat controversial,[9] though it was understood by 1884 that the potentials must propagate at the speed of light like the fields, unlike the concept of instantaneous action-at-a-distance like the then conception of gravitational potential.[7] Modern analysis of, for example, radio antennas, makes full use of Maxwell's vector and scalar potentials to separate the variables, a common technique used in formulating the solutions of differential equations. However the potentials can be introduced by algebraic manipulation of the four fundamental equations.

The net result of Heaviside's work was the symmetrical duplex set of four equations,[6] all of which originated in Maxwell's previous publications, in particular Maxwell's 1861 paper *On Physical Lines of Force*, the 1865 paper *A Dynamical Theory of the Electromagnetic Field* and the Treatise. The fourth was a partial time derivative version of Faraday's law of induction that doesn't include motionally induced EMF; this version is often termed the *Maxwell-Faraday equation* or *Faraday's law in differential form* to keep clear the distinction from Faraday's law of induction, though it expresses the same law.[10][11]

[] Maxwell's *On Physical Lines of Force* (1861)

The four modern day Maxwell's equations appeared throughout Maxwell's 1861 paper *On Physical Lines of Force*:

i. Equation (56) in Maxwell's 1861 paper is $\nabla \cdot \mathbf{B} = 0$.
ii. Equation (112) is Ampère's circuital law with Maxwell's displacement current added. It is the addition of displacement current that is the most significant aspect of Maxwell's work in electromagnetism, as it enabled him to later derive the electromagnetic wave equation in his 1865 paper A Dynamical Theory of the Electromagnetic Field, and hence show that light is an electromagnetic wave. It is therefore this aspect of Maxwell's work which gives the equations their full significance. (Interestingly, Kirchhoff derived the telegrapher's equations in 1857 without using displacement current. But he did use Poisson's equation and the equation of continuity which are the mathematical ingredients of the displacement current. Nevertheless, Kirchhoff believed his equations to be applicable only inside an electric wire and so he is not cred with having discovered that light is an electromagnetic wave).
iii. Equation (115) is Gauss's law.
iv. Equation (54) is an equation that Oliver Heaviside referred to as 'Faraday's law'. This equation caters for the time varying aspect of electromagnetic induction, but not for the motionally induced aspect, whereas Faraday's original flux law caters for both aspects. Maxwell deals with the motionally dependent aspect of electromagnetic induction, $\mathbf{v} \times \mathbf{B}$, at equation (77). Equation (77) which is the same as equation (D) in the original eight Maxwell's equations listed below, corresponds to all intents and purposes to the modern

day force law $\mathbf{F} = q\,(\,\mathbf{E} + \mathbf{v} \times \mathbf{B}\,)$ which sits adjacent to Maxwell's equations and bears the name Lorentz force, even though Maxwell derived it when Lorentz was still a young boy.

The difference between the $\mathbf{B}$ and the $\mathbf{H}$ vectors can be traced back to Maxwell's 1855 paper entitled *On Faraday's Lines of Force* which was read to the Cambridge Philosophical Society. The paper presented a simplified model of Faraday's work, and how the two phenomena were related. He reduced all of the current knowledge into a linked set of differential equations.
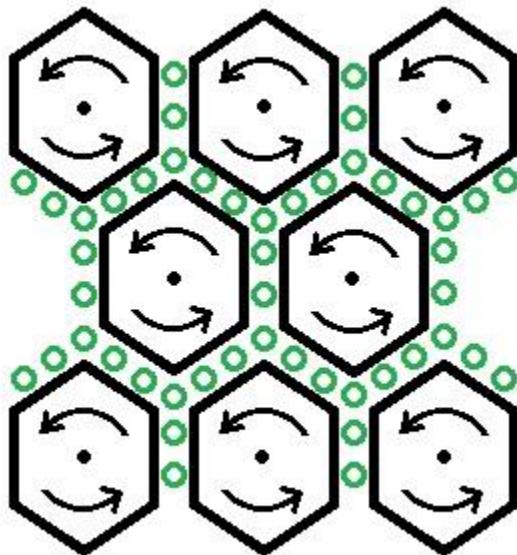
Figure of Maxwell's molecular vortex model. For a uniform magnetic field, the field lines point outward from the display screen, as can be observed from the black dots in the middle of the hexagons. The vortex of each hexagonal molecule rotates counter-clockwise. The small green circles are clockwise rotating particles sandwiching between the molecular vortices.

It is later clarified in his concept of a sea of molecular vortices that appears in his 1861 paper On Physical Lines of Force - 1861. Within that context, $\mathbf{H}$ represented pure vorticity (spin), whereas $\mathbf{B}$ was a weighted vorticity that was weighted for the density of the vortex sea. Maxwell considered magnetic permeability µ to be a measure of the density of the vortex sea. Hence the relationship,

(1) **Magnetic induction current** causes a magnetic current density

$$\mathbf{B} = \mu \mathbf{H}$$

was essentially a rotational analogy to the linear electric current relationship,

(2) **Electric convection current**

$$\mathbf{J} = \rho \mathbf{v}$$

where ρ is electric charge H density. B was seen as a kind of magnetic current of vortices aligned in their axial planes, with being the circumferential velocity of H the vortices. With μ representing vortex density B, it follows that the product of μ with vorticity leads to the magnetic field denoted as .

The electric current equation can be viewed as a convective current of electric charge that involves linear motion. By analogy, the magnetic equation is an inductive current involving spin. There is no linear motion in the inductive current along the direction of the B vector. The magnetic inductive current represents lines of force. In particular, it represents lines of inverse square law force.

The extension of the above considerations confirms that where B is to H, and where J is to ρ, then it necessarily follows from Gauss's law and from the equation of continuity of charge that E is to D. i.e. B parallels with E, whereas H parallels with D.

[] Maxwell's *A Dynamical Theory of the Electromagnetic Field* (1864)

Main article: A Dynamical Theory of the Electromagnetic Field

In 1864 Maxwell published **A Dynamical Theory of the Electromagnetic Field** in which he showed that light was an electromagnetic phenomenon. Confusion over the term "Maxwell's equations" is exacerbated because it is also sometimes used for a set of eight equations that appeared in Part III of Maxwell's 1864 paper A Dynamical Theory of the Electromagnetic Field, entitled "General Equations of the Electromagnetic Field,"[12] a confusion compounded by the writing of six of those eight equations as three separate equations (one for each of the Cartesian axes), resulting in twenty equations and twenty unknowns. (As noted above, this terminology is not common: Modern references to the term "Maxwell's equations" refer to the Heaviside restatements.)

The eight original Maxwell's equations can be written in modern vector notation as follows:

(A) The law of total currents
$$\mathbf{J}_{tot} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$
(B) The equation of magnetic force
$$\mu \mathbf{H} = \nabla \times \mathbf{A}$$
(C) Ampère's circuital law
$$\nabla \times \mathbf{H} = \mathbf{J}_{tot}$$
(D) Electromotive force created by convection, induction, and by static electricity. (This is in effect the Lorentz force)
$$\mathbf{E} = \mu \mathbf{v} \times \mathbf{H} - \frac{\partial \mathbf{A}}{\partial t} - \nabla \phi$$
(E) The electric elasticity equation
$$\mathbf{E} = \frac{1}{\epsilon} \mathbf{D}$$

(F) Ohm's law

$$\mathbf{E} = \frac{1}{\sigma}\mathbf{J}$$

(G) Gauss's law

$$\nabla \cdot \mathbf{D} = \rho$$

(H) Equation of continuity

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$$

Notation

$\mathbf{H}$ is the magnetizing field, which Maxwell called the *magnetic intensity*.

$\mathbf{J}$ is the electric current density (with $\mathbf{J}_{tot}$ being the total current including displacement current).[note 2]

$\mathbf{D}$ is the displacement field (called the *electric displacement* by Maxwell).

ρ is the free charge density (called the *quantity of free electricity* by Maxwell).

$\mathbf{A}$ is the magnetic vector potential (called the *angular impulse* by Maxwell).

$\mathbf{E}$ is called the *electromotive force* by Maxwell. The term electromotive force is nowadays used for voltage, but it is clear from the context that Maxwell's meaning corresponded more to the modern term electric field.

ϕ is the electric potential (which Maxwell also called *electric potential*).

σ is the electrical conductivity (Maxwell called the inverse of conductivity the *specific resistance*, what is now called the resistivity).

It is interesting to note the $\mu\mathbf{v} \times \mathbf{H}$ term that appears in equation D. Equation D is therefore effectively the Lorentz force, similarly to equation (77) of his 1861 paper (see above).

When Maxwell derives the electromagnetic wave equation in his 1865 paper, he uses equation D to cater for electromagnetic induction rather than Faraday's law of induction which is used in modern textbooks. (Faraday's law itself does not appear among his equations.) However, Maxwell drops the $\mu\mathbf{v} \times \mathbf{H}$ term from equation D when he is deriving the electromagnetic wave equation, as he considers the situation only from the rest frame.

[] *A Treatise on Electricity and Magnetism* (1873)

English Wikisource has original text related to this article:
A Treatise on Electricity and Magnetism

In *A Treatise on Electricity and Magnetism*, an 1873 textbook on electromagnetism written by James Clerk Maxwell, the equations are compiled into two sets.

The first set is

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}$$
$$\mathbf{B} = \nabla \times \mathbf{A}.$$

The second set is

$$\nabla \cdot \mathbf{D} = \rho$$
$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}.$$

[] Maxwell's equations and matter

[] Bound charge and current

Main articles: Bound charge#Bound charge and Bound current#Magnetization current
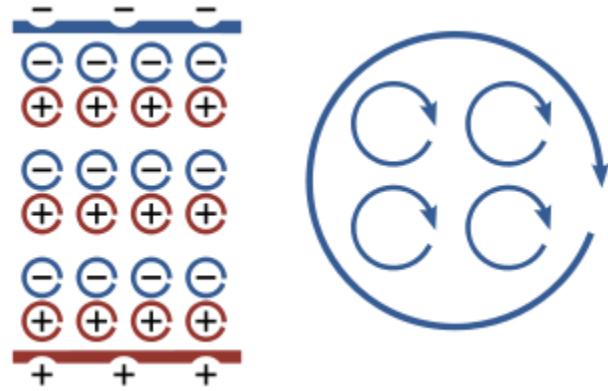
Left: A schematic view of how an assembly of microscopic dipoles appears like a macroscopically separated pair of charged sheets, as shown at top and bottom (these sheets are not intended to be viewed as originating the electric field that causes the dipole alignment, but as a representation equivalent to the dipole array); *Right:* How an assembly of microscopic current loops appears as a macroscopically circulating current loop. Inside the boundaries, the individual contributions tend to cancel, but at the boundaries no cancellation occurs.

If an electric field is applied to a dielectric material, each of the molecules responds by forming a microscopic electric dipole—its atomic nucleus will move a tiny distance in the direction of the field, while its electrons will move a tiny distance in the opposite direction. This is called polarization of the material. In an idealized situation like that shown in the figure, the distribution of charge that results from these tiny movements turns out to be identical (outside the material) to having a layer of positive charge on one side of the material, and a layer of negative charge on the other side (a macroscopic separation of charge) even though all of the charges involved are bound to individual molecules. The volume polarization **P** is a result of bound charge. (Mathematically, once physical approximation has established the electric dipole density **P** based upon the underlying behavior of atoms, the surface charge that is equivalent to the material with its internal polarization is provided by the divergence theorem applied to a region straddling the interface between the material and the surrounding vacuum.)[13][14]

Somewhat similarly, in all materials the constituent atoms exhibit magnetic moments that are intrinsically linked to the angular momentum of the atoms' components, most notably their electrons. The connection to angular momentum suggests the picture of an assembly of

microscopic current loops. Outside the material, an assembly of such microscopic current loops is not different from a macroscopic current circulating around the material's surface, despite the fact that no individual magnetic moment is traveling a large distance. The *bound currents* can be described using **M**. (Mathematically, once physical approximation has established the magnetic dipole density based upon the underlying behavior of atoms, the surface current that is equivalent to the material with its internal magnetization is provided by Stokes' theorem applied to a path straddling the interface between the material and the surrounding vacuum.)[15][16]

These ideas suggest that for some situations the microscopic details of the atomic and electronic behavior can be treated in a simplified fashion that ignores many details on a fine scale that may be unimportant to understanding matters on a grosser scale. That notion underlies the bound/free partition of behavior.

[] Proof that the two general formulations are equivalent

In this section, a simple proof is outlined which shows that the two alternate general formulations of Maxwell's equations given in Section 1 are mathematically equivalent.

The relation between polarization, magnetization, bound charge, and bound current is as follows:

$$\rho_b = -\nabla \cdot \mathbf{P}$$
$$\mathbf{J}_b = \nabla \times \mathbf{M} + \frac{\partial \mathbf{P}}{\partial t}$$
$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$
$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$$
$$\rho = \rho_b + \rho_f$$
$$\mathbf{J} = \mathbf{J}_b + \mathbf{J}_f$$

where **P** and **M** are polarization and magnetization, and ρ_b and $\mathbf{J}_b$ are bound charge and current, respectively. Plugging in these relations, it can be easily demonstrated that the two formulations of Maxwell's equations given in Section 2 are precisely equivalent.

[] Constitutive relations

In order to apply Maxwell's equations (the formulation in terms of free/bound charge and current using **D** and **H**), it is necessary to specify the relations between **D** and **E**, and **H** and **B**.

Finding relations between these fields is another way to say that to solve Maxwell's equations by employing the free/bound partition of charges and currents, one needs the properties of the materials relating the response of bound currents and bound charges to the fields applied to these materials.[note 3] These relations may be empirical (based directly upon measurements), or theoretical (based upon statistical mechanics, transport theory or other tools of condensed matter physics). The detail employed may be macroscopic or microscopic, depending upon the level necessary to the problem under scrutiny. These material properties specifying the response of bound charge and current to the field are called constitutive relations, and correspond physically

to how much polarization and magnetization a material acquires in the presence of electromagnetic fields.

Once the responses of bound currents and charges are related to the fields, Maxwell's equations can be fully formulated in terms of the **E**- and **B**-fields alone, with only the *free* charges and currents appearing explicitly in the equations.

[] Case without magnetic or dielectric materials

In the absence of magnetic or dielectric materials, the relations are simple:

$$\mathbf{D} = \epsilon_0 \mathbf{E}, \quad \mathbf{H} = \mathbf{B}/\mu_0$$

where ε_0 and μ_0 are two universal constants, called the permittivity of free space and permeability of free space, respectively.

[] Case of linear materials

In a linear, isotropic, nondispersive, uniform material, the relations are also straightforward:

$$\mathbf{D} = \epsilon \mathbf{E}, \quad \mathbf{H} = \mathbf{B}/\mu$$

where ε and μ are constants (which depend on the material), called the permittivity and permeability, respectively, of the material.

[] General case

For real-world materials, the constitutive relations are not simple proportionalities, except approximately. The relations can usually still be written:

$$\mathbf{D} = \epsilon \mathbf{E}, \quad \mathbf{H} = \mathbf{B}/\mu$$

but ε and μ are not, in general, simple constants, but rather functions. For example, ε and μ can depend upon:

- The strength of the fields (the case of *nonlinearity*, which occurs when ε and μ are functions of **E** and **B**; see, for example, Kerr and Pockels effects),
- The direction of the fields (the case of *anisotropy*, *birefringence*, or *dichroism*; which occurs when ε and μ are second-rank tensors),
- The frequency with which the fields vary (the case of *dispersion*, which occurs when ε and μ are functions of frequency; see, for example, Kramers–Kronig relations).

If further there are dependencies on:

- The position inside the material (the case of a *nonuniform material*, which occurs when the response of the material varies from point to point within the material, an effect called

spatial inhomogeneity; for example in a domained structure, heterostructure or a liquid crystal, or most commonly in the situation where there are simply multiple materials occupying different regions of space),

- The history of the fields—in a linear time-invariant material, this is equivalent to the material dispersion mentioned above (a frequency dependence of the ε and μ), which after Fourier transforming turns into a convolution with the fields at past times, expressing a non-instantaneous response of the material to an applied field; in a nonlinear or time-varying medium, the time-dependent response can be more complicated, such as the example of a hysteresis response,

then the constitutive relations take a more complicated form:[17][18]

$$\mathbf{D}(\mathbf{r}, t) = \epsilon_0 \mathbf{E}(\mathbf{r}, t) + \mathbf{P}(\mathbf{r}, t)$$

$$\mathbf{H}(\mathbf{r}, t) = \frac{1}{\mu_0} \mathbf{B}(\mathbf{r}, t) - \mathbf{M}(\mathbf{r}, t)$$

$$\mathbf{P}(\mathbf{r}, t) = \epsilon_0 \int d^3 r' dt' \, \hat{\chi}_{\text{elec}}(\mathbf{r}, \mathbf{r}', t, t'; \mathbf{E}) \, \mathbf{E}(\mathbf{r}', t')$$

$$\mathbf{M}(\mathbf{r}, t) = \frac{1}{\mu_0} \int d^3 r' dt' \, \hat{\chi}_{\text{magn}}(\mathbf{r}, \mathbf{r}', t, t'; \mathbf{B}) \, \mathbf{B}(\mathbf{r}', t')$$
,

in which the permittivity and permeability functions are replaced by integrals over the more general electric and magnetic susceptibilities.

It may be noted that man-made materials can be designed to have customized permittivity and permeability, such as metamaterials and photonic crystals.

[] Maxwell's equations in terms of E and B for linear materials

Substituting in the constitutive relations above, Maxwell's equations in linear, dispersionless, time-invariant materials (differential form only) are:

$$\nabla \cdot (\epsilon \mathbf{E}) = \rho_f$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times (\mathbf{B}/\mu) = \mathbf{J}_f + \epsilon \frac{\partial \mathbf{E}}{\partial t}.$$

These are formally identical to the *general* formulation in terms of **E** and **B** (given above), except that the permittivity of free space was replaced with the permittivity of the material (see also displacement field, electric susceptibility and polarization density), the permeability of free space was replaced with the permeability of the material (see also magnetization, magnetic susceptibility and magnetic field), and only free charges and currents are included (instead of all

charges and currents). Unless that material is homogeneous in space, ε and μ cannot be factored out of the derivative expressions on the left-hand sides.

[] Calculation of constitutive relations

See also: Computational electromagnetics

The fields in Maxwell's equations are generated by charges and currents. Conversely, the charges and currents are affected by the fields through the Lorentz force equation:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}),$$

where *q* is the charge on the particle and **v** is the particle velocity. (It also should be remembered that the Lorentz force is not the *only* force exerted upon charged bodies, which also may be subject to gravitational, nuclear, *etc.* forces.) Therefore, in both classical and quantum physics, the precise dynamics of a system form a set of coupled differential equations, which are almost always too complicated to be solved exactly, even at the level of statistical mechanics.[note 4] This remark applies to not only the dynamics of free charges and currents (which enter Maxwell's equations directly), but also the dynamics of bound charges and currents, which enter Maxwell's equations through the constitutive equations, as described next.

Commonly, real materials are approximated as continuous media with bulk properties such as the refractive index, permittivity, permeability, conductivity, and/or various susceptibilities. These lead to the *macroscopic* Maxwell's equations, which are written (as given above) in terms of free charge/current densities and **D**, **H**, **E**, and **B** (rather than **E** and **B** alone) along with the constitutive equations relating these fields. For example, although a real material consists of atoms whose electronic charge densities can be individually polarized by an applied field, for most purposes behavior at the atomic scale is not relevant and the material is approximated by an overall polarization density related to the applied field by an electric susceptibility.

Continuum approximations of atomic-scale inhomogeneities cannot be determined from Maxwell's equations alone, but require some type of quantum mechanical analysis such as quantum field theory as applied to condensed matter physics. See, for example, density functional theory, Green-Kubo relations and Green's function (many-body theory). Various approximate transport equations have evolved, for example, the Boltzmann equation or the Fokker-Planck equation or the Navier-Stokes equations. Some examples where these equations are applied are magnetohydrodynamics, fluid dynamics, electrohydrodynamics, superconductivity, plasma modeling. An entire physical apparatus for dealing with these matters has developed. A different set of *homogenization methods* (evolving from a tradition in treating materials such as conglomerates and laminates) are based upon approximation of an inhomogeneous material by a homogeneous *effective medium*[19][20] (valid for excitations with wavelengths much larger than the scale of the inhomogeneity).[21][22][23][24]

Theoretical results have their place, but often require fitting to experiment. Continuum-approximation properties of many real materials rely upon measurement,[25] for example, ellipsometry measurements.

In practice, some materials properties have a negligible impact in particular circumstances, permitting neglect of small effects. For example: optical nonlinearities can be neglected for low field strengths; material dispersion is unimportant where frequency is limited to a narrow bandwidth; material absorption can be neglected for wavelengths where a material is transparent; and metals with finite conductivity often are approximated at microwave or longer wavelengths as perfect metals with infinite conductivity (forming hard barriers with zero skin depth of field penetration).

And, of course, some situations demand that Maxwell's equations and the Lorentz force be combined with other forces that are not electromagnetic. An obvious example is gravity. A more subtle example, which applies where electrical forces are weakened due to charge balance in a solid or a molecule, is the Casimir force from quantum electrodynamics.[26]

The connection of Maxwell's equations to the rest of the physical world is via the fundamental charges and currents. These charges and currents are a response of their sources to electric and magnetic fields and to other forces. The determination of these responses involves the properties of physical materials.

[] In vacuum

Further information: Electromagnetic wave equation and Sinusoidal plane-wave solutions of the electromagnetic wave equation

Starting with the equations appropriate in the case without dielectric or magnetic materials, and assuming that there is no current or electric charge present in the vacuum, we obtain the Maxwell equations in free space:[note 5]

$$
\nabla \cdot \mathbf{E} = 0
$$
$$
\nabla \cdot \mathbf{B} = 0
$$
$$
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}
$$
$$
\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}.
$$

One set of solutions to these equations is in the form of traveling sinusoidal plane waves, with the electric and magnetic field directions orthogonal to one another and the direction of travel, and with the two fields in phase, traveling at the speed[note 6]

$$
c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}.
$$

In fact, Maxwell's equations explains specifically how these waves can physically propagate through space. The changing magnetic field creates a changing electric field through Faraday's law. That electric field, in turn, creates a changing magnetic field through Maxwell's correction

to Ampère's law. This perpetual cycle allows these waves, known as electromagnetic radiation, to move through space, always at velocity c.

Maxwell knew from an 1856 leyden jar experiment by Wilhelm Eduard Weber and Rudolf Kohlrausch, that c was very close to the measured speed of light in vacuum (known from early experiments), and concluded (correctly) that light is a form of electromagnetic radiation.

[] With magnetic monopoles

Maxwell's equations of electromagnetism relate the electric and magnetic fields to the motions of electric charges. The standard form of the equations provide for an electric charge, but posit no magnetic charge. There is no known magnetic analog of an electron, however recently scientists have described behavior in a crystalline state of matter known as spin-ice which have macroscopic behavior like magnetic monopoles.[28][29] (in accordance with the fact that magnetic charge has never been seen and may not exist). Except for this, the equations are symmetric under interchange of electric and magnetic field. In fact, symmetric equations can be written when all charges are zero, and this is how the wave equation is derived (see immediately above).

Fully symmetric equations can also be written if one allows for the possibility of magnetic charges.[30] With the inclusion of a variable for these magnetic charges, say ρ_m, there will also be a "magnetic current" variable in the equations, $\mathbf{J}_m$. The extended Maxwell's equations, simplified by nondimensionalization via Planck units, are as follows:

Name	Without magnetic monopoles	With magnetic monopoles (hypothetical)
Gauss's law:	$\nabla \cdot \mathbf{E} = 4\pi\rho_e$	$\nabla \cdot \mathbf{E} = 4\pi\rho_e$
Gauss's law for magnetism:	$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 4\pi\rho_m$
Maxwell–Faraday equation (Faraday's law of induction):	$-\nabla \times \mathbf{E} = \dfrac{\partial \mathbf{B}}{\partial t}$	$-\nabla \times \mathbf{E} = \dfrac{\partial \mathbf{B}}{\partial t} + 4\pi\mathbf{j}_m$
Ampère's law (with Maxwell's extension):	$\nabla \times \mathbf{B} = \dfrac{\partial \mathbf{E}}{\partial t} + 4\pi\mathbf{j}_e$	$\nabla \times \mathbf{B} = \dfrac{\partial \mathbf{E}}{\partial t} + 4\pi\mathbf{j}_e$

Note: the Bivector notation embodies the sign swap, and these four equations can be written as only one equation.

If magnetic charges do not exist, or if they exist but where they are not present in a region, then the new variables are zero, and the symmetric equations reduce to the conventional equations of electromagnetism such as $\nabla \cdot \mathbf{B} = 0$.

[] Boundary conditions: using Maxwell's equations

Although Maxwell's equations apply throughout space and time, practical problems are finite and solutions to Maxwell's equations inside the solution region are joined to the remainder of the universe through boundary conditions[31][32][33] and started in time using initial conditions.[34]

In particular, in a region without any free currents or free charges, the electromagnetic fields in the region originate elsewhere, and are introduced via boundary and/or initial conditions. An example of this type is a an electromagnetic scattering problem, where an electromagnetic wave originating outside the scattering region is scattered by a target, and the scattered electromagnetic wave is analyzed for the information it contains about the target by virtue of the interaction with the target during scattering.[35]

In some cases, like waveguides or cavity resonators, the solution region is largely isolated from the universe, for example, by metallic walls, and boundary conditions at the walls define the fields with influence of the outside world confined to the input/output ends of the structure.[36] In other cases, the universe at large sometimes is approximated by an artificial absorbing boundary,[37][38][39] or, for example for radiating antennas or communication satellites, these boundary conditions can take the form of *asymptotic limits* imposed upon the solution.[40] In addition, for example in an optical fiber or thin-film optics, the solution region often is broken up into subregions with their own simplified properties, and the solutions in each subregion must be joined to each other across the subregion interfaces using boundary conditions.[41][42][43] A particular example of this use of boundary conditions is the replacement of a material with a volume polarization by a charged surface layer, or of a material with a volume magnetization by a surface current, as described in the section Bound charge and current.

Following are some links of a general nature concerning boundary value problems: Examples of boundary value problems, Sturm-Liouville theory, Dirichlet boundary condition, Neumann boundary condition, mixed boundary condition, Cauchy boundary condition, Sommerfeld radiation condition. Needless to say, one must choose the boundary conditions appropriate to the problem being solved. See also Kempel[44] and the extraordinary book by Friedman.[45]

[] CGS units

The above equations are given in the International System of Units, or **SI** for short. In a related unit system, called **CGS** (short for centimeter-gram-second), there are several ways of defining the unit of electric current in terms of the CGS base units (cm, g, s). In one of those variants, called Gaussian units, the equations take the following form:[46]

$$\nabla \cdot \mathbf{D} = 4\pi\rho_f$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} + \frac{4\pi}{c}\mathbf{J}_f$$

where c is the speed of light in a vacuum. For the electromagnetic field in a vacuum, assuming that there is no current or electric charge present in the vacuum, the equations become:

$$\nabla \cdot \mathbf{E} = 0$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}.$$

In this system of units the relation between displacement field, electric field and polarization density is:

$$\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P}.$$

And likewise the relation between magnetic induction, magnetic field and total magnetization is:

$$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}.$$

In the linear approximation, the electric susceptibility and magnetic susceptibility can be defined so that:

$$\mathbf{P} = \chi_e \mathbf{E}, \quad \mathbf{M} = \chi_m \mathbf{H}.$$

(Note that although the susceptibilities are dimensionless numbers in both cgs and SI, they have different values in the two unit systems, by a factor of 4π.) The permittivity and permeability are:

$$\epsilon = 1 + 4\pi\chi_e, \quad \mu = 1 + 4\pi\chi_m$$

so that

$$\mathbf{D} = \epsilon\mathbf{E}, \quad \mathbf{B} = \mu\mathbf{H}.$$

In vacuum, one has the simple relations $\epsilon = \mu = 1$, D=E, and B=H.

The force exerted upon a charged particle by the electric field and magnetic field is given by the Lorentz force equation:

$$\mathbf{F} = q\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right),$$

where q is the charge on the particle and $\mathbf{v}$ is the particle velocity. This is slightly different from the SI-unit expression above. For example, here the magnetic field $\mathbf{B}$ has the same units as the electric field $\mathbf{E}$.

Some equations in the article are given in Gaussian units but not SI or vice-versa. Fortunately, there are general rules to convert from one to the other; see the article Gaussian units for details.

[] Special relativity

Main article: Classical electromagnetism and special relativity

Maxwell's equations have a close relation to special relativity: Not only were Maxwell's equations a crucial part of the historical development of special relativity, but also, special relativity has motivated a compact mathematical formulation of Maxwell's equations, in terms of covariant tensors.

[] Historical developments

Main article: History of special relativity

Maxwell's electromagnetic wave equation only applied in what he believed to be the rest frame of the luminiferous medium because he didn't use the $\mathbf{v} \times \mathbf{B}$ term of his equation (D) when he derived it. Maxwell's idea of the luminiferous medium was that it consisted of aethereal vortices aligned solenoidally along their rotation axes.

The American scientist A.A. Michelson set out to determine the velocity of the earth through the luminiferous medium aether using a light wave interferometer that he had invented. When the Michelson-Morley experiment was conducted by Edward Morley and Albert Abraham Michelson in 1887, it produced a null result for the change of the velocity of light due to the Earth's motion through the hypothesized aether. This null result was in line with the theory that was proposed in 1845 by George Stokes which suggested that the aether was entrained with the Earth's orbital motion.

Hendrik Lorentz objected to Stokes' aether drag model and along with George FitzGerald and Joseph Larmor, he suggested another approach. Both Larmor (1897) and Lorentz (1899, 1904) derived the Lorentz transformation (so named by Henri Poincaré) as one under which Maxwell's equations were invariant. Poincaré (1900) analyzed the coordination of moving clocks by exchanging light signals. He also established mathematically the group property of the Lorentz transformation (Poincaré 1905).

This culminated in Albert Einstein's theory of special relativity, which postulated the absence of any absolute rest frame, dismissed the aether as unnecessary (a bold idea, which did not come to Lorentz nor to Poincaré), and established the invariance of Maxwell's equations in all inertial frames of reference, in contrast to the famous Newtonian equations for classical mechanics. But the transformations between two different inertial frames had to correspond to Lorentz' equations and not - as formerly believed - to those of Galileo (called Galilean transformations).[47] Indeed, Maxwell's equations played a key role in Einstein's famous paper on special relativity; for example, in the opening paragraph of the paper, he motivated his theory by noting that a description of a conductor moving with respect to a magnet must generate a consistent set of

fields irrespective of whether the force is calculated in the rest frame of the magnet or that of the conductor.[48]

General relativity has also had a close relationship with Maxwell's equations. For example, Kaluza and Klein showed in the 1920s that Maxwell's equations can be derived by extending general relativity into five dimensions. This strategy of using higher dimensions to unify different forces continues to be an active area of research in particle physics.

[] Covariant formulation of Maxwell's equations

Main article: Covariant formulation of classical electromagnetism

In special relativity, in order to more clearly express the fact that Maxwell's equations in *vacuum* take the same form in any inertial coordinate system, Maxwell's equations are written in terms of four-vectors and tensors in the "manifestly covariant" form. The purely spatial components of the following are in SI units.

One ingredient in this formulation is the electromagnetic tensor, a rank-2 covariant antisymmetric tensor combining the electric and magnetic fields:

$$F_{\alpha\beta} = \begin{pmatrix} 0 & \frac{-E_x}{c} & \frac{-E_y}{c} & \frac{-E_z}{c} \\ \frac{E_x}{c} & 0 & B_z & -B_y \\ \frac{E_y}{c} & -B_z & 0 & B_x \\ \frac{E_z}{c} & B_y & -B_x & 0 \end{pmatrix}$$

and the result of raising its indices

$$F^{\mu\nu} \stackrel{\text{def}}{=} \eta^{\mu\alpha} F_{\alpha\beta} \eta^{\beta\nu} = \begin{pmatrix} 0 & \frac{E_x}{c} & \frac{E_y}{c} & \frac{E_z}{c} \\ \frac{-E_x}{c} & 0 & B_z & -B_y \\ \frac{-E_y}{c} & -B_z & 0 & B_x \\ \frac{-E_z}{c} & B_y & -B_x & 0 \end{pmatrix}.$$

The other ingredient is the four-current: $J^{\alpha} = (c\rho, \vec{J})$ where ρ is the charge density and J is the current density.

With these ingredients, Maxwell's equations can be written:

$$\mu_0 J^{\beta} = \frac{\partial F^{\beta\alpha}}{\partial x^{\alpha}} \stackrel{\text{def}}{=} \partial_{\alpha} F^{\beta\alpha} \stackrel{\text{def}}{=} F^{\beta\alpha}{}_{,\alpha}$$

and

$$0 = \partial_\gamma F_{\alpha\beta} + \partial_\beta F_{\gamma\alpha} + \partial_\alpha F_{\beta\gamma} \overset{\text{def}}{=} F_{\alpha\beta,\gamma} + F_{\gamma\alpha,\beta} + F_{\beta\gamma,\alpha}.$$

The first tensor equation is an expression of the two inhomogeneous Maxwell's equations, Gauss's law and Ampere's law with Maxwell's correction. The second equation is an expression of the two homogeneous equations, Faraday's law of induction and Gauss's law for magnetism. The second equation is equivalent to

$$0 = \epsilon^{\delta\alpha\beta\gamma} F_{\beta\gamma,\alpha}$$

where $\epsilon^{\alpha\beta\gamma\delta}$ is the contravariant version of the Levi-Civita symbol, and

$$\frac{\partial}{\partial x^\alpha} \overset{\text{def}}{=} \partial_\alpha \overset{\text{def}}{=} {}_{,\alpha} \overset{\text{def}}{=} \left(\frac{1}{c} \frac{\partial}{\partial t}, \nabla \right)$$

is the 4-gradient. In the tensor equations above, repeated indices are summed over according to Einstein summation convention. We have displayed the results in several common notations. Upper and lower components of a vector, v^α and v_α respectively, are interchanged with the fundamental tensor g, e.g., $g=\eta=$diag(-1,+1,+1,+1).

Alternative covariant presentations of Maxwell's equations also exist, for example in terms of the four-potential; see Covariant formulation of classical electromagnetism for details.

[] Potentials

Main article: Mathematical descriptions of the electromagnetic field

Maxwell's equations can be written in an alternative form, involving the electric potential (also called scalar potential) and magnetic potential (also called vector potential), as follows.[18] (The following equations are valid in the absence of dielectric and magnetic materials; or if such materials are present, they are valid as long as bound charge and bound current are included in the total charge and current densities.)

First, Gauss's law for magnetism states:

$$\nabla \cdot \mathbf{B} = 0.$$

By Helmholtz's theorem, $\mathbf{B}$ can be written in terms of a vector field $\mathbf{A}$, called the magnetic potential:

$$\mathbf{B} = \nabla \times \mathbf{A}.$$

Second, plugging this into Faraday's law, we get:

$$\nabla \times \left(\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} \right) = 0.$$

By Helmholtz's theorem, the quantity in parentheses can be written in terms of a scalar function φ, called the electric potential:

$$\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\nabla \varphi.$$

Combining these with the remaining two Maxwell's equations yields the four relations:

$$\mathbf{E} = -\nabla \varphi - \frac{\partial \mathbf{A}}{\partial t}$$
$$\mathbf{B} = \nabla \times \mathbf{A}$$
$$\nabla^2 \varphi + \frac{\partial}{\partial t} (\nabla \cdot \mathbf{A}) = -\frac{\rho}{\varepsilon_0}$$
$$\left(\nabla^2 \mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} \right) - \nabla \left(\nabla \cdot \mathbf{A} + \frac{1}{c^2} \frac{\partial \varphi}{\partial t} \right) = -\mu_0 \mathbf{J}.$$

These equations, taken together, are as powerful and complete as Maxwell's equations. Moreover, if we work only with the potentials and ignore the fields, the problem has been reduced somewhat, as the electric and magnetic fields each have three components which need to be solved for (six components altogether), while the electric and magnetic potentials have only four components altogether.

Many different choices of $\mathbf{A}$ and φ are consistent with a given $\mathbf{E}$ and $\mathbf{B}$, making these choices physically equivalent – a flexibility known as gauge freedom. Suitable choice of $\mathbf{A}$ and φ can simplify these equations, or can adapt them to suit a particular situation. For more information, see the article gauge freedom.

[] Four-potential

The two equations that represent the potentials can be reduced to one manifestly Lorentz invariant equation, using four-vectors: the four-current defined by

$$j^\mu = (\rho c, \mathbf{j})$$

formed from the current density $\mathbf{j}$ and charge density ρ, and the electromagnetic four-potential defined by

$$A^\mu = (\varphi, \mathbf{A}c)$$

formed from the vector potential **A** and the scalar potential φ. The resulting single equation, due to Arnold Sommerfeld, a generalization of an equation due to Bernhard Riemann and known as the Riemann-Sommerfeld equation[49] or the covariant form of the Maxwell-Lorentz equations,[50] is:

$$\Box A^{\mu} = \mu_0 j^{\mu},$$

where $\Box = \partial^2 = \partial_\alpha \partial^\alpha$ is the d'Alembertian operator, or four-Laplacian, $\left(\dfrac{\partial^2}{\partial t^2} - \nabla^2 \right)$, sometimes written $\Box^2$, or $\Box \cdot \Box$, where $\Box$ is the four-gradient.

[] Differential forms

In free space, where $\varepsilon = \varepsilon_0$ and $\mu = \mu_0$ are constant everywhere, Maxwell's equations simplify considerably once the language of differential geometry and differential forms is used. In what follows, cgs-Gaussian units, not SI units are used. (To convert to SI, see here.) The electric and magnetic fields are now jointly described by a 2-form **F** in a 4-dimensional spacetime manifold. Maxwell's equations then reduce to the Bianchi identity

$$\mathrm{d}\mathbf{F} = 0$$

where d denotes the exterior derivative — a natural coordinate and metric independent differential operator acting on forms — and the source equation

$$\mathrm{d} * \mathbf{F} = \mathbf{J}$$

where the (dual) Hodge star operator * is a linear transformation from the space of 2-forms to the space of (4-2)-forms defined by the metric in Minkowski space (in four dimensions even by any metric conformal to this metric), and the fields are in natural units where $1 / 4\pi\varepsilon_0 = 1$. Here, the 3-form **J** is called the *electric current form* or *current 3-form* satisfying the continuity equation

$$\mathrm{d}\mathbf{J} = 0.$$

The current 3-form can be integrated over a 3-dimensional space-time region. The physical interpretation of this integral is the charge in that region if it is spacelike, or the amount of charge that flows through a surface in a certain amount of time if that region is a spacelike surface cross a timelike interval. As the exterior derivative is defined on any manifold, the differential form version of the Bianchi identity makes sense for any 4-dimensional manifold, whereas the source equation is defined if the manifold is oriented and has a Lorentz metric. In particular the differential form version of the Maxwell equations are a convenient and intuitive formulation of the Maxwell equations in general relativity.

In a linear, macroscopic theory, the influence of matter on the electromagnetic field is described through more general linear transformation in the space of 2-forms. We call

$$C : \Lambda^2 \ni \mathbf{F} \mapsto \mathbf{G} \in \Lambda^{(4-2)}$$

the constitutive transformation. The role of this transformation is comparable to the Hodge duality transformation. The Maxwell equations in the presence of matter then become:

$$\mathrm{d}\mathbf{F} = 0$$
$$\mathrm{d}\mathbf{G} = \mathbf{J}$$

where the current 3-form $\mathbf{J}$ still satisfies the continuity equation $\mathrm{d}\mathbf{J} = 0$.

When the fields are expressed as linear combinations (of exterior products) of basis forms θ^p,

$$\mathbf{F} = \frac{1}{2}F_{pq}\theta^p \wedge \theta^q .$$

the constitutive relation takes the form

$$G_{pq} = C_{pq}^{mn} F_{mn}$$

where the field coefficient functions are antisymmetric in the indices and the constitutive coefficients are antisymmetric in the corresponding pairs. In particular, the Hodge duality transformation leading to the vacuum equations discussed above are obtained by taking

$$C_{pq}^{mn} = \frac{1}{2}g^{ma}g^{nb}\epsilon_{abpq}\sqrt{-g}$$

which up to scaling is the only invariant tensor of this type that can be defined with the metric.

In this formulation, electromagnetism generalises immediately to any 4-dimensional oriented manifold or with small adaptations any manifold, requiring not even a metric. Thus the expression of Maxwell's equations in terms of differential forms leads to a further notational and conceptual simplification. Whereas Maxwell's Equations could be written as two tensor equations instead of eight scalar equations, from which the propagation of electromagnetic disturbances and the continuity equation could be derived with a little effort, using differential forms leads to an even simpler derivation of these results.

[] Conceptual insight from this formulation

On the conceptual side, from the point of view of physics, this shows that the second and third Maxwell equations should be grouped together, be called the homogeneous ones, and be seen as geometric *identities* expressing nothing else than: the *field* $\mathbf{F}$ derives from a more "fundamental" *potential* $\mathbf{A}$. While the first and last one should be seen as the dynamical *equations of motion*, obtained via the Lagrangian principle of least action, from the "interaction term" $\mathbf{A}\,\mathbf{J}$ (introduced through gauge covariant derivatives), coupling the field to matter.

Often, the time derivative in the third law motivates calling this equation "dynamical", which is somewhat misleading; in the sense of the preceding analysis, this is rather an artifact of breaking relativistic covariance by choosing a preferred time direction. To have physical degrees of freedom propagated by these field equations, one must include a kinetic term **F** ***F** for **A**; and take into account the non-physical degrees of freedom which can be removed by gauge transformation **A**→**A'** = **A**-dα: see also gauge fixing and Fadeev-Popov ghosts.

[] Classical electrodynamics as the curvature of a line bundle

An elegant and intuitive way to formulate Maxwell's equations is to use complex line bundles or principal bundles with fibre U(1). The connection ∇ on the line bundle has a curvature $\mathbf{F} = \nabla^2$ which is a two-form that automatically satisfies $\mathrm{d}\mathbf{F} = 0$ and can be interpreted as a field-strength. If the line bundle is trivial with flat reference connection d we can write $\nabla = \mathrm{d} + \mathbf{A}$ and **F** = d**A** with **A** the 1-form composed of the electric potential and the magnetic vector potential.

In quantum mechanics, the connection itself is used to define the dynamics of the system. This formulation allows a natural description of the Aharonov-Bohm effect. In this experiment, a static magnetic field runs through a long magnetic wire (e.g., an iron wire magnetized longitudinally). Outside of this wire the magnetic induction is zero, in contrast to the vector potential, which essentially depends on the magnetic flux through the cross-section of the wire and does not vanish outside. Since there is no electric field either, the Maxwell tensor **F = 0** throughout the space-time region outside the tube, during the experiment. This means by definition that the connection ∇ is flat there.

However, as mentioned, the connection depends on the magnetic field through the tube since the holonomy along a non-contractible curve encircling the tube is the magnetic flux through the tube in the proper units. This can be detected quantum-mechanically with a double-slit electron diffraction experiment on an electron wave traveling around the tube. The holonomy corresponds to an extra phase shift, which leads to a shift in the diffraction pattern. *(See* Michael Murray*, Line Bundles,* 2002 (PDF web link) *for a simple mathematical review of this formulation. See also* R. Bott*, On some recent interactions between mathematics and physics,* Canadian Mathematical Bulletin, **28** (1985) no. 2 pp 129–164.)

[] Curved spacetime

Main article: Maxwell's equations in curved spacetime

[] Traditional formulation

Matter and energy generate curvature of spacetime. This is the subject of general relativity. Curvature of spacetime affects electrodynamics. An electromagnetic field having energy and momentum will also generate curvature in spacetime. Maxwell's equations in curved spacetime can be obtained by replacing the derivatives in the equations in flat spacetime with covariant

derivatives. (Whether this is the appropriate generalization requires separate investigation.) The sourced and source-free equations become (cgs-Gaussian units):

$$\frac{4\pi}{c}j^{\beta} = \partial_{\alpha}F^{\alpha\beta} + \Gamma^{\alpha}{}_{\mu\alpha}F^{\mu\beta} + \Gamma^{\beta}{}_{\mu\alpha}F^{\alpha\mu} \stackrel{\text{def}}{=} D_{\alpha}F^{\alpha\beta} \stackrel{\text{def}}{=} F^{\alpha\beta}{}_{;\alpha}$$

and

$$0 = \partial_{\gamma}F_{\alpha\beta} + \partial_{\beta}F_{\gamma\alpha} + \partial_{\alpha}F_{\beta\gamma} = D_{\gamma}F_{\alpha\beta} + D_{\beta}F_{\gamma\alpha} + D_{\alpha}F_{\beta\gamma}.$$

Here,

$$\Gamma^{\alpha}{}_{\mu\beta}$$

is a Christoffel symbol that characterizes the curvature of spacetime and D_{γ} is the covariant derivative.

[] Formulation in terms of differential forms

The formulation of the Maxwell equations in terms of differential forms can be used without change in general relativity. The equivalence of the more traditional general relativistic formulation using the covariant derivative with the differential form formulation can be seen as follows. Choose local coordinates x^{α} which gives a basis of 1-forms dx^{α} in every point of the open set where the coordinates are defined. Using this basis and cgs-Gaussian units we define

- The antisymmetric infinitesimal field tensor $F_{\alpha\beta}$, corresponding to the field 2-form $\mathbf{F}$

$$\mathbf{F} := \frac{1}{2}F_{\alpha\beta}\,\mathrm{d}\,x^{\alpha} \wedge \mathrm{d}\,x^{\beta}.$$

- The current-vector infinitesimal 3-form $\mathbf{J}$

$$\mathbf{J} := \frac{4\pi}{c}j^{\alpha}\sqrt{-g}\,\epsilon_{\alpha\beta\gamma\delta}\mathrm{d}\,x^{\beta} \wedge \mathrm{d}\,x^{\gamma} \wedge \mathrm{d}\,x^{\delta}.$$

Here g is as usual the determinant of the metric tensor $g_{\alpha\beta}$. A small computation that uses the symmetry of the Christoffel symbols (i.e., the torsion-freeness of the Levi Civita connection) and the covariant constantness of the Hodge star operator then shows that in this coordinate neighborhood we have:

- the Bianchi identity

$$\mathrm{d}\mathbf{F} = 2(\partial_{\gamma}F_{\alpha\beta} + \partial_{\beta}F_{\gamma\alpha} + \partial_{\alpha}F_{\beta\gamma})\mathrm{d}x^{\alpha} \wedge \mathrm{d}\,x^{\beta} \wedge \mathrm{d}\,x^{\gamma} = 0,$$

- the source equation

$$\mathrm{d} * \mathbf{F} = F^{\alpha\beta}{}_{;\alpha}\sqrt{-g}\,\epsilon_{\beta\gamma\delta\eta}\mathrm{d}\,x^{\gamma} \wedge \mathrm{d}\,x^{\delta} \wedge \mathrm{d}\,x^{\eta} = \mathbf{J},$$

- the continuity equation

$$\mathrm{d}\mathbf{J} = \frac{4\pi}{c}j^{\alpha}{}_{;\alpha}\sqrt{-g}\,\epsilon_{\alpha\beta\gamma\delta}\mathrm{d}\,x^{\alpha} \wedge \mathrm{d}\,x^{\beta} \wedge \mathrm{d}\,x^{\gamma} \wedge \mathrm{d}\,x^{\delta} = 0.$$

ÉLECTRICITÉ ET MAGNÉTISME

Lectures Obligatoires

Source: Wikipedia.org

Table des matières

Magnétisme

Le **magnétisme** est un [phénomène](#) physique, par lequel se manifestent des forces attractives ou répulsives d'un objet sur un autre, ou avec des [charges électriques](#) en mouvement. Ces objets, dits magnétisables, sont susceptibles de réagir au [champ magnétique](#) par une réaction d'orientation et/ou de déplacement dépendante de la force et de l'orientation. Cette force s'effectue par l'intermédiaire du [champ magnétique](#), et est produite par des charges en mouvement ou des [aimants](#).

Expérience d'Ørsted []

En 1820, le danois [Hans Christian Ørsted](#) montre qu'à proximité d'un fil rectiligne parcouru par un courant électrique, l'aiguille d'une boussole dévie.

« Le déplacement de charges électriques crée un [champ magnétique](#). »

Caractéristiques du vecteur d'induction (densité de flux) du champ magnétique $\vec{B}$[]

Il règne un champ magnétique lorsqu'une aiguille aimantée prend une direction déterminée.

- direction : celle de l'aiguille aimantée qui détecte le flux magnétique du champ.
- sens : choisi selon le sens sud-nord de l'aiguille aimantée.
- norme : [unité SI](#), le [tesla](#) (T).

Magnétisme dans la matière []

[Faraday](#) a montré que toute substance est aimantable mais le plus souvent l'effet n'est appréciable que dans un champ magnétique intense ; en plaçant dans un champ magnétique non uniforme des barreaux de substances différentes :

- certains sont attirés vers les régions de champ intense en s'orientant parallèlement aux lignes de champ comme le ferait un barreau de fer doux ;
- d'autres sont repoussées vers les régions où le champ magnétique est faible et s'orientent perpendiculairement aux lignes de champ ; de telles substances sont dites [diamagnétiques](#) ([argent](#), [or](#), [cuivre](#), [mercure](#), [plomb](#), presque tous les composés organiques…).

Les substances qui sont comparables au [fer](#) sont dites [ferromagnétiques](#) (fer, [cobalt](#), [nickel](#) et un grand nombre de leurs alliages en particulier les [aciers](#)) et certain de leurs composés ainsi que certaines combinaisons d'éléments non ferromagnétiques.

Les substances qui subissent des actions de même nature que le fer mais beaucoup moins intenses sont dites paramagnétiques (aluminium, chrome, platine… et certains composés d'éléments ferromagnétiques par exemple l'alliage 68% fer 32% de nickel).

Description macroscopique []

Un solénoïde (enroulement cylindrique) parcouru par un courant d'intensité I crée un champ magnétique noté $\vec{B}_0$. Si, à l'intérieur de ce solénoïde on place un matériau, on constate une modification du module du vecteur champ magnétique que l'on notera maintenant $\vec{B}$.

Remarque : dans certains ouvrages anciens ou certains livres techniques $\vec{B}$ est appelé *vecteur induction magnétique*

Excitation magnétique []

B_0 Champ magnétique crée dans le vide

B Champ magnétique crée dans le matériau
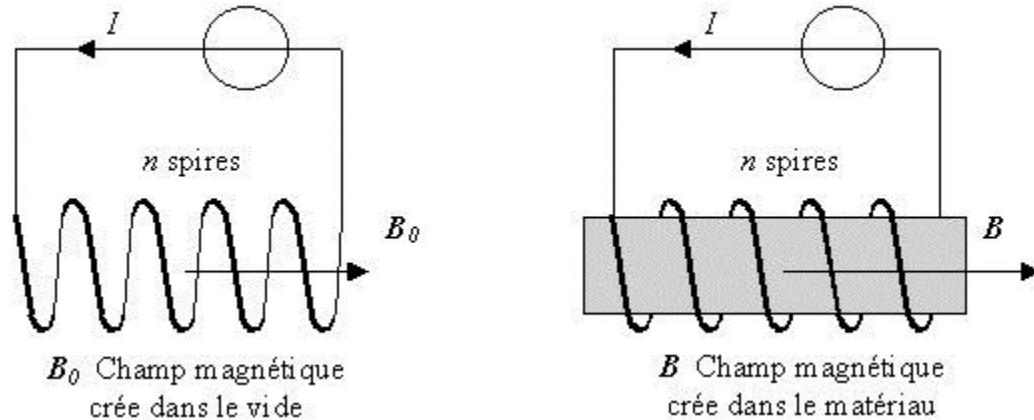
$$\vec{B}_0 = \mu_0 \vec{H}$$

On pose : $\vec{H} = \dfrac{\vec{B}}{\mu_0} - \vec{M}$, avec μ_0 : *perméabilité* du vide, et $\vec{M}$, aimantation

Perméabilité et susceptibilité magnétiques []

La présence du matériau modifie le champ magnétique. On pose :

- $\vec{B} = \mu . \vec{H}$ avec μ : *perméabilité magnétique* du matériau

On définit par $\vec{M}$ le *vecteur aimantation* acquise par la matière

- $\vec{M} = \chi . \vec{H}$ avec χ : *susceptibilité magnétique* du matériau
    - d'où : $\vec{B} = \mu_0(\vec{H} + \vec{M}) = (1 + \chi)\vec{B}_0$

On pose aussi :

- $$\mu_r = \frac{\mu}{\mu_0} = (1 + \chi)$$ avec μ_r : *[perméabilité relative](#)* du matériau.

Classification des effets magnétiques []

- [Diamagnétisme](#) : matériaux pour lesquels χ est négatif mais toujours extrêmement faible, de l'ordre de 10^{-5}
- [Paramagnétisme](#) : matériaux pour lesquels χ est positif mais toujours très faible, de l'ordre de 10^{-3}
- [Ferromagnétisme](#) et [ferrimagnétisme](#) : matériaux pour lesquels χ est positif et très grand, il peut atteindre 10^5 ! En [électrotechnique](#) seuls ces matériaux sont importants car ce sont les seuls à produire des augmentations du champ magnétique qui sont significatives (voir ci-dessous).

Origine microscopique du magnétisme []

Mouvement des électrons []

Le mouvement des électrons dans le nuage électronique est responsable de l'existence d'un magnétisme dit *orbital*, alors que la rotation sur eux-mêmes est responsable du *magnétisme de spin*. Il n'est pas possible d'ignorer l'aspect [quantique](#) de ces phénomènes : en 1919, dans sa thèse de Doctorat, J. H. van Leeuwen prouva qu'il était impossible d'expliquer le magnétisme uniquement à l'aide de l'électrodynamique de Maxwell et de la mécanique statistique classique.

Origine du diamagnétisme []

L'effet d'un champ magnétique est de donner à l'ensemble du mouvement électronique une vitesse angulaire de rotation autour de la direction du champ magnétique appliqué : phénomène classique d'[induction](#). Ce moment magnétique *induit* est proportionnel au champ appliqué et s'oppose à ce dernier. C'est l'origine du diamagnétisme qui est donc un phénomène tout à fait général mais qui peut être masqué par les autres phénomènes dont l'effet est plus important.

Remarque : on emploi le terme de *diamagnétisme parfait* pour désigner le comportement des supraconducteurs qui créent en leur sein des courants induits qui s'opposent à toute variation de champ magnétique. Cette propriété est utilisée pour produire la lévitation magnétique des supraconducteurs.

Origine du paramagnétisme []

Lorsque les atomes possèdent leur propre [moment magnétique](#) permanent, le diamagnétisme (toujours présent) est masqué par le paramagnétisme. Sous l'effet d'un champ magnétique extérieur, ces atomes, petits aimants permanents, s'orientent selon le champ appliqué et

l'amplifient. Ce phénomène est limité par l'agitation thermique et dépend fortement de la température : (loi de Curie : $\mathbf{M} = C \cdot \dfrac{\mathbf{B}}{T}$)

Ce phénomène est lié à l'existence du spin de l'électron.

- Pour les atomes : un atome dont les couches électroniques sont totalement remplies ne possède pas de moment magnétique. Lorsque les couches sont incomplètes, il y a toujours un déséquilibre qui produit un moment magnétique de spin.
- Pour les solides cela peut être très différent : les électrons externes participent aux liaisons chimiques. Dans les liaisons covalentes les électrons appariés sont de spin opposé. Les ions des cristaux ioniques ont des couches complètes. On peut donc avoir une disparition du magnétisme propre. L'existence du paramagnétisme subsiste pour les solides composés d'atomes ayant des couches électroniques internes incomplètes : métaux de transitions et Lanthanides (terres rares) par exemple.

Ferromagnétisme []

Article détaillé : Ferromagnétisme.

C'est la propriété qu'ont certains corps de s'aimanter *très fortement* sous l'effet d'un champ magnétique extérieur, et pour certains d'entre-eux, appelés aimants (*ie.* les matériaux magnétiques durs), de garder une aimantation importante même après la disparition du champ extérieur (aimantation rémanente).

Corps ferromagnétiques []

Pour l'usage industriel, seul le fer, le cobalt et le nickel sont des ferromagnétiques intéressants. Certaines terres rares (les Lanthanides dans la classifiation périodique) sont également ferromagnétiques à basse température.

En ce qui concerne les alliages, la situation est très complexe : certains alliages de fer et de nickel ne sont pas ferromagnétiques, alors que l'alliage d'Heussler, constitué uniquement de métaux non ferromagnétiques (61 % Cu, 24 % Mn, 15 % Al), est ferromagnétique.

Enfin, il faut ajouter les ferrites, dont la composition est de la forme (MO ; Fe_2O_3) où M est un métal divalent et dont le représentant le plus ancien est la magnétite Fe_3O_4 (FeO ; Fe_2O_3).

Courbe de première aimantation []

Courbe de l'aimantation

Courbe du champ magnétique
dans le matériau
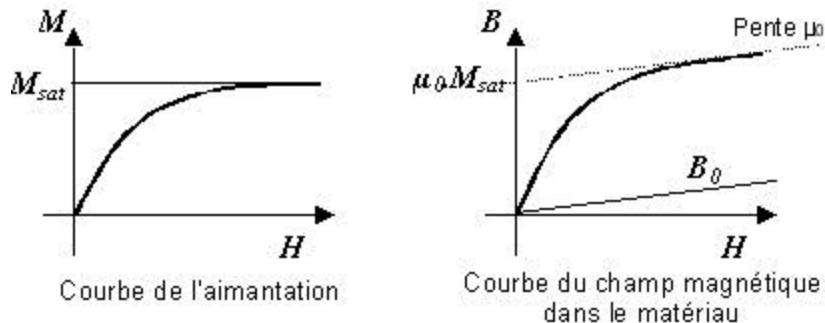
Cycles d'hystéresis []

Article détaillé : Hystérésis.

Lorsqu'on a magnétisé un échantillon de matériau jusqu'à la saturation et que l'on fait décroître l'excitation H, on constate que B décroît également, mais en suivant une courbe différente qui se situe au-dessus de la courbe de première aimantation. Ceci est le fait d'un *retard* à la désaimantation. On dit qu'il y a hystérésis.

Lorsque H est ramené à 0, il subsiste un champ magnétique B_r appelé *champ rémanent* (du latin *remanere*, rester). Pour annuler ce champ rémanent, il est nécessaire d'inverser le courant dans le solénoïde, c'est-à-dire d'imposer à H une valeur négative. Le champ magnétique s'annule alors pour une valeur de l'excitation H_c appelée *excitation coercitive*.

Conséquences de l'hystérésis []

L'aimantation de la matière absorbe de l'énergie qui n'est que partiellement restituée au cours de la désaimantation. Cette énergie est dissipée sous forme calorifique : le matériau s'échauffe. On démontre que les pertes par hystérésis sont proportionnelles à l'aire du cycle d'hystérésis.

Dans le cas où la substance ferromagnétique doit décrire un grand nombre de cycles d'hytérésis (machines tournantes, transformateurs…), il faut choisir des matériaux tels que l'aire du cycle soit aussi petite que possible. Ces matériaux sont dits magnétiquement « doux. »

À l'opposé, c'est grâce à une hystérésis importante que l'on peut réaliser des aimants permanents. On utilise pour leur fabrication des matériaux magnétiquement durs : certains aciers à l'aluminium, au nickel ou au cobalt conviennent parfaitement. On réalise aussi des aimants avec de la poudre de fer agglomérée dans un isolant.

Matériaux magnétiques doux []
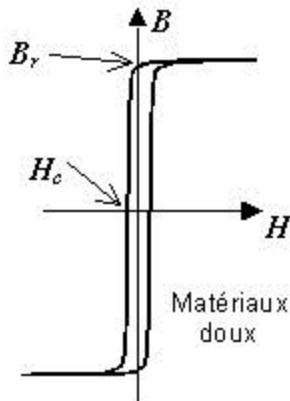
Ce sont en général des matériaux doux mécaniquement. Ces matériaux ont des cycles très étroits : l'excitation cœrcitive ne dépasse pas 100 A.m^{-1}. Ils possèdent une grande perméabilité.

Quelques exemples :

- SuperMalloy (fer, nickel, molybdène...) : $H_c = 0,16$ A.m^{-1} ; $B_r = 1,2$ T (l'un des plus doux) ;
- Fer + 3 % de Silicium, grains orientés : $H_c = 8$ A.m^{-1} ; $B_r = 1,0$ T

Les matériaux magnétiques doux sont utilisés pour réaliser des électroaimants (leur aimantation doit pouvoir facilement être annulée) ou des circuits magnétiques fonctionnant en régime alternatif (machines électriques, transformateurs).

Matériaux magnétiques durs []
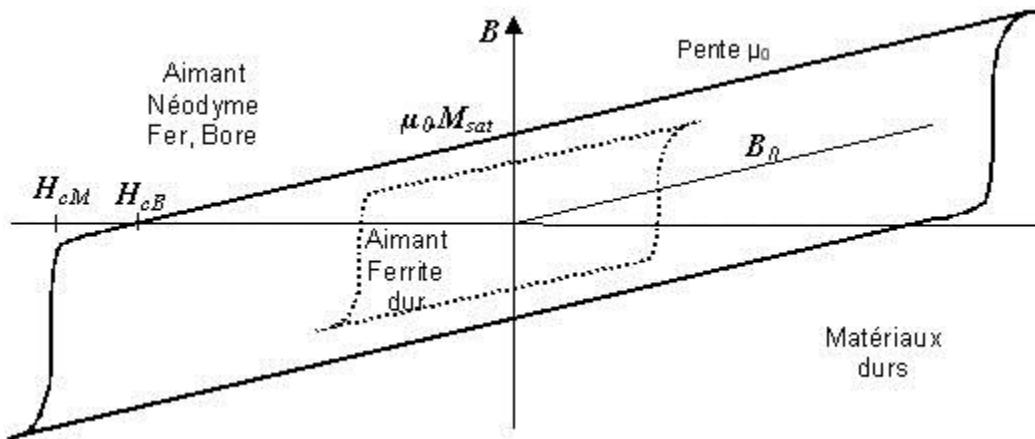
Contrairement aux précédents, les cycles sont extrêmement larges : plusieurs centaines de kA.m^{-1}. Il est impossible de les dessiner dans un même repère que les précédents.

Certains de ces matériaux à base de terres rares (alliages samarium-cobalt ou néodyme-fer-bore) ne se désaimantent pas, même lorsqu'on annule le champ magnétique interne (l'excitation vaut

alors H_{cB}). Pour annuler (en fait inverser) l'aimantation, il est nécessaire de fournir une excitation magnétique que l'on appelle H_{cM} : excitation de désaimantation irréversible.

L'application de ces matériaux est la réalisation d'aimants permanents de très forte puissance. Les ferrofluides sont des suspensions de particules aimantées de taille micronique dans un liquide. Ces liquides réagissent à un champ magnétique extérieur (par exemple, leur surface se hérisse de pointes).

Origine microscopique du ferromagnétisme []

La théorie des intégrales (ou interactions) d'échange proposée par Heisenberg en 1928 constitue le fondement théorique des explications de ce phénomène. Lorsqu'un solide est constitué d'atomes paramagnétiques (chaque atome peut être assimilé à un petit aimant), il se produit un couplage entre ces derniers.

Ferromagnétisme []

Article détaillé : Ferromagnétisme.

Lorsque les atomes sont éloignés les uns des autres dans la structure cristalline, le couplage favorise un alignement de ces aimants élémentaires. C'est le cas du Fer α (structure cubique centrée), du nickel, du cobalt et, plus faiblement, de certains métaux de la famille des terres rares comme le Gadolinium. Quelques alliages dont les mailles sont grandes peuvent avoir cette propriété.

Antiferromagnétisme []

Article détaillé : Antiferromagnétisme.

Lorsque les atomes sont plus proches les uns des autres, comme c'est le cas pour le chrome, le manganèse ou l'hématite, la configuration la plus stable correspond à des aimants en antiparallèle. Il n'y a alors plus d'aimantation apparente à grande distance car chaque aimant élémentaire est compensé par son voisin.

Ferrimagnétisme []

Article détaillé : Ferrimagnétisme.

Il s'observe dans des matériaux comportant deux types d'atomes différents, produisant chacun des aimants élémentaires de force différente et orientés en tête-bêche.

Domaines de Weiss []

Article détaillé : Domaine de Weiss.

Lorsqu'un matériau est ferro ou ferrimagnétique, il est divisé en domaines, appelés *domaines de Weiss*, à l'intérieur duquel l'orientation magnétique est identique. Ce domaine se comporte alors comme un aimant. Ces domaines sont séparés par des parois dites *parois de Bloch*.

- Ces domaines n'existent pas lorsque les dimensions du matériau sont très faibles (quelques nm). Ces matériaux sont dits nanocristallins.
- Le déplacement de ces parois est responsable des phénomènes d'hystérésis.

Champ magnétique

Un article de Wikipédia, l'encyclopédie libre.

Aller à : Navigation, rechercher
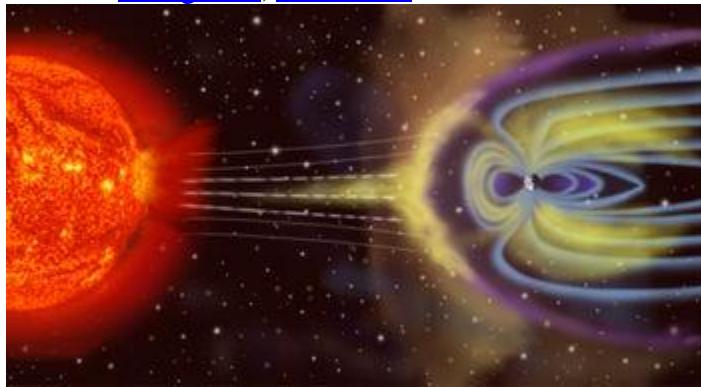
Vue d'artiste de la magnétosphère terrestre.
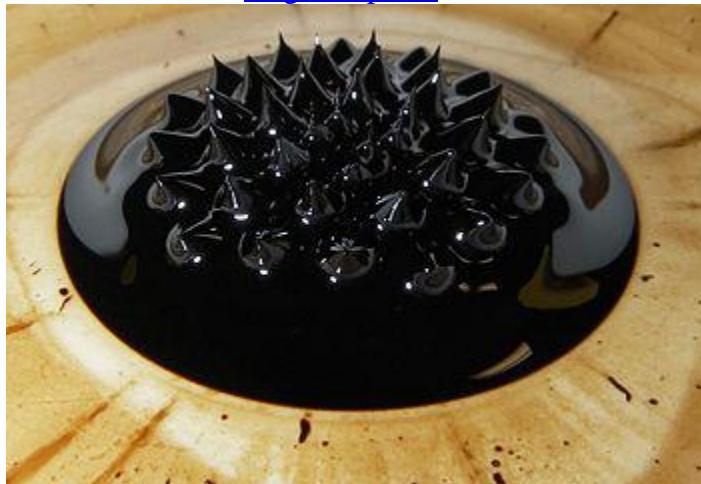
Ferrofluide soumis à un **champ magnétique** dont l'instabilité provoque des pointes qui l'emportent sur la gravité et la tension superficielle du fluide.

En physique, le **champ magnétique** (ou **induction magnétique**, ou **densité de flux magnétique**) est une grandeur caractérisée par la donnée d'une intensité et d'une direction, définie en tout point de l'espace, et déterminée par la position et l'orientation d'aimants,

d'électroaimants et le déplacement de charges électriques. La présence de ce champ se traduit par l'existence d'une force agissant sur les charges électriques en mouvement (dite force de Lorentz), et divers effets affectant certains matériaux (paramagnétisme, diamagnétisme ou ferromagnétisme selon les cas). La grandeur qui détermine l'interaction entre un matériau et un champ magnétique est la susceptibilité magnétique.

Le champ magnétique forme, avec le champ électrique les deux composantes du champ électromagnétique décrit par l'électromagnétisme. Des ondes de champs électrique et magnétique mêlées peuvent se propager librement dans l'espace, et dans la plupart des matériaux. Ces ondes sont appelées ondes électromagnétiques, et correspondent à toutes les manifestations de la lumière, dans tous les domaines de longueur d'onde (ondes radio, domaine micro-onde, infrarouge, domaine visible, ultraviolet, rayons X et rayons gamma). La discipline qui étudie les champs magnétiques statiques (ne dépendant pas du temps) est la magnétostatique.

Les applications de la maîtrise de ce champ sont nombreuses, même dans la vie courante : outre le fait que celui-ci est une composante de la lumière, il explique l'attraction des aimants, l'orientation des boussoles et permet entre autres la construction d'alternateurs et de moteurs électriques. Le stockage d'informations sur bandes magnétiques ou disques durs se fait à l'aide de champs magnétiques. Des champs magnétiques de très forte intensité sont utilisés dans les accélérateurs de particules ou les tokamaks pour focaliser un faisceau de particules très énergétiques dans le but de les faire entrer en collision. Les champs magnétiques sont également omniprésents en astronomie, où ils sont à l'origine de nombreux phénomènes comme le rayonnement synchrotron et le rayonnement de courbure, ainsi que la formation de jets dans les régions où l'on observe un disque d'accrétion. Le rayonnement synchrotron est également abondamment utilisé dans de nombreuses applications industrielles.

Mathématiquement, le champ magnétique est décrit par un champ pseudo vectoriel[1], qui se rapproche d'un champ de vecteurs par plusieurs aspects, mais présente quelques subtilités au niveau des symétries. Les équations décrivant l'évolution du champ magnétique sont appelées équations de Maxwell, en l'honneur de James Clerk Maxwell qui les a finalisées en 1873. C'est cependant Albert Einstein qui en 1905 en a proposé le premier la vision la plus cohérente, dans le cadre de la relativité restreinte qu'il venait de découvrir et qui en est indissociable.

Historique []

Article détaillé : Histoire de l'électricité.

Dès le VIe siècle av. J.-C., les philosophes grecs décrivaient — et tentaient d'expliquer — l'effet de minerais riches en magnétite. Ces roches étaient issues entre autres de la cité de Magnésie : elle donna son nom au phénomène.

La magnétite, minerai présentant des propriétés magnétiques, intriguait déjà les Grecs il y a 2 600 ans.

L'aiguille « *Montre-sud* » est mentionnée pour la première fois au XIe siècle par Chen Koua et, même s'il y a des attestations de la connaissance de l'aimant en Chine[2] dès le IIIe siècle av. J.-C., le problème du magnétisme terrestre apparaît beaucoup plus tard. L'utilisation de la boussole dans les techniques de navigation daterait du XIIe siècle et son usage exact reste à préciser du fait d'une navigation essentiellement côtière à cette époque[2]. Les boussoles faisaient usage du champ magnétique terrestre, qui se trouve être aujourd'hui à peu près aligné avec l'axe de rotation terrestre, raison pour laquelle une boussole, en indiquant le pôle magnétique, indique aussi (quoique approximativement) la direction du pôle géographique terrestre.

En Occident, Pierre de Maricourt fut l'un des premiers à travailler sur le magnétisme et publia son *Epistola de magnete* à peu près à la même époque que les savants chinois. Au-delà du simple problème des priorités, il serait intéressant de savoir comment certaines techniques ont pu voyager et s'il n'est pas possible que des développements parallèles, et chronologiquement presque concomitants, se soient produits[2].

Pour les encyclopédistes des *Lumières*[3], « *le magnétisme est le nom général qu'on donne aux différentes propriétés de l'aimant* ». Ils attribuent ses effets à une « *matière subtile*[4], *différente de l'air* » (parce que ces phénomènes ont également lieu dans le vide) qu'ils appellent *magnétique*. Plus loin ils affirment que « *c'est encore une question non moins difficile que de savoir s'il y a quelque rapport entre la cause du magnétisme & celle de l'électricité, car on ne connoît guère mieux l'une que l'autre.* »

Jusqu'au début des années 1820, on ne connaissait que le magnétisme des aimants naturels à base de magnétite. Hans Christian Ørsted montra en 1821 qu'un courant électrique parcourant un fil influence l'aiguille d'une boussole située à proximité. Il fut cependant incapable d'expliquer ce phénomène à la lumière des connaissances de l'époque. La même année, Michael Faraday énonce la loi de Faraday, qui trace un premier lien entre électricité et magnétisme.

En 1822, le premier moteur électrique est inventé : la roue de Barlow.

André-Marie Ampère proposa peu après une loi phénoménologique, aujourd'hui démontrée dans le cadre général de l'électromagnétisme, appelé théorème d'Ampère, qui relie le champ magnétique aux courants. Peu après, en 1825, l'électricien William Sturgeon crée le premier électroaimant.

En 1873, James Clerk Maxwell unifie le champ magnétique et le champ électrique, au sein de la théorie de l'électromagnétisme. Ce faisant, il découvre une incompatibilité entre les lois de la mécanique classique et les lois de l'électromagnétisme. Ces dernières prédisent que la vitesse de la lumière est indépendante de la vitesse d'un observateur par rapport à la source qui émet la lumière, hypothèse incompatible avec les lois de la mécanique classique.

En 1873, l'ingénieur belge Zénobe Gramme découvre le premier moteur électrique à courant continu, utilisable à grande échelle.

En 1887, les Américains Albert A. Michelson et Edward Morley vérifient expérimentalement (expérience de Michelson-Morley) les prédictions de Maxwell.

En 1905, Albert Einstein résout le paradoxe découvert par Maxwell en montrant que les lois de la mécanique classique doivent en réalité être remplacées par d'autres lois, celles de la relativité restreinte.

En 1933, Walther Meissner et Robert Ochsenfeld découvrent qu'un échantillon supraconducteur plongé dans un champ magnétique a tendance à expulser celui-ci de son intérieur (effet Meissner).

En 1944, Lars Onsager propose le premier modèle (dit modèle d'Ising) décrivant le phénomène de ferromagnétisme.

En 1966, le docteur Karl Strnat découvre les premiers aimants samarium-cobalt, d'une énergie phénoménale (18 à 30 MGOe)[5].

En 1968 sont découverts les pulsars, cadavres d'étoiles extraordinairement denses, siège des champs magnétiques les plus intenses existant aujourd'hui dans la nature (4×10^8 teslas pour le pulsar du Crabe, par exemple).

En 1983, une équipe internationale crée des aimants néodyme-fer-bore, les plus puissants aimants permanents connus à ce jour (35 MGOe soit environ 1,25 tesla[5]).

En 1998, une équipe russe crée un champ magnétique pulsé par une explosion qui atteint 2 800 T[6].

Le 12 décembre 1999, une équipe américaine crée un champ magnétique continu d'une intensité de 45 T[7].

En 2006, des champs magnétiques pulsés ont atteint 100 T sans destruction[8].

Expression du champ magnétique []

Notation []

On note généralement le champ magnétique avec la lettre B, écrite en caractère gras ou surmontée d'une flèche, ces deux notations indiquant qu'il s'agit d'un vecteur (ou en l'occurrence d'un [pseudovecteur](#)) : $\vec{B}$ ou **B**. Cette lettre, empruntée à [James Clerk Maxwell](#), vient de ses notations : il décrivait les trois composantes du champ magnétique indépendamment, par les lettres *B*, *C*, *D*. Les composantes du [champ électrique](#) étant, dans les notations de Maxwell les lettres *E*, *F*, *G*.

Le champ étant défini dans tout l'espace, c'est en fait une fonction des coordonnées, en général notées par le rayon vecteur *r*, et éventuellement du temps *t*, aussi est-il noté **B**(*r*) ou **B**(*r*, *t*). Cependant, on utilise souvent la notation **B**, la dépendance spatiale et/ou temporelle étant implicite.

Unités []

Article détaillé : [Intensité de champ magnétique](#).

Des aimants NdFeB, créant un champ de 1,25 tesla (en haut et entre les sphères), supportant
1 300 fois leur propre poids.

L'unité moderne utilisée pour quantifier l'intensité du champ magnétique est le tesla, défini en
1960[9]. C'est une unité dérivée du système SI. On définit un tesla par un flux d'induction
magnétique d'un weber par mètre carré :

$$1 \text{ T} = 1 \text{ Wb·m}^{-2} = 1 \text{ kg·s}^{-2}\text{·A}^{-1} = 1 \text{ N·A}^{-1}\text{·m}^{-1} = 1 \text{ kg·s}^{-1}\text{·C}^{-1}.$$

Pour diverses raisons historiques remontant aux travaux de Charles de Coulomb, certains auteurs
préfèrent utiliser des unités hors du système SI, comme le gauss[10] ou le gamma[11]. On a :

- 1 tesla ≈ 10 000 gauss (approximation) ;
- 1 tesla = 1 000 000 gamma.

Enfin, on utilise également parfois l'œrsted, notamment pour quantifier la « force » des aimants
naturels, dont l'équivalent SI est l'ampère par mètre A.m^{-1} par la relation :

$$1 \text{ Oe} = \frac{10^3}{4\pi} \text{A} \cdot \text{m}^{-1}.$$

Ordres de grandeur []

Dans l'espace interplanétaire, le champ magnétique est compris entre 10^{-10} et 10^{-8} T[12]. Des
champs magnétiques à plus grande échelle, par exemple au sein de la Voie lactée sont également
mesurés, par l'intermédiaire du phénomène de rotation de Faraday, en particulier grâce à
l'observation des pulsars. L'origine et l'évolution des champs magnétiques aux échelles
galactiques et au-delà est à l'heure actuelle (2007) un problème ouvert en astrophysique. Les
étoiles, à l'instar des planètes, possèdent aussi un champ magnétique, qui peut être mis en
évidence par spectroscopie (effet Zeeman). Une étoile en fin de vie a tendance à se contracter,
laissant à l'issue de la phase où elle est le siège de réactions nucléaires un résidu plus ou moins
compact. Cette phase de contraction augmente considérablement le champ magnétique à la
surface de l'astre compact. Ainsi, une naine blanche possède un champ magnétique pouvant aller
jusqu'à 10^4 teslas, alors qu'une étoile à neutrons jeune, bien plus compacte qu'une naine blanche
a un champ mesuré à 10^8 voire 10^9 teslas. Certaines étoiles à neutrons appelées pulsars X
anormaux et magnétars semblent être dotées d'un champ magnétique jusqu'à 100 fois plus
élevé[13],[14].

Un aimant NdFeB (néodyme-fer-bore) de la taille d'une pièce de monnaie (créant un champ de
l'ordre de 1,25 T[5]) peut soulever un objet de 9 kg et effacer les informations stockées sur une
carte de crédit ou une disquette. Les utilisations médicales, comme l'IRM, impliquent des
champs d'intensité allant jusqu'à 6 T. Les spectromètres RMN peuvent atteindre jusqu'à 23,5 T
(1 GHz résonance du proton).

Étant une composante du [champ électromagnétique](#), l'intensité du champ magnétique décroît avec la distance à sa source, mais est de portée infinie. Ceci est intimement lié au fait que la [particule élémentaire](#) vecteur de l'[interaction électromagnétique](#), le [photon](#), est de [masse](#) nulle.

Manifestations du champ magnétique []

Articles détaillés : [Magnétisme](#) et [Électromagnétisme](#).

En physique classique, les champs magnétiques sont issus de [courants électriques](#). Au niveau microscopique, un [électron](#) en « orbite » autour d'un noyau atomique peut être vu comme une minuscule boucle de courant, générant un faible champ magnétique et se comportant comme un [dipôle magnétique](#). Selon les propriétés des matériaux, ces structures magnétiques microscopiques vont donner lieu à essentiellement trois types de phénomènes :

- Dans certains cas, les champs générés par des électrons d'atomes voisins présentent une certaine tendance à s'aligner les uns par rapport aux autres, un champ magnétique macroscopique, c'est-à-dire une [aimantation](#) spontanée, est susceptible d'apparaître. C'est le phénomène de [ferromagnétisme](#), expliquant l'existence d'aimants permanents. Il est possible de détruire le champ magnétique d'un aimant en le chauffant au-delà d'une certaine température. L'[agitation thermique](#) générée par le chauffage brise les interactions entre atomes proches qui étaient responsables de l'alignement des champs magnétiques atomiques. En pratique, le phénomène de ferromagnétisme disparaît au-delà d'une certaine température appelée [température Curie](#). Elle est de 770 [degrés Celsius](#) pour le [fer](#).
- En l'absence de ferromagnétisme, ou à une température trop élevée pour que celui-ci apparaisse, la présence d'un champ magnétique externe peut amener les champs microscopiques à s'aligner dans le sens du champ. Ce phénomène est appelé [paramagnétisme](#). La transition entre l'état ferromagnétique et l'état paramagnétique se fait par l'intermédiaire d'une [transition de phase](#) dite de second ordre (c'est-à-dire que l'aimantation tend continûment vers 0 à mesure que la température approche la température de Curie, mais que sa dérivée par rapport à la température diverge à la transition). Le premier modèle mathématique permettant de reproduire un tel comportement s'appelle le [modèle d'Ising](#), dont la résolution, considérée comme un tour de force mathématique, a été effectuée par le [prix Nobel de chimie](#) [Lars Onsager](#) en [1944](#).
- À l'inverse, certains matériaux tendent à réagir en alignant leurs champs magnétiques microscopiques de façon antiparallèle avec le champ, c'est-à-dire s'efforçant de diminuer le champ magnétique imposé de l'extérieur. Un tel phénomène est appelé [diamagnétisme](#).

Courants électriques []
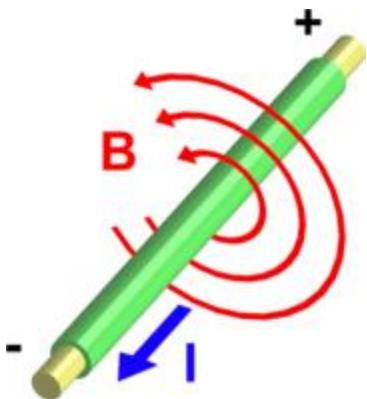
Un [courant électrique](), d'intensité **I** (I > 0), parcourant un fil crée un champ magnétique **B** autour de celui-ci.

Tout [courant électrique]() génère un champ magnétique, ce qu'a montré l'expérience historique d'Ørsted.

La présence d'un courant permet donc d'influencer localement le champ magnétique, c'est le principe des [électroaimants](). Ce champ magnétique est d'autant plus intense que le courant l'est. Réciproquement, un champ magnétique variable est susceptible de générer un courant électrique. C'est le principe de l'[induction magnétique]() qu'utilisent toutes les [machines électriques]().

Champs magnétiques des planètes []
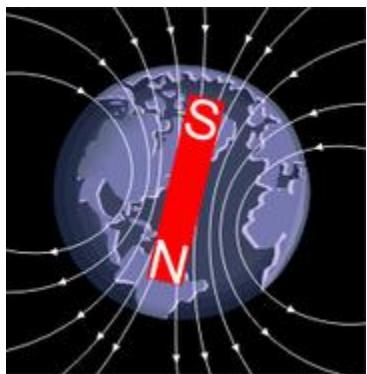
Représentation d'artiste du champ magnétique terrestre.
Articles détaillés : [Magnétosphère](), [champ magnétique terrestre]() et [aurore polaire]().

La [Terre](), comme la plupart des [planètes]() du [système solaire](), possède un champ magnétique. Ce [champ magnétique terrestre]() — qui protège la Terre en déviant les particules chargées issues du [Soleil]() dans une région appelée [magnétosphère]() — est principalement d'origine interne. On suppose qu'il est issu d'effets de [convection]() de la matière située dans le noyau interne de la Terre, principalement composé de [fer]() et de [nickel]() liquide. En particulier, des [courants]() (bien que très faibles), parcourant le noyau induiraient ce champ magnétique, par un processus appelé [effet dynamo]().

La valeur moyenne du champ magnétique terrestre est d'environ 0,5 gauss (soit 5×10⁻⁵ T). Le champ magnétique terrestre fluctue au cours du temps : sa direction et son intensité ne sont pas constantes. De plus, il n'est pas homogène en tout point du globe[15].
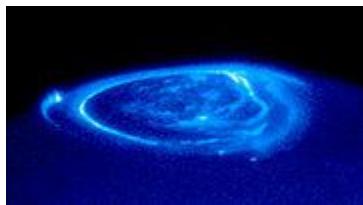
Aurores au pôle de Jupiter.

En particulier, les champs magnétiques des planètes Jupiter et Saturne, les plus intenses après celui du Soleil[16] sont actuellement beaucoup étudiés afin notamment de comprendre le décalage entre l'orientation du champ magnétique et l'axe de rotation de la planète, ainsi que ses variations[17]. La mesure du champ magnétique de Saturne est l'un des objectifs de la mission Cassini-Huygens[18], tandis que celui de Jupiter sera étudié par la sonde JUNO[19]. L'origine de ces champs est supposée liée aux mouvements du noyau d'hydrogène métallique qu'elles abritent.

Au niveau des pôles magnétiques de ces planètes, le champ a tendance à guider les particules chargées, issues par exemple du vent solaire. Celles-ci, très énergétiques, interagissent parfois avec l'atmosphère de la planète : c'est ce que l'on peut observer sous la forme des aurores polaires.

Monopôles magnétiques []

Article détaillé : Monopôle magnétique.

Une des différences fondamentales entre le champ électrique et le champ magnétique est que l'on observe dans la nature des particules possédant une charge électrique, alors que l'on n'observe ni particule ni objet possédant une charge magnétique. En pratique cela se traduit par l'absence de configurations possédant un champ magnétique purement radial, ce qui mathématiquement correspond au fait que le champ magnétique est de divergence nulle.

En particulier, tout aimant possède un pôle nord et un pôle sud magnétique. Si l'on casse cet aimant en deux, on se retrouve avec deux aimants ayant chacun un pôle nord et un pôle sud magnétique. Mathématiquement, cette propriété se traduit par le fait que la divergence du champ magnétique est nulle, propriété formalisée par l'une des équations de Maxwell. Des objets hypothétiques ne possédant qu'un seul pôle magnétique sont appelés monopôles magnétiques.

L'existence de monopôles magnétiques n'a pour l'heure pas été prouvée. D'un point de vue physique, rien n'interdit cependant leur existence. Dans cette hypothèse, l'électrodynamique quantique prédit certaines de leurs propriétés, à savoir que la charge électrique et la charge magnétique sont deux entités nécessairement discrètes, dont le produit de la plus petite valeur positive est égal au produit d'un nombre entier par la constante de Planck réduite. On parle dans

ce cas de monopôles de Dirac, nommés en l'honneur du physicien anglais Paul Dirac qui a prouvé cette propriété de discrétisation.

Dans la théorie de Yang-Mills, on fait intervenir un monopôle de 't Hooft-Polyakov.

Origine relativiste []
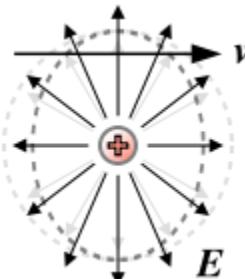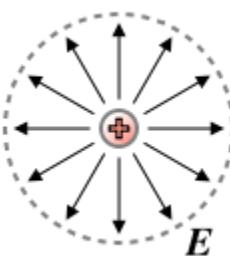
Une particule chargée, au repos dans le vide, engendre un champ électrique isotrope, identique dans toutes les directions de l'espace.

En revanche, son déplacement brise cette symétrie, à cause d'effets relativistes : cette déformation est à l'origine du champ magnétique.

Article détaillé : Transformations de Lorentz du champ électromagnétique.
En 1905, Albert Einstein montra comment le champ magnétique apparaît, comme un des aspects relativistes du champ électrique[20], plus précisément dans le cadre de la relativité restreinte.

Il se présente comme le résultat de la transformation lorentzienne d'un champ électrique d'un premier référentiel à un second en mouvement relatif.

Lorsqu'une charge électrique se déplace, le champ électrique engendré par cette charge n'est plus perçu par un observateur au repos comme à symétrie sphérique, à cause de la dilatation du temps prédite par la relativité. On doit alors employer les transformations de Lorentz pour calculer l'effet de cette charge sur l'observateur, qui donne une composante du champ qui n'agit que sur les charges se déplaçant : ce que l'on appelle « champ magnétique ».

On peut ainsi décrire les champs magnétique et électrique comme deux aspects d'un même objet physique, représenté en théorie de la relativité restreinte par un tenseur de rang 2.

Champ magnétique, excitation magnétique et aimantation []

Un matériau plongé dans un champ magnétique est susceptible de générer un nouveau champ magnétique au sein de celui-ci. Plus précisément, il peut générer une densité de dipôles magnétiques suffisante pour que celle-ci génère un champ magnétique mesurable. Un tel phénomène est appelé aimantation.

Un matériau ferromagnétique possède une aimantation spontanée, c'est-à-dire existant même en l'absence de champ extérieur, mais même dans ce cas, la valeur de l'aimantation est influencée par l'intensité du champ magnétique extérieur. Le champ généré par l'aimantation s'ajoute au champ initial, et c'est la somme des deux qui est observée. Dans un tel cas, le terme de champ magnétique désigne le champ total, et le champ initial prend le nom d'excitation magnétique.

En d'autres termes, on est parfois amené à distinguer le champ initial, l'excitation magnétique, notée H, du champ total, noté B, reliés l'un à l'autre par la relation :

$$B = \mu_0 \left(H + M \right),$$

μ_0 étant la perméabilité magnétique du vide et M l'aimantation du milieu.

L'aimantation est une conséquence de l'excitation magnétique, et il existe donc une fonction (éventuellement compliquée)

$$M = M \left(H \right).$$

Dans les cas les plus simples, ces deux quantités sont proportionnelles l'une de l'autre, et l'on définit la susceptibilité magnétique χ, quantité sans unité, par :

$$M = \chi H$$

ce qui permet d'écrire :

$$B = \mu_0 \left(1 + \chi \right) H = \mu_0 \mu_r H$$

avec

$$\mu_r = 1 + \chi,$$

qui est appelé la perméabilité magnétique (ou perméabilité) du milieu.

Quand le matériau est anisotrope, la relation entre excitation et aimantation peut être plus complexe. En particulier, ces deux quantités peuvent ne pas avoir même direction. Dans ce cas, si l'on suppose qu'elles sont reliées par une relation linéaire, celle-ci est sous forme matricielle.

Différence entre champ magnétique et excitation magnétique []

Il est bien souvent difficile de distinguer un champ magnétique B d'une excitation magnétique H d'autant qu'ils sont souvent donnés comme étant liés par une perméabilité magnétique relativement complexe à estimer (car fonction du matériau et de la norme d'un éventuel courant électrique embrassant celui-ci).

En fait, l'induction magnétique est un champ au sens physique du terme c'est-à-dire qu'en un point de l'espace soumis à ce champ, une force d'attraction (ou de répulsion) magnétique est susceptible de s'appliquer.

Quant à l'excitation magnétique, c'est une fonction mathématique qui fait le lien entre un courant électrique et un champ magnétique. Elle est parfaitement définie par l'équation de Maxwell-Ampère :

$$\nabla \wedge \boldsymbol{H} = \boldsymbol{j} \text{(formulation locale),}$$

ou bien

$$\oint_C \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l} = nI$$
(formulation intégrale).

Dans l'air, les vecteurs $\boldsymbol{H}$ et $\boldsymbol{B}$ sont égaux à une constante multiplicative près (perméabilité magnétique de l'air, proche de celle du vide). On peut choisir d'étudier indifféremment l'un ou l'autre du fait de la relation rigide qui les lie. Par contre, dans un matériau quelconque, en particulier pour les matériaux ferromagnétiques, les deux vecteurs doivent être clairement distingués car en plus des différences que nous venons de voir, des phénomènes de non-linéarités (saturation magnétique et hystérésis) compliquent les relations entre $\boldsymbol{B}$ et $\boldsymbol{H}$.

Le champ d'excitation $\boldsymbol{H}$ est parfois appelé « champ courant » pour mieux souligner son origine.

Visualisation du champ magnétique

Lignes de champ

Article détaillé : Ligne de champ.
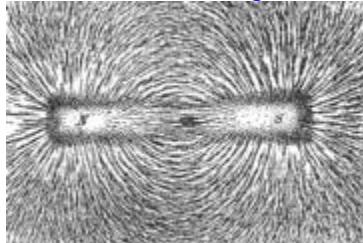
Mise en évidence de lignes de champ magnétique par des brindilles d'acier et de la limaille.

Par définition, les lignes de champ du champ magnétique sont l'ensemble des courbes « en tout point » tangentes à $\boldsymbol{B'}$.

Ces lignes relient les pôles magnétiques, et par convention on les oriente de sorte que les lignes de champ d'un aimant entrent par le sud et ressortent par le nord. Leur expression locale est telle que :

$$\boldsymbol{B} \wedge \mathrm{d}\boldsymbol{l} = 0.$$

où d$\boldsymbol{l}$, de coordonnées (dx, dy, dz), est un vecteur infinitésimal. Une équation paramétrique décrivant les lignes de champ se déduit de la formule ci-dessus en choisissant une variable d'intégration (par exemple x si la composante B_x est non nulle) et en intégrant les équations, qui en coordonnées cartésiennes donnent

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{B_y}{B_x},$$
$$\frac{\mathrm{d}z}{\mathrm{d}x} = \frac{B_z}{B_x}.$$

Observation []

Lorsqu'on approche un aimant d'une poudre de fer, on observe des formes géométriques particulières. Le ferromagnétisme de la limaille de fer fait qu'elle s'aimante légèrement en présence du champ magnétique. Ainsi, la limaille s'orientera de sorte qu'on observera les lignes de champ magnétique.

La forme précise de ces lignes dépend de la forme de l'aimant.

Dans une bobine suffisamment longue, on observe et on montre que le champ magnétique est pratiquement uniforme à l'intérieur : les lignes de champ sont portées par des droites parallèles et de même écart, selon l'axe du solénoïde.

Décomposition []

Le champ magnétique étant de divergence nulle (on parle parfois de champ solénoïdal), il est possible de le décomposer en deux champs appelés champ toroïdal et champ poloïdal. Une telle décomposition est particulièrement appropriée dans les configurations de forme sphérique, et se trouve donc fréquemment utilisée en géophysique et en physique stellaire. Elle est également utilisée pour décrire le champ magnétique qui règne dans un tokamak.

Effets du champ magnétique []

Effets physiques []

Force de Lorentz []

Article détaillé : Force de Lorentz.
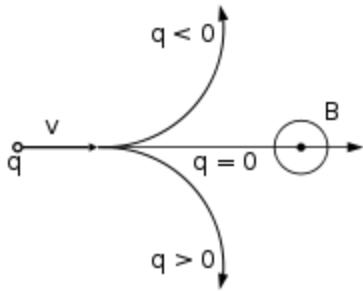
La force de Lorentz.

Le champ magnétique influence les particules chargées au travers de la force de Lorentz.

En l'absence de champ électrique, l'expression de cette force est, pour une particule de charge q animée d'une vitesse $\mathbf{v}$:

$$\boldsymbol{F} = q\boldsymbol{v} \wedge \boldsymbol{B}$$

où on a noté le produit vectoriel par une croix, et où les quantités sont exprimées dans les unités du système international.

On peut réécrire cette relation sous forme différentielle pour un fil, en introduisant le courant électrique :

$$\mathrm{d}\boldsymbol{F} = I\mathrm{d}\boldsymbol{l} \wedge \boldsymbol{B}$$

avec I l'intensité du courant électrique, $\boldsymbol{B}$ le champ magnétique et $\mathrm{d}\boldsymbol{l}$ une portion infinitésimale de fil, symbolisée par un vecteur tangent à celui-ci.

Cette expression se généralise aux distributions de courants bidimensionnelles (surfaces et courants surfaciques) aussi bien que tridimensionnelles (volumes et courants volumiques). On introduit dans ces cas la notion d'« élément de courant » d$\boldsymbol{C}$, définie par :

- $\mathrm{d}\boldsymbol{C} = \boldsymbol{j}_\mathrm{s}\cdot\mathrm{d}S$ pour une surface, où $\boldsymbol{j}_\mathrm{s}$ est le courant surfacique ;
- $\mathrm{d}\boldsymbol{C} = \boldsymbol{j}\,\mathrm{d}\tau$ pour un volume, où $\boldsymbol{j}$ est le courant volumique.

On a ainsi une expression générale :

$$\mathrm{d}\boldsymbol{F} = \mathrm{d}\boldsymbol{C} \wedge \boldsymbol{B}.$$

Force de Laplace []

Article détaillé : Force de Laplace.

La force de Laplace est simplement un cas particulier de la force de Lorentz, pour un barreau homogène et conducteur, parcouru par un courant électrique et placé dans un champ magnétique.

Contrairement à la [force de Lorentz](#), elle ne traite pas des particules constituantes du barreau, mais de l'effet macroscopique : si son expression est similaire, le sens physique des objets considérés diffère. En particulier, la [force](#) n'est pas toujours orthogonale à la vitesse.

L'expression de la force de Laplace est :

$$\mathrm{d}\boldsymbol{F} = I\mathrm{d}\boldsymbol{l} \wedge \boldsymbol{B},$$

où I est l'intensité du courant, $\boldsymbol{B}$ le champ magnétique et $\mathrm{d}\boldsymbol{l'}$ un élément infinitésimal du barreau.

Supraconducteurs []
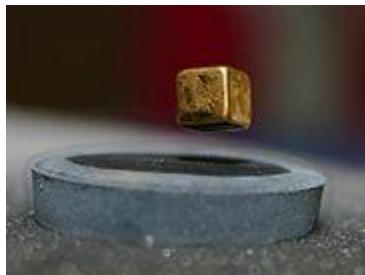
L'effet Meissner résulte de l'expulsion des champs magnétiques par un matériau supraconducteur.
Article détaillé : [Effet Meissner](#).

Les matériaux [supraconducteurs](#) ont la propriété intéressante de ne pas pouvoir être pénétrés par un champ magnétique : on parle d'expulsion du champ magnétique. On observe ce phénomène par exemple au travers de l'[effet Meissner](#).

Une des interprétations possibles consiste à fournir une [masse](#) aux [photons](#), porteurs du champ magnétique, ce qui diminue la portée de ce champ à l'intérieur du matériau. Il est ainsi possible de faire des analogies avec des processus comme le [mécanisme de Higgs](#), qui explique la masse des porteurs des interactions nucléaires.

On traduit cela par une expression particulière du [potentiel vecteur](#).

Cet effet ne saurait par ailleurs être observé entre deux aimants : la lévitation statique serait alors interdite par le [théorème d'Earnshaw](#).

Dans la [théorie BCS](#), qui traite des supraconducteurs, on peut montrer que le potentiel vecteur est de la forme :

$$\boldsymbol{A}(x) = \boldsymbol{A}_0 e^{-\frac{x}{\lambda}},$$

ou x la profondeur de pénétration dans le supraconducteur et λ est la longueur de pénétration caractéristique, qui vaut

$$\lambda = \sqrt{\frac{m}{2\mu_0 e^2 \rho_s}},$$

où m est la [masse](#) d'un [électron](#), e sa [charge électrique](#) et ρ_s la [densité superfluide](#) du supraconducteur, supposée uniforme et constante. Ainsi, le [potentiel vecteur](#) — donc le champ magnétique — ne pénètre que sur une épaisseur de quelques λ à l'intérieur du matériau.

Si le champ magnétique environnant le matériau supraconducteur est trop intense, celui-ci ne peut expulser le champ dans sa totalité. Certaines régions du matériau supraconducteur vont devenir non supraconductrices et canaliser le champ magnétique. Le supraconducteur a tendance à minimiser la taille de telles régions, qui prennent la forme de tubes alignés le long du champ magnétique. Ces régions sont appelées, pour des raisons évidentes, tubes de flux.

Induction, induction mutuelle et ondes []

Article détaillé : [Induction magnétique](#).

Chauffage par induction d'une bouteille en métal : la variation d'un champ magnétique induit des courants dans le corps de l'objet, qui échauffent celui-ci par effet Joule.

Le phénomène d'induction électromagnétique (ou induction magnétique ou, simplement, induction) a pour résultat la production d'une différence de potentiel aux bornes d'un conducteur électrique soumis à un champ électromagnétique variable. Cela s'exprime au travers de l'[équation locale de Maxwell-Faraday](#) :

$$\boldsymbol{\nabla} \wedge \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t},$$

$\boldsymbol{E}$ étant le [champ électrique](#), $\boldsymbol{B}$ le champ magnétique.

Ce champ électrique peut à son tour engendrer un champ magnétique, propageant ainsi une [onde électromagnétique](#).

Lorsqu'un matériau est placé dans un champ magnétique variant, il apparaît dans celui-ci un [champ électrique](#) (dont la circulation est appelée [force électromotrice](#)) qui génère à son tour des [courants](#), appelés [courants de Foucault](#). C'est d'une part le principe des [alternateurs](#), qui

produisent de l'électricité en déplaçant des aimants. C'est d'autre part le principe des chauffages et plaques à induction, car la dissipation par effet Joule de ces courants échauffe le métal.

Par ailleurs, deux systèmes magnétiques, comme des bobines, peuvent être couplés au travers du champ magnétique. On parle d'induction mutuelle (ou de mutuelle induction). Cet effet modifie le comportement individuel de chaque circuit.

On peut aborder cet effet par un modèle très simple : un conducteur ohmique de conductivité électrique γ est parcouru par un champ magnétique sinusoïdal, d'intensité B_0 et de pulsation ω. Ce champ est, à tout instant t, d'intensité B donnée par :

$$B = B_0 \sin(\omega t + \varphi).$$

Ce champ induit dans le conducteur, d'après la loi de Faraday, un champ électrique E d'intensité E donnée par

$$E = -\omega B_0 \cos(\omega t + \varphi).$$

D'après la loi d'Ohm, il se dissipe donc une puissance moyenne volumique, par effet Joule :

$$\mathcal{P} = \gamma \langle E^2 \rangle = \frac{\gamma \omega^2 B_0^2}{2}.$$

Effet Hall []

Article détaillé : Effet Hall.

Un conducteur, parcouru par un courant électrique selon une direction, soumis à un champ magnétique dirigé dans une seconde direction, présente une différence de potentiel selon la troisième direction. Ce phénomène est connu sous le nom d'effet Hall, en l'honneur du physicien américain Edwin Herbert Hall.
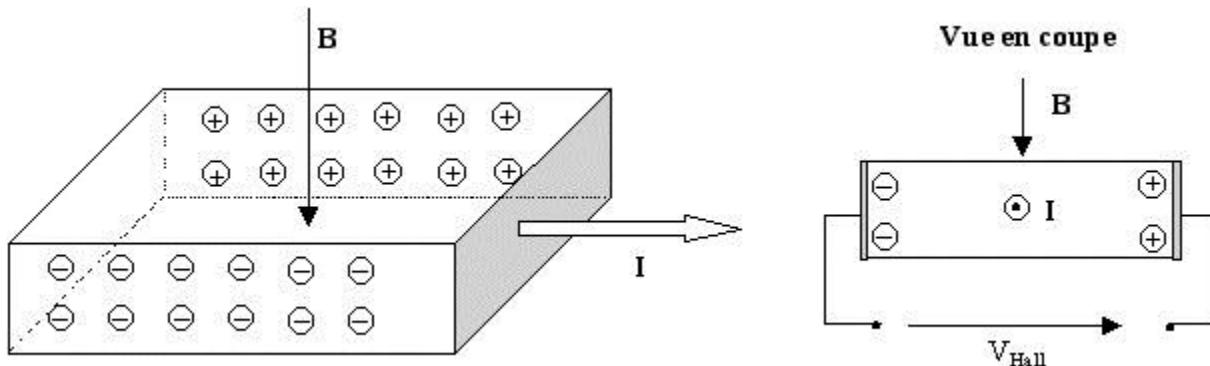
On peut expliquer cet effet au travers de la physique classique, en considérant que les porteurs de charge (par exemple les électrons) qui se déplacent dans le corps du conducteur sont soumis à la

[force de Lorentz](#), donc déviés, de sorte que leur répartition est différente d'une part et d'autre du conducteur — d'où la différence de potentiel. On peut l'expliquer de manière plus fondamentale du point de vue de la [mécanique quantique](#).

Cet effet est à la base de nombreux dispositifs de mesure du champ magnétique et du [courant électrique](#).

Magnétorésistance []

Articles détaillés : [Magnétorésistance](#), [Magnétorésistance géante](#), [Magnétorésistance colossale](#) et [Magnétorésistance à effet tunnel](#).

En présence d'un champ magnétique, certains conducteurs voient leur [résistance électrique](#) varier. Cet effet est appelé [magnétorésistance](#), et présente de nombreuses applications, par exemple dans les [disques durs](#) qui équipent les [ordinateurs](#) modernes.

Il n'existe pas à ce jour d'explication définitive de tous les phénomènes de magnétorésistance, mais des théories distinctes qui régissent les principales manifestations de cet effet : la [magnétorésistance classique](#), « [géante](#) », « [colossale](#) » et la [magnétorésistance à effet tunnel](#).

Dipôles magnétiques []

Articles détaillés : [Dipôle magnétique](#) et [moment magnétique](#).

Un dipôle magnétique, caractérisé par son moment magnétique, est analogue à un aimant droit.

Parfois, on peut introduire la notion de [moment magnétique](#), qui permet de travailler avec des [dipôles](#).

En particulier, on utilise ce modèle au niveau microscopique, lorsqu'un ensemble de molécules ou de particules est parcouru par un courant. Pour une boucle ceinturant une surface orientée S et parcourue par un courant I, on définit le moment magnétique M par :

$$M = IS.$$

Ceci revient à assimiler l'objet à un [aimant](#) droit infiniment fin. On peut alors introduire une [énergie potentielle](#) dipolaire :

$$E_p = -M \cdot B.$$

28

Ainsi, elle est minimale lorsque le dipôle est aligné avec le champ. On montre de même que, dans une chaîne de dipôles, ils s'orientent tous dans une même direction pour minimiser leur énergie. Dans les cas (fréquents) où on ne sait pas modéliser la structure d'un dipôle magnétique par une boucle de courant, le moment magnétique est *défini* par la relation ci-dessus, c'est-à-dire par l'énergie qu'il faut fournir pour tourner un dipôle magnétique dans un champ magnétique donné.

Dans les matériaux, lorsqu'on considère des moments magnétiques de particules, le fait qu'ils s'orientent tous de la même manière ne peut être expliqué que d'un point de vue quantique (principe d'exclusion de Pauli et hamiltonien de Heisenberg).
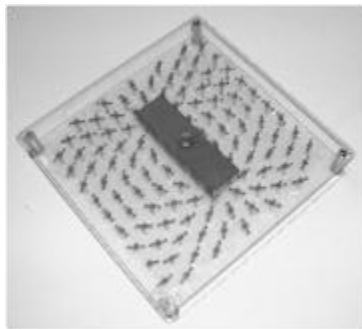
En présence d'un champ magnétique, le fer s'aimante à son tour et devient un dipôle. Il est alors soumis aux forces créées par un aimant droit et s'oriente selon les lignes de champ.

Dans le cadre d'un dipôle magnétique de moment M soumis à un champ B, lorsque le champ est homogène, le torseur des forces se réduit au moment car la résultante des forces est nulle. On a donc :

$$\mathbf{\Gamma} = \mathbf{M} \wedge \mathbf{B},$$

où $\mathbf{\Gamma}$ le moment résultant, M le moment magnétique du dipôle et B le champ magnétique.

Cela explique notamment l'effet d'un champ magnétique sur une boussole : il a tendance à aligner l'aiguille de celle-ci avec le champ.

Si en revanche le champ est inhomogène, alors le dipôle subit de plus une force, dont l'expression est :

$$\mathbf{F} = (\mathbf{M} \cdot \mathbf{\nabla}) \mathbf{B},$$

avec les mêmes notations que précédemment.

Cela explique notamment le fait que deux aimants s'attirent : cette force s'exerce sur le premier de sorte à l'approcher des champs plus intenses, donc plus près de l'autre aimant. En supposant cette fois que les pôles sont ponctuels, alors l'intensité de la force F s'exerçant d'un pôle sur l'autre est donnée par[21] :

$$F = \frac{\mu g_1 g_2}{4\pi r^2},$$

où g_1 et g_2 représentent l'intensité de ces pôles en (A·m si elles sont exprimées dans le système international d'unités), μ la perméabilité magnétique du milieu, et r la distance entre les pôles.

Effets géologiques []

Certaines roches sont riches en matériaux ferromagnétiques, qui sont sensibles au champ magnétique. En particulier, ils perdent leurs propriétés magnétiques au-delà d'une certaine température, dite température de Curie.
Les roches basaltiques issues par exemple des volcans ou des rifts océaniques, sont chauffées au-delà de cette température dans le magma. Lorsqu'elles refroidissent, elles regagnent leurs propriétés magnétiques, et *figent* l'orientation du champ magnétique terrestre. On observe cet effet au travers des anomalies magnétiques des roches. C'est par l'analyse de ces roches que l'on a observé les inversions du champ terrestre[22],[23].
Il existe également des roches, comme l'hématite, dont les propriétés magnétiques sont telles qu'on observe les variations de champ au cours de leur formation. L'étude de ces roches est également un élément déterminant qui appuie la tectonique des plaques.

Effets biologiques []

Effet des champs magnétostatiques []

Les différentes espèces connues ne sont pas identiquement sensibles aux champs électromagnétiques. Les données concernant les êtres humains sont encore sporadiques[24]. Les champs statiques inférieurs à 8 teslas n'ont vraisemblablement pas d'effets physiologiques notables, si ce n'est l'apparition chez certaines personnes de phosphènes lorsqu'ils sont exposés à des champs de plus de 4 T[25]. L'organisation mondiale de la santé mène encore aujourd'hui des études[26] sur les risques potentiels.

Des champs continus aussi intenses sont relativement difficiles à obtenir en dehors des laboratoires spécialisés, les applications courantes impliquant généralement des champs inférieurs au tesla.

Les recherches actuelles s'orientent davantage sur les champs non ionisants de très basse fréquence (EMF : *extremely low frequency*), qui ne sont pas statiques, mais semblent agir sur les systèmes biologiques ou parfois provoquer des cancers[27].

Effet des champs magnétiques pulsés []

Les champs pulsés, que l'on peut créer beaucoup plus intenses, provoquent de plus par induction un rayonnement électromagnétique. Celui-ci peut interagir avec les systèmes biologiques, et son effet dépend de la radiorésistance des espèces exposées. Notamment, selon la fréquence, de tels champs peuvent provoquer des radiations ionisantes : ultraviolets, rayons X ou gamma. Ceux-ci sont dangereux pour la santé, et provoquent en particulier la brûlure des tissus.

Récemment, des médecines alternatives faisant intervenir des champs magnétiques faibles pulsés prétendent limiter les cancers ou la sclérose en plaques. Si de tels champs ne semblent pas dangereux, aucune étude scientifique sérieuse n'appuie à ce jour ces allégations[28],[29]. En revanche, les champs magnétiques pulsés peuvent influencer l'équilibre[30] et semblent diminuer les symptômes du trouble bipolaire[31].

Les effets, principalement liés à l'induction dans les nerfs, permettent ainsi *via* la stimulation magnétique transcranienne, le diagnostic de pathologies neurologiques.

Énergie magnétique []

Article détaillé : Énergie électromagnétique.

La présence d'un champ magnétique s'exprime globalement par une énergie, dite « énergie magnétique ». Elle s'exprime par :

$$\mathcal{E}_B = \int \frac{|\boldsymbol{B}(\boldsymbol{x})|^2}{2\mu}\,\mathrm{d}\boldsymbol{x}\,,$$

avec $|\boldsymbol{B}|$ étant la norme du champ magnétique et μ la perméabilité magnétique en chacun des points considérés.

En pratique, on définit une énergie volumique, appelée dans ce contexte pression magnétique :

$$e_B = \frac{|\boldsymbol{B}|^2}{2\mu}\,.$$

Calcul du champ magnétique []

Propriétés mathématiques []

Symétries []

En tant que champ pseudovectoriel, le champ magnétique a un comportement particulier par rapport aux symétries. En effet, contrairement au champ (vectoriel) électrique, les champs magnétiques ne suivent pas la symétrie de leurs sources. On parle ainsi de vecteur « axial » ou de « pseudovecteur ».

Par exemple, pour une spire circulaire parcourue par un courant :

- un plan de symétrie Π^+ est celui qui contient la spire ;
- un plan d'antisymétrie Π^- est tout plan passant par le centre de la spire et orthogonal au premier plan.

Respectivement, Π^+ et Π^- sont un plan d'antisymétrie et de symétrie pour le champ magnétique.

Changement de référentiel []

En mécanique classique, où l'on considère des vitesses relatives très inférieures à la vitesse de la lumière, le champ magnétique mesuré est identique dans deux systèmes de coordonnées en translation rectiligne et uniforme l'un par rapport à l'autre (référentiels galiléens). Cette propriété n'est pas partagée par le champ électrique, dont la valeur change d'un référentiel à l'autre si le champ magnétique est non nul.

Calcul du champ []

Article détaillé : Équation aux dérivées partielles.

Le calcul du champ magnétique créé par un système demande de résoudre des équations différentielles assez complexes. Il existe pour cela une multitude de méthodes numériques comme la méthode des éléments finis, la méthode des différences finies et la méthode des volumes finis pour ne citer que les méthodes les plus répandues. Toutefois, il est possible de calculer analytiquement le champ magnétique dans certains cas simples. Sauf mention contraire, les expressions données pour le calcul du champ magnétique sont exprimées dans les unités SI. Cela explique notamment le facteur $1/4\,\pi$.

Théorème d'Ampère []

Article détaillé : Théorème d'Ampère.

À partir des observations révélant un lien entre courants électriques et champ magnétique, André-Marie Ampère énonça une loi d'abord phénoménologique, qui décrivait l'effet observé. Démontrée depuis, dans le cadre plus général de l'électromagnétisme, cette relation est devenue le théorème d'Ampère. Elle n'est valable, en toute rigueur, que dans les cas magnétostatiques.

La formulation originelle de ce théorème est la suivante :

$$\oint_C \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{\ell} = \mu_0 I$$

,

$\boldsymbol{B}$ étant le champ magnétique, C une courbe fermée et orientée et I l'intensité qui traverse une surface délimitée par C.

Cete équation peut être écrite localement, on a alors :

$$\boldsymbol{\nabla} \wedge \boldsymbol{B} = \mu_0 \boldsymbol{j}$$

où μ_0 est la perméabilité magnétique du vide, et $\boldsymbol{j}$ le vecteur densité de courant.

Cette relation étant mise en défaut dans le cas de champs magnétiques ou électriques dépendant du temps, Maxwell introduisit en 1861 les « courants de déplacement », dont la variation corrigeait cette relation : c'est l'équation locale de Maxwell-Ampère[32]. On peut l'écrire localement sous la forme :

$$\boldsymbol{\nabla} \wedge \boldsymbol{B} = \mu_0 \boldsymbol{j} + \mu_0 \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t},$$

$\boldsymbol{E}$ étant le champ électrique et ε_0 la perméabilité électrique du vide.

On peut *a posteriori* réécrire cette loi sous forme intégrale, également appelée théorème d'Ampère :

$$\oint_C \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{\ell} = \epsilon_0 (I + I_D),$$

avec

$$I_D = \int_S \frac{\partial \boldsymbol{E}}{\partial t} \cdot \mathrm{d}\boldsymbol{S},$$

où S est la surface délimitée par le contour C.

Ceci se comprend aisément grâce au théorème de Green-Stokes :

$$\int_S (\boldsymbol{\nabla} \wedge \boldsymbol{B}) \cdot \mathrm{d}\boldsymbol{S} = \oint_C \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{\ell}.$$

Loi de Biot-Savart locale []

Article détaillé : Loi de Biot-Savart.

La loi de Biot-Savart permet de donner l'expression du champ magnétique dans un milieu de perméabilité magnétique isotrope et homogène.

Le champ $\boldsymbol{B}$ généré en un point de coordonnées $\boldsymbol{r}$ par une charge q en mouvement, située en un point $\boldsymbol{r'}$ et se déplaçant à la vitesse $\boldsymbol{v}$, est donné par la relation suivante :

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu}{4\pi} \frac{q\boldsymbol{v} \wedge (\boldsymbol{r} - \boldsymbol{r'})}{|\boldsymbol{r} - \boldsymbol{r'}|^3}.$$

Loi de Biot-Savart intégrale []

Si on a affaire à une distribution de courants, qui est connue en tout point, alors on peut intégrer la relation locale.

Avec les notations précédentes, cela donne :

$$B(r) = \frac{\mu}{4\pi} \int \frac{j(r') \wedge (r - r')}{|r - r'|^3} dx$$
.

Potentiel vecteur []

Article détaillé : Potentiel vecteur du champ magnétique.

L'absence de monopôles magnétique implique que la divergence du champ magnétique est nulle :

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0.$$

Ceci implique, d'après les théorèmes de l'analyse vectorielle, qu'il existe un champ vectoriel A, dont le rotationnel est égal à B :

$$\boldsymbol{B} = \boldsymbol{\nabla} \wedge \boldsymbol{A}.$$

Un tel champ A est appelé potentiel vecteur, par analogie au potentiel électrique, dit « potentiel scalaire », du champ électrique.

Ce potentiel n'est toutefois pas unique : il est défini à un gradient près. En effet, le rotationnel d'un gradient est identiquement nul, aussi le potentiel vecteur **A'** défini par :

$$\boldsymbol{A'} = \boldsymbol{A} + \boldsymbol{\nabla}\phi$$

vérifie-t-il également la relation :

$$\boldsymbol{B} = \boldsymbol{\nabla} \wedge \boldsymbol{A'}.$$

De façon quelque peu étrange, la quantité fondamentale n'est pas le champ magnétique mais le potentiel vecteur, alors que ce dernier ne peut être défini de façon univoque. Une telle situation est appelée en physique invariance de jauge : des phénomènes identiques, ici le champ B, peuvent être générés par plusieurs configurations, appelées pour diverses raisons historiques « jauges » de l'objet fondamental, ici le champ A. D'un point de vue mathématique, l'invariance de jauge est la cause d'une loi fondamentale de l'électromagnétisme, la conservation de la charge électrique. Cette loi, expérimentalement vérifiée à une très grande précision implique en effet que l'objet fondamental apparaissant en électromagnétisme n'est ni le champ magnétique ni le champ électrique, mais le potentiel vecteur et le potentiel électrique.

Connaissant A, on peut facilement en déduire B. Le fait que le potentiel vecteur soit plus fondamental que le champ magnétique transparaît en mécanique quantique, où en présence de champ magnétique, c'est en fait le potentiel vecteur qui apparaît dans l'équation de Schrödinger, qui décrit l'évolution des particules élémentaires. L'illustration la plus manifeste de la prééminence du potentiel vecteur se trouve dans l'effet Aharonov-Bohm, où l'on est amené à considérer des configurations dans lesquelles le champ B s'annule dans certaines régions alors que le potentiel vecteur A n'est pas nul (mis de rotationnel nul) et influence explicitement le comportement des particules.

Il est d'ailleurs possible de calculer le potentiel vecteur A directement à partir de la donnée des courants :

$$A(r) = \frac{\mu}{4\pi} \int \frac{j(r')}{|r - r'|} \mathrm{d}x$$

l'expression ci-dessus n'étant valable que lorsque les courants — donc les champs — ne dépendent pas du temps. En pratique, ces variations peuvent souvent être négligées tant que l'on n'étudie pas les ondes et leur propagation.

Dans ces derniers cas, il faut remplacer l'expression ci-dessus par une expression plus complexe, faisant appel au concept de potentiels retardés pour tenir compte du temps de propagation du champ magnétique.

Applications []

Déviation de particules []

On peut montrer qu'un champ magnétique affecte le déplacement de particules chargées, en infléchissant leur trajectoire, mais sans la valeur de leur vitesse. Il est ainsi utilisé pour courber leur trajectoire dans les accélérateurs de particules.

En effet, d'après la loi de Lorentz, la force F qu'exerce un champ magnétique B sur une particule de charge q se déplaçant à la vitesse v est :

$$F = qv \wedge B$$

Ainsi, cette force est toujours orthogonale à la vitesse, donc son travail δW exercé lors d'un petit déplacement dr est nul :

$$\delta W \equiv F \cdot \mathrm{d}r = 0$$

Par conséquent, la norme de la vitesse n'est pas influencée par le champ magnétique. En revanche, cette force modifie la direction de celle-ci dès que vitesse et champ magnétique ne sont pas colinéaires.

Chambres à bulles []

Article détaillé : [Chambre à bulles](#).
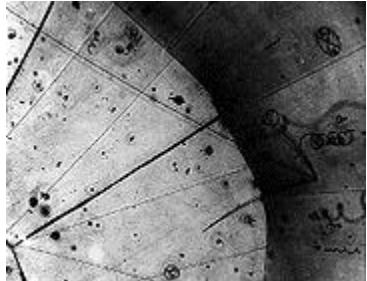
Photographie d'une chambre à bulles. Des trajectoires, on peut trouver les particules ayant interagi : ici, la première « photographie » d'un neutrino, le 13 novembre 1970.

Le champ magnétique dévie les particules chargées. Si, de plus, le milieu présente une certaine viscosité, alors ces particules décrivent des spirales, desquelles on peut déduire la [charge électrique](#) (le sens de l'enroulement) et la [masse](#) (au travers de la décélération) des particules.

C'est le principe des chambres à bulles, inventées au début du [XXe siècle](#) pour observer, en particulier, les constituants de la matière ([protons](#), [neutrons](#) et [électrons](#)), les [positrons](#) et les [neutrinos](#). On préfère cependant aujourd'hui, depuis leur invention dans les années 1970, utiliser les [chambres à fils](#).

En pratique, il existe toujours un [champ électrique](#), qui dévie les particules.

Une particule dans une chambre à bulles est idéalement soumise uniquement à la force magnétique et aux forces de frottement. Elle vérifie donc :

$$m\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = q\boldsymbol{v} \wedge \boldsymbol{B} - \eta\boldsymbol{v},$$

où η est le coefficient intervenant dans la [force de frottement](#), colinéaire mais opposée à la vitesse. Cette équation peut se réécrire de façon équivalente :

$$\dot{\boldsymbol{v}} - \frac{q}{m}\boldsymbol{v} \wedge \boldsymbol{B} + \frac{\eta}{m}\boldsymbol{v} = 0.$$

Résonance magnétique : IRM et RMN []

Articles détaillés : [Imagerie par résonance magnétique](#) et [Résonance magnétique nucléaire](#).

IRM encéphalique (coupe sagittale passant par la ligne médiane).

La résonance magnétique est un phénomène qui apparaît lorsque certains atomes sont placés dans un champ magnétique et reçoivent un rayonnement radio adapté.

En effet, les atomes dont le noyau est composé d'un nombre impair de constituants — en particulier l'hydrogène, dont le noyau se résume à un proton — présentent une sorte de moment magnétique, appelé moment magnétique de spin. Lorsqu'un noyau est placé dans un champ magnétique — mécanique quantique oblige — il ne peut se placer que dans deux états distincts. On peut toutefois faire passer un noyau d'un état à l'autre avec un photon de pulsation adaptée : on parle de résonance. Ce phénomène affectant le noyau d'un atome, on parle de résonance magnétique nucléaire.

Un noyau affecté retourne à l'équilibre en reprenant son état d'origine et en émettant un photon. Ce rayonnement, en plus d'indiquer la présence du noyau, peut également informer sur son voisinage au sein d'une molécule. En effet, il se produit des couplages, qui influencent notamment sa fréquence. En RMN, on appelle ces écarts à un solvant de référence les « déplacements ».

L'imagerie par résonance magnétique nucléaire (IRM) est l'application de cet effet en imagerie médicale, permettant d'avoir une vue 2D ou 3D d'une partie du corps, notamment du cerveau.

Transformateurs électriques []
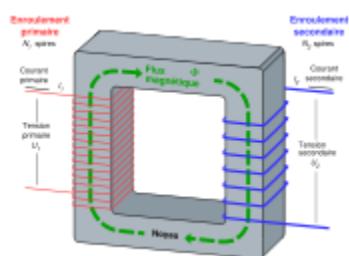
Modèle simplifié d'un transformateur électrique idéal.
Article détaillé : Transformateur électrique.

Un [transformateur électrique](#) est un convertisseur, qui permet de les valeurs de la [tension](#) et de [l'intensité du courant](#) délivrées par une source d'énergie électrique alternative en un système de tension et de courant de valeurs différentes, mais de même [fréquence](#) et de même forme. Il effectue cette transformation avec un excellent rendement. Il est analogue à un engrenage en mécanique (le [couple](#) sur chacune des roues dentées étant l'analogue de la tension et la vitesse de rotation étant l'analogue du courant).

Un transformateur est constitué de deux parties : le [circuit magnétique](#) et les enroulements. Les enroulements créent ou sont traversés par un [flux magnétique](#) que le circuit magnétique permet de canaliser afin de limiter les pertes. Dans le cas d'un transformateur [monophasé](#) parfait pour lequel toutes les pertes et les fuites de flux sont négligées, le rapport du nombre de [spires](#) primaires et secondaires détermine totalement le rapport de transformation du transformateur. Ainsi, si on note respectivement n_1 et n_2 le nombre de spires au primaire et au secondaire, on obtient :

$$\frac{U_2}{U_1} = \frac{n_2}{n_1}$$

Avec U_1 la tension primaire et U_2 la tension secondaire.

Moteurs électriques []

Articles détaillés : [Roue de Barlow](#) et [Machine électrique](#).
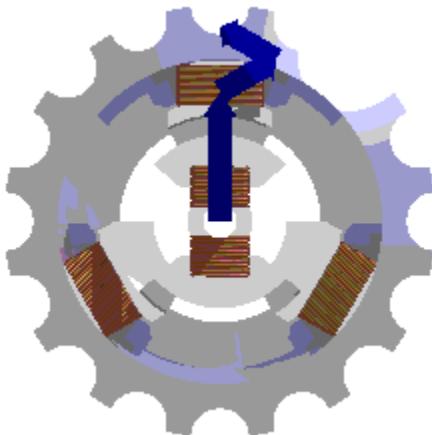
Champ magnétique tournant au sein d'un moteur à courant alternatif triphasé.

Une *machine électrique* est un dispositif permettant la conversion d'[énergie électrique](#) en [travail](#) ou énergie [mécanique](#) : les moteurs rotatifs produisent d'un [couple](#) par un déplacement [angulaire](#) tandis que les moteurs linéaires produisent d'une [force](#) par un déplacement [linéaire](#).

Les forces engendrées par les champs magnétiques, formulées par la [relation de Lorentz](#), permettent d'envisager des dispositifs qui utilisent un tel champ pour transformer l'[énergie électromagnétique](#) en [énergie mécanique](#).

Le premier moteur électrique fut construit par [Peter Barlow]() : une roue, soumise à un champ magnétique permanent, est parcourue par un [courant électrique](). Il s'exerce donc une [force]() sur cette roue, qui se met alors en rotation : c'est la [roue de Barlow](). Elle constitue de fait le premier [moteur électrique à courant continu]().

Les liens entre champ magnétique et champ électrique, exprimés par les [équations de Maxwell](), font qu'il est possible de construire des systèmes qui créent un champ magnétique non permanent — à partir d'une source de courant, au moyen d'[électroaimants]().

Au sein de tels appareils, on crée un champ magnétique tournant[33], c'est-à-dire un champ dont la direction varie en tournant dans un sens ou dans l'autre avec une fréquence de rotation déterminée.

L'une des possibilités est de créer un tel champ à l'aide d'électroaimants fixes — ils constituent le « *stator* » — parcourus par un [courant électrique]() d'intensité variable, par exemple [triphasé](). Au centre, une partie mobile et sensible au champ magnétique, constituée par exemple d'aimants permanents, est ainsi mise en mouvement : c'est le « *rotor* », dont le mouvement de rotation est transmis à un [arbre](). Ce principe est par exemple mis en œuvre pour les [machines synchrones]() et les [machines asynchrones]().

Une autre possibilité est de créer un champ permanent au stator à l'aide d'aimants permanents ou d'enroulements parcourus par un courant continu et de réaliser un champ magnétique tournant au rotor par un système de connexions glissantes afin que ce champ rotorique reste en quadrature avec le champ statorique. C'est le principe mis en œuvre pour la [machine à courant continu]().

Loi d'Ohm

La **loi d'Ohm** est une [loi physique]() permettant de relier l'[intensité du courant électrique]() traversant un [dipôle électrique]() à la [tension]() à ses bornes (elle permet de déterminer la valeur d'une résistance). La loi d'Ohm a été nommée ainsi en l'honneur du physicien allemand [Georg Ohm]().

Point de vue macroscopique []

En [courant continu]() []

La [différence de potentiel]() ou [tension]() U (en [volts]()) aux bornes d'une [résistance]() R (en [ohms]()) est proportionnelle à l'[intensité du courant électrique]() I (en [ampères]()) qui la traverse.
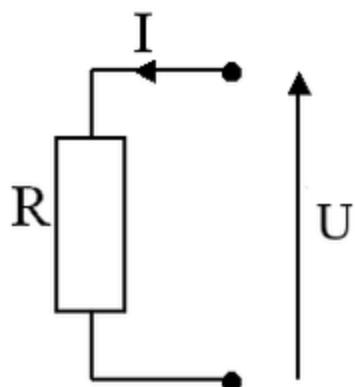
Représentation schématique d'une résistance parcourue par un courant. La loi d'Ohm relie l'intensité *i* du courant à la valeur *R* de la résistance et à la tension *U* entre ses bornes par la relation $U = R.I$

$$U = R \cdot I$$

On peut en déduire :

- $I = \dfrac{U}{R}$ si *R* est non nul

- $R = \dfrac{U}{I}$ si *I* est non nul

La résistance s'exprime en ohms (symbole : Ω).

Cette loi porte le nom de Georg Ohm qui a travaillé sur le comportement des conducteurs métalliques. Elle s'applique de manière satisfaisante aux conducteurs métalliques **thermostatés**, c'est-à-dire maintenus à une température constante. Lorsque la température change, la valeur de la résistance change également de manière plus ou moins simple, ce qui impose d'introduire des termes correctifs. Par convention, on conserve la loi et on introduit les termes correctifs dans la valeur de la résistance du conducteur.

En courant alternatif []

La loi précédente se généralise au cas des courants sinusoïdaux en utilisant les notations complexes. On note $\underline{U}, \underline{I}$ la tension et le courant complexes. La loi d'Ohm s'écrit alors :

$$\underline{U} = \underline{Z} \cdot \underline{I}$$

Avec $\underline{Z}$: impédance complexe du dipôle considéré, qui peut être constitué de dipôles linéaires (résistances, condensateurs et inductances).

Point de vue local (mésoscopique) []

Énoncé de la loi d'Ohm locale []

D'un point de vue local, c'est-à-dire mésoscopique, la loi (locale) d'Ohm s'énonce en disant que la mobilité des porteurs de charge est indépendante de $||\vec{E}||$.

Si on note μ la mobilité des porteurs de charge, leur vitesse s'écrit alors $\vec{v} = \pm\mu\vec{E}$ (la direction du mouvement dépend du signe des porteurs) ; la densité de courant $\vec{j}$ associée à une densité de porteurs n vaut quant à elle :

$$\vec{j} = qn\vec{v} = qn\mu\vec{E}$$, où q est la charge électrique du porteur (en valeur absolue).

On note $\sigma = qn\mu$ la conductivité électrique du matériau (pour un seul type de porteur).

On a alors la loi locale d'Ohm pour un seul type de porteur :

$$\vec{j} = \sigma\vec{E}.$$

Si on a plusieurs types de porteurs, comme par exemple les électrons et les trous dans un semi-conducteur ou des ions différents dans un électrolyte, la densité de courant devient :

$$\vec{j} = \sum_k n_k q_k \vec{v}_k,$$

avec $\vec{v}_k = \mu_k\vec{E}$,

donc $$\vec{j} = \left[\sum_k n_k q_k \mu_k\right]\vec{E}.$$

On a alors la conductivité totale :

$$\sigma = \sum_k n_k q_k \mu_k$$

Voir aussi Loi de Nernst-Einstein.

Rapport avec la loi d'Ohm macroscopique : définition de la résistance []

Considérons une portion de conducteur d'un point **A** à un point **B** et de section droite *S*, on a alors la différence de potentiel qui vaut :

$$V_A - V_B = \int_A^B \vec{E} \cdot d\vec{l}$$

et l'intensité :

$$i = \iint_S \vec{j} \cdot d\vec{S} = \iint_S \sigma\vec{E} \cdot d\vec{S} = \sigma \iint_S \vec{E} \cdot d\vec{S}$$

Multiplions par une constante la différence de potentiel $V_A - V_B$, alors les conditions aux limites sont inchangées ainsi que les lignes de champ de $\vec{E}$, et l'expression $\iint_S \vec{E} \cdot d\vec{S}$ est multipliée par la même constante, par conséquent le rapport :

$$\frac{V_A - V_B}{i}$$ est indépendant de cette constante, c'est une "constante" (il dépend quand même de divers paramètres telle la température) appelée **résistance électrique** et notée R.

$$R = \frac{V_A - V_B}{i} = \frac{\int_A^B \vec{E}.d\vec{l}}{\sigma \iint_S \vec{E} \cdot d\vec{S}}$$

Cette formule permet de calculer la résistance de diverses géométries de matériaux (filiforme, cylindrique, sphérique, ...).